

1 DYNAMIC ANALYSIS

1.1 Overview

The dynamic analysis option permits two-dimensional, plane-strain, plane-stress or axisymmetric, fully dynamic analysis with *FLAC*. The calculation is based on the explicit finite difference scheme (as discussed in [Section 1.1.2](#) in **Theory and Background**) to solve the full equations of motion, using lumped gridpoint masses derived from the real density of surrounding zones (rather than fictitious masses used for static solution). This formulation can be coupled to the structural element model, thus permitting analysis of soil-structure interaction brought about by ground shaking. The dynamic feature can also be coupled to the groundwater flow model. This allows, for example, analyses involving time-dependent pore pressure change associated with liquefaction. (See [Section 1.4.4](#).) The dynamic model can likewise be coupled to the optional thermal model in order to calculate the combined effect of thermal and dynamic loading. The dynamic option expands *FLAC*'s analysis capability to a wide range of dynamic problems in disciplines such as earthquake engineering, seismology and mine rockbursts.

The fully nonlinear analysis method used by *FLAC* contrasts with the more commonly accepted “equivalent-linear” method used in earthquake engineering. [Section 1.2](#) compares the two methods and provides a review of recent applications of the fully nonlinear method. Background information on the dynamic formulation of the fully nonlinear method implemented in *FLAC* is also provided. (See [Section 1.3](#).)

This volume includes discussions on the various features and considerations associated with the dynamic option in *FLAC* (i.e., dynamic loading and boundary conditions, wave transmission and mechanical damping). These features are described separately in [Section 1.4](#).

The user is strongly encouraged to become familiar with the operation of *FLAC* for simple mechanical, static problems before attempting to solve problems involving dynamic loading. Dynamic analysis is often very complicated and requires a considerable amount of insight to interpret correctly. A recommended procedure for conducting dynamic numerical analysis with *FLAC* is provided in [Section 1.5](#).

An example application of a seismic analysis using the fully nonlinear method is given in [Section 1.6](#). This example illustrates the recommended procedure for dynamic analysis and covers several of the features and considerations described in [Section 1.4](#).

Validation problems illustrating the accuracy of the dynamic model are provided in [Sections 1.7*](#).

* The data files in this volume are stored in the directory “ITASCA\FLAC600\Dynamic” with the extension “.DAT.” A project file is also provided for each example. In order to run an example and compare the results to plots in this volume, open a project file in the *GIIC* by clicking on the **FILE/OPEN PROJECT** menu item and selecting the project file name (with extension “.PRJ”). Click on the *Project Options* icon at the top of the *Project Tree Record*, select *Rebuild unsaved states*, and the example data file will be run and plots created.

1.2 Relation to Equivalent-Linear Methods

The “equivalent-linear” method is common in earthquake engineering for modeling wave transmission in layered sites and dynamic soil-structure interaction. Since this method is widely used, and the fully nonlinear method embodied in *FLAC* is not, it is worth pointing out some of the differences between the two methods.

In the equivalent-linear method (Seed and Idriss 1969), a linear analysis is performed, with some initial values assumed for damping ratio and shear modulus in the various regions of the model. The maximum cyclic shear strain is recorded for each element and used to determine new values for damping and modulus, by reference to laboratory-derived curves that relate damping ratio and secant modulus to amplitude of cycling shear strain. Some empirical scaling factor is usually used when relating laboratory strains to model strains. The new values of damping ratio and shear modulus are then used in a new numerical analysis of the model. The whole process is repeated several times, until there is no further change in properties. At this point, it is said that “strain-compatible” values of damping and modulus have been found, and the simulation using these values is representative of the response of the real site.

In contrast, only one run is done with a fully nonlinear method (apart from parameter studies, which are done with both methods), because nonlinearity in the stress-strain law is followed directly by each element as the solution marches on in time. Provided that an appropriate nonlinear law is used, the dependence of damping and apparent modulus on strain level are automatically modeled.

Both methods have their strengths and weaknesses. The equivalent-linear method takes drastic liberties with physics but is user-friendly and accepts laboratory results from cyclic tests directly. The fully nonlinear method correctly represents the physics but demands more user involvement and needs a comprehensive stress-strain model in order to reproduce some of the more subtle dynamic phenomena. Important characteristics of the two methods are examined in [Sections 1.2.1](#) and [1.2.2](#).

FLAC contains an optional form of damping, *hysteretic damping*, that incorporates strain-dependent damping ratio and secant modulus functions, allowing direct comparisons between the equivalent-linear method and the fully nonlinear method. This form of damping is described in [Section 1.4.3.4](#).

There is a comparison between *FLAC* and *SHAKE* (a one-dimensional equivalent-linear program – Schnabel, Lysmer and Seed 1972) in [Section 1.7.2](#) for the case of a linear elastic, layered system, and in [Section 1.7.3](#) for the case of a nonlinear elastic, layered system.

1.2.1 Characteristics of the Equivalent-Linear Method

The equivalent-linear method is distinguished by the following characteristics:

1. The method uses linear properties for each element that remain constant throughout the history of shaking, and are estimated from the mean level of dynamic motion. During quiet periods in the excitation history, elements will be overdamped and too soft; during strong shaking, elements will be underdamped and too stiff. However, there *is* a spatial variation in properties that corresponds to different levels of motion at different locations.

2. The interference and mixing phenomena that occur between different frequency components in a nonlinear material are missing from an equivalent-linear analysis.
3. The method does not directly provide information on irreversible displacements and the permanent changes that accompany liquefaction, because only oscillatory motion is modeled. These effects may be estimated empirically, however.
4. It is commonly accepted that, during plastic flow, the strain-increment tensor is related to some function of the stress tensor, giving rise to the “flow rule” in plasticity theory. However, elasticity theory (as used by the equivalent-linear method) relates the strain tensor (not *increments*) to the stress tensor. Plastic yielding, therefore, is modeled somewhat inappropriately.
5. The material constitutive model is built into the method: it consists of a stress-strain curve in the shape of an ellipse (see Cundall 1976). Although this pre-choice relieves the user of the need to make any decisions, the flexibility to substitute alternative shapes is removed. However, the effects of a different shape to the curve are *partially* allowed for by the iteration procedure used in the method. It should be pointed out that a frequency-independent hysteresis curve in the form of an ellipse is physically impossible, since the continuous change in slope prior to reversal implies preknowledge (and *rate* information is not available to the model because the model is defined as being rate-independent).
6. In the case where both shear and compressional waves are propagated through a site, the equivalent-linear method typically treats these motions independently. Therefore, no interaction is allowed between the two components of motion.
7. Equivalent linear methods cannot be formulated in terms of effective stresses to allow the generation and dissipation of pore pressures during and following earthquake shaking.

1.2.2 Characteristics of the Fully Nonlinear Method

The following characteristics of the fully nonlinear method should be compared to the corresponding points listed in [Section 1.2.1](#).

1. The method follows any prescribed nonlinear constitutive relation. If a hysteretic-type model is used and no extra damping is specified, then the damping and tangent moduli are appropriate to the level of excitation at each point in time and space, since these parameters are embodied in the constitutive model. If Rayleigh or local damping is used, the associated damping coefficients remain constant throughout shaking. Consult [Section 1.4.3](#) for more details on damping.

2. Using a nonlinear material law, interference and mixing of different frequency components occur naturally.
3. Irreversible displacements and other permanent changes are modeled automatically.
4. A proper plasticity formulation is used in all of the built-in models whereby plastic strain increments are related to stresses.
5. The effects of using different constitutive models may be easily studied.
6. Both shear and compressional waves are propagated together in a single simulation, and the material responds to the combined effect of both components. For strong motion, the coupling effect can be very important. For example, normal stress may be reduced dynamically, thus causing the shearing strength to be reduced in a frictional material.
7. The formulation for the nonlinear method can be written in terms of effective stresses. Consequently, the generation and dissipation of pore pressures during and following shaking can be modeled.

Although the method follows any stress-strain relation in a realistic way, it turns out that the results are quite sensitive to seemingly small details in the assumed constitutive model (see Cundall 1976, and Dames and Moore and SAI 1978). The various nonlinear models built into *FLAC* are intended primarily for use in quasi-static loading, or in dynamic situations where the response is mainly monotonic (e.g., extensive plastic flow caused by seismic excitation). A good model for dynamic soil/structure interaction would capture the hysteresis curves and energy-absorbing characteristics of real soil. In particular, energy should be absorbed from each component of a complex wave form composed of many component frequencies. (In many models, high frequencies remain undamped in the presence of a low frequency.) It is possible to add additional damping into the existing *FLAC* constitutive models in order to simulate the inelastic cyclic behavior. This procedure is described in [Section 1.4.3.11](#).

A comprehensive model for dynamic soil behavior may not yet exist. A review of current models is provided in [Section 1.4.4.3](#). Also, the user is free to experiment with candidate models, either using *FISH* to incorporate the new model into *FLAC* (see [Section 2.8](#) in the ***FISH* volume**), or writing a model in C++ and loading as a DLL (dynamic link library) file. (See [Section 3](#) in **Theory and Background**.)

It is possible to simulate cyclic laboratory tests on the new model, and derive modulus and damping curves that may be compared with those from a real target material. The model parameters may then be adjusted until the two sets of curves match. This approach is discussed in [Section 1.4.3.4](#). Even standard elastic/plastic models (e.g., Mohr-Coulomb) can produce such curves. An example is shown in [Section 1.4.3.11](#).

1.2.3 Applications of the Fully Nonlinear Method in Dynamic Analysis

The standard practice for dynamic analysis of earth structures, and especially analyses dealing with liquefaction, is based primarily upon the equivalent-linear method. The nonlinear numerical method has not been applied as often in practical design. However, as more emphasis is placed on making a reliable prediction of permanent deformations and liquefaction-induced damage of earth structures, practical applications with nonlinear numerical codes have increased. Byrne et al. (2006) provide an overview of the different methods used for liquefaction assessment, and discuss the benefit of the nonlinear numerical method over the equivalent-linear method for different practical applications.

There are several publications describing applications of nonlinear numerical models for analysis and design of earth structures subjected to seismic loading.* Many of the publications describing nonlinear numerical models pertain to back-analyses of geotechnical case histories which recorded large permanent ground deformations and failures of earth dams. These studies revisit analyses previously performed with equivalent-linear models. The response of the Upper and Lower San Fernando dams to the 1971 San Fernando earthquake is one of the most commonly cited case histories. See Beaty and Byrne (2001) for a review of the observed response of both dams, and an assessment of the key parameters affecting the response. An important observation from this case history is that although the characteristics of the dams were similar, the earthquake-induced responses were quite different. While the Upper San Fernando dam experienced large lateral displacements of approximately 2 meters, a flow slide occurred at the upstream face and crest of the Lower San Fernando dam some 20 to 30 seconds after the earthquake, and nearly resulted in a catastrophic failure. Beaty and Byrne (2000) describe nonlinear numerical analyses of both dams using *FLAC*, incorporating a liquefaction constitutive model based upon a total stress procedure. The analyses directly consider the triggering of liquefaction and post-liquefaction response of the dam material. Beaty and Byrne (2000) conclude that the total stress approach is a logical extension of the equivalent-linear method because it incorporates both liquefaction-triggering and residual strength charts in the approach. The approach calculates progressive liquefaction-induced ground deformations that compare reasonably well with observed response, especially for the Upper San Fernando dam. However, excess pore pressures are not computed directly in the total stress approach, and Beaty and Byrne (2000) state that an effective stress analysis is warranted to investigate the response of the Lower San Fernando dam properly.

Dawson et al. (2001) present a back-analysis of the Lower San Fernando dam based upon an effective stress analysis with *FLAC* and a semi-empirical constitutive model. The constitutive model is described as a “decoupled” effective stress model, because it generates pore pressure directly in response to the number of shear stress cycles required to trigger liquefaction. Pore pressures are generated incrementally in relation to the cyclic strength of the material as defined by a cyclic strength curve. The same modeling approach is also applied to a back-analysis of the Upper San Fernando dam as described in Inel et al. (1993).

* It is interesting to note that the proceedings of the “Geotechnical Earthquake Engineering and Soil Dynamics IV” conference, held May 18-22, 2008 in Sacramento, California, contains more than 20 publications that describe nonlinear numerical analysis related to geotechnical earthquake engineering. (See Zeng et al. 2008.)

The delayed failure of the Upper San Fernando dam was also observed in the Mochikoshi tailings dam failure, which occurred in 1978 in Izu-Ohshim-Kinkai, Japan as a result of a magnitude M7 earthquake followed by a magnitude M5.8 aftershock. Two dams failed: Dam No. 1 failed during the main shaking, and Dam No. 2 failed approximately 24 hours after the main shock. Byrne and Seid-Karbasi (2004) suggest that the delayed failure of Dam No. 2 may be related to the low permeability silt layers contained within the sands of the tailings dam. These layers could impede vertical drainage of excess pore pressures and greatly reduce stability because they cause a water bubble to develop beneath the layers. Byrne and Seid-Karbasi performed coupled, nonlinear effective stress analyses to evaluate the excess pore pressures and deformations that develop during the earthquake and help assess the suggested failure mode.

Back-analyses of full-scale case histories are subject to many uncertainties with respect to material behavior and input motions, which make it difficult to verify nonlinear numerical analyses. Confidence in the accuracy of the nonlinear seismic deformation analysis is primarily subject to the uncertainty related to the understanding of liquefaction. Mitchell (2008) lists four difficulties that contribute to this uncertainty:

Difficulties in the constitutive modeling of liquefiable soils, in estimating the extent of liquefaction, in determining the time at which liquefaction is triggered during shaking and in estimating the post-liquefaction residual strength...

Centrifuge model tests are commonly used to attempt to address these difficulties, and permit verification of nonlinear numerical models. The VELACS (Verification of Liquefaction Analysis by Centrifuge Studies) project (Arulmoli et al. 1992) is one example that has provided experimental data for use in the verification of nonlinear liquefaction analysis. Comparisons are typically made in terms of excess pore pressure, acceleration and displacement time histories. Publications by Inel et al. (1993), Byrne et al. (2003), Andrianopoulos et al. (2006) and Kutter et al. (2008) describe different constitutive models that have been tested in *FLAC* by comparison to results from centrifuge tests.

Nonlinear numerical analyses are presently being applied to provide seismic vulnerability assessments and evaluate remedial measures for dam rehabilitation projects. The application of the decoupled effective stress model to assess liquefaction potential of the Pleasant Valley dam in California is described by Roth et al. (1991). Deformation analyses using this constitutive model helped determine a safe operating level for the reservoir, and supported the renewal of Pleasant Valley dam's operating license for the lower pool level. Seismic retrofitting of the Success Dam in Southern California is being guided by a combination of deformation analysis methods, ranging from simplified procedures based on the equivalent-linear method and limit equilibrium analyses, to decoupled and fully coupled effective-stress analyses with *FLAC*. Perlea et al. (2008) provide an overview of the analyses and remediation design. Salah-Mars et al. (2008) report the use of nonlinear deformation analyses with *FLAC* as part of a probabilistic seismic-hazard analysis to estimate the seismic hazard of the Sacramento-San Joaquin Delta levees in California.

In addition to seismic analyses for earthfill dams and levees, nonlinear numerical models have been used to assess the seismic stability of concrete gravity dams (e.g., Bureau et al. 2005), concrete water reservoirs (e.g., Roth et al. 2008), mechanically stabilized earth (MSE) walls (e.g., Lindquist 2008) and bridge foundations (e.g., Yegian et al. 2008). Several other applications of the fully nonlinear method can also be found in the proceedings edited by Zeng et al. (2008).

1.3 Dynamic Formulation

1.3.1 Dynamic Timestep

The finite difference formulation is similar to that described in [Section 1.3](#) in **Theory and Background** except that “real” masses are used at gridpoints rather than the fictitious masses used to improve convergence speed when a static solution is required. Each triangular subzone contributes one-third of its mass (computed from zone density and area) to each of the three associated gridpoints. The final gridpoint mass is then divided by two in the case of a quadrilateral zone that contains two overlays. In finite-element terminology, *FLAC* uses lumped masses and a diagonal mass matrix.

The calculation of critical timestep involves contributions of stiffness and mass at each degree of freedom, so that the effects of nonuniform grids, structural members, interfaces and fluid can be accommodated. For each triangular subzone, the following stiffness contribution (in units of force/distance) is made from each of the three gridpoints of the subzone:

$$k = \left(K + \frac{4}{3}G \right) \left\{ \frac{(L^{\max})^2}{6A_{\Delta}} \right\} T \quad (1.1)$$

where $L^{\max}$ is the maximum edge-length of the triangle, A_{Δ} is the area of the triangle and T is the out-of-plane dimension, equal to 1.0 for a plane-strain analysis. Thus for the full quadrilateral zone, the total contribution to each of the four gridpoints is the summation of those for the three triangles meeting at the gridpoint. For example, for the northwest gridpoint (assuming two overlays, with notation as illustrated in [Figure 1.3](#) in **Theory and Background**),

$$k_{\text{nw}} = \frac{(K + \frac{4}{3}G)}{6} \left\{ \frac{(L_a^{\max})^2}{A_a} + \frac{(L_c^{\max})^2}{A_c} + \frac{(L_d^{\max})^2}{A_d} \right\} T \quad (1.2)$$

where A_n is the area of triangle n , and $L_n^{\max}$ is the maximum edge-length of triangle n . For a complete *rectangular* zone, comprising four triangular subzones, the stiffness term reduces to

$$k_z = \left(K + \frac{4}{3}G \right) \frac{L_d^2}{A_z} T \quad (1.3)$$

where A_z is the area of the rectangular zone, and L_d the length of its diagonal. Note that [Eq. \(1.3\)](#) only applies in the specific case of a rectangular full-zone, and is provided for interest only; the general form of the stiffness contribution is given by expressions similar to [Eq. \(1.2\)](#).

Masses are also accumulated at zone gridpoints from each triangular subzone. As an example, for the northwest gridpoint (assuming two overlays),

$$M_{nw} = \frac{m_a + m_c + m_d}{6} \quad (1.4)$$

where m_a , m_c and m_d are the masses of triangles a, c and d, respectively. For the case of a rectangular full-zone (containing four triangular subzones), the mass contributed to each gridpoint is

$$M_{gp} = m_z/4 \quad (1.5)$$

where m_z is the mass of each triangle.

The stiffness and mass contributions from all zones surrounding each gridpoint are made, according to equations of the form [Eqs. \(1.2\)](#) and [\(1.4\)](#), and summed, giving a total stiffness term of k and total mass term of M , respectively. The critical timestep is then calculated as the minimum (over all gridpoints) of the following expression, which is the critical timestep for a single mass-spring system:

$$\Delta t_{crit} = 2\sqrt{\frac{M}{k}} \quad (1.6)$$

For the case of a *rectangular* zone, we can substitute stiffness and mass values from [Eqs. \(1.3\)](#) and [\(1.5\)](#):

$$\Delta t_{crit} = 2\sqrt{\frac{m_z A_z}{4(K + \frac{4}{3}G)L_d^2 T}} \quad (1.7)$$

Substituting $m_z = A_z \rho T$,

$$\Delta t_{crit} = \frac{A_z}{L_d} \sqrt{\frac{\rho}{K + \frac{4}{3}G}} = \frac{A_z}{L_d C_p} \quad (1.8)$$

where C_p is the speed of longitudinal waves. This expression is identical to that given in [Section 1.3.5](#) in **Theory and Background**. However, the more general form (based on [Eq. \(1.6\)](#)) is used in deriving the dynamic timestep, Δt_d , using a safety factor of 0.5 (to allow for the fact that the calculation of timestep is an estimate only). Thus,

$$\Delta t_d = \min \left\{ \sqrt{\frac{\Sigma M}{\Sigma k}} \right\} \cdot \frac{1}{2} \quad (1.9)$$

where the **min()** function is taken over all gridpoints and structural degrees of freedom, and Σ is a summation over all contributions to the gridpoint or structural degree-of-freedom. For a simple grid consisting of only rectangular zones, the computed timestep may be verified using Eq. (1.8), noting that $\Delta t_d = \Delta t_{crit}/2$. However, a more complicated model will contain unequal zones, different materials connected to common gridpoints, structural elements, interfaces and the added stiffness of coupled fluid. Each of these objects or conditions will contribute to the summations of Eq. (1.9), so that the final timestep will be a combined function of all items. Note that stiff or small zones may control the timestep chosen by *FLAC*, due to the **min()** function and the division by stiffness. The above derivation is for plane strain; related expressions are obtained for axisymmetric analysis, accounting for the effects of the varying “out-of-plane” thickness on masses and stiffnesses. For zones containing only one overlay, the contribution from two subzones (instead of four) is summed as above, but a divisor of 3 instead of 6 is used in Eqs. (1.1), (1.2) and (1.4).

If stiffness-proportional damping is used (see Section 1.4.3.1), the timestep must be reduced for stability. Belytschko (1983) provides a formula for critical timestep, Δt_β , that includes the effect of stiffness-proportional damping:

$$\Delta t_\beta = \left\{ \frac{2}{\omega_{\max}} \right\} (\sqrt{1 + \lambda^2} - \lambda) \quad (1.10)$$

where $\omega_{\max}$ is the highest eigenfrequency of the system, and λ is the fraction of critical damping at this frequency. Both $\omega_{\max}$ and λ are estimated in *FLAC*, since an eigenvalue solution is not performed. The estimates are

$$\omega_{\max} = \frac{2}{\Delta t_d} \quad (1.11)$$

$$\lambda = \frac{0.4 \beta}{\Delta t_d} \quad (1.12)$$

given

$$\beta = \xi_{\min} / \omega_{\min} \quad (1.13)$$

where $\xi_{\min}$ and $\omega_{\min}$ are the damping fraction and angular frequency specified for Rayleigh damping (see Section 1.4.3.1). The resulting value of Δt_β is used as the dynamic timestep if stiffness-proportional damping is in operation.

1.3.2 Dynamic Multi-stepping

The maximum stable timestep for dynamic analysis is determined by the largest material stiffness and smallest zone in the model (see Eq. (1.1)). Often, the stiffness and zone size can vary widely in a model (e.g., in the case of a finely zoned concrete structure located in a soft soil). A few zones will then determine the critical timestep for a dynamic analysis, even though the major portion of the model can be run at a significantly larger timestep.

A procedure known as *dynamic multi-stepping* is available in *FLAC* to reduce the computation time required for a dynamic calculation. In this procedure, zones and gridpoints in a model are ordered into classes of similar maximum timesteps. Each class is then run at its timestep, and information is transferred between zones at the appropriate time.

Dynamic multi-stepping uses a local timestep for each individual gridpoint and zone. At the start of an analysis, the grid is scanned and the local stable timestep for each gridpoint, Δt_{gp} , is determined and stored. The value of Δt_{gp} depends on the size, stiffness and mass of the neighboring subzones (as shown in Eq. (1.1)), attached structural elements and interfaces. The global timestep, Δt_G , is determined as the minimum of all Δt_{gp} , as in the standard formulation.

Integer multipliers, M_{gp} , to the global timestep are then determined for each gridpoint according to the algorithm illustrated by the flow chart in Figure 1.1. This algorithm ensures that multipliers are powers of 2. In the current implementation, M_{gp} is set to 1 for nodes that are assigned a null material model, connected to structural elements, attached to other gridpoints, or part of a quiet boundary. All zones are then scanned, and an integer multiplier, M_z , is calculated for each zone as the minimum of the multipliers for the four surrounding gridpoints.

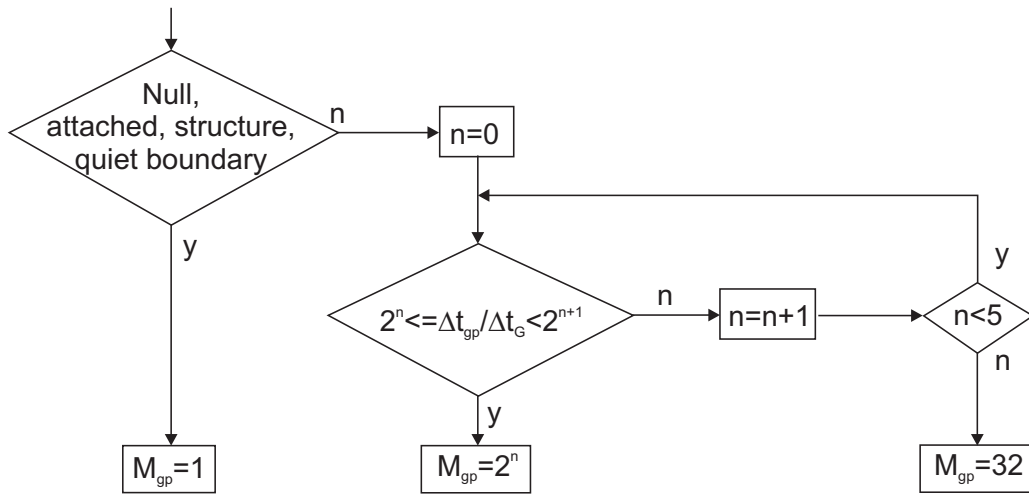


Figure 1.1 Flow chart for determination of gridpoint multiplier, M_{gp}

Calculations for a zone (i.e., derivation of new stresses from surrounding gridpoint velocities and accumulation of gridpoint force sums from stress components) are only performed every M_z timesteps. In all expressions involving a timestep, the global timestep is replaced by $\Delta t_G M_z$.

Calculations for a gridpoint (i.e., derivation of new velocities and displacements from gridpoint force sums) are only performed every M_{gp} timesteps; otherwise, the force sums are reset to zero, which is normally done after every motion calculation. In all expressions involving a timestep, the global timestep is replaced by $\Delta t_G M_{gp}$.

The effect of the prescriptions described above is to skip calculation of selected gridpoints and zones, thereby speeding up the overall calculation. The use of gridpoint and zone multipliers (M_{gp} and M_z , respectively) ensures the following characteristics:

1. The force sum at each gridpoint is composed of component forces from each connected zone that exist at the same point in time. The simultaneous nature of the component forces is guaranteed by the fact that multipliers are powers of two. Arbitrary integral multipliers would not have this characteristic.
2. Velocities seen by a zone (at the four surrounding gridpoints) are not updated between zone updates. This is guaranteed by the fact that the zone multiplier is the minimum of the surrounding gridpoint multipliers. Since stress increments are derived from strain and displacement increments, the displacement contribution of a gridpoint is felt by a zone at each update, even though the gridpoint is updated less frequently than the zone. In essence, the total displacement increment of the gridpoint is divided into M_{gp}/M_z equal parts.

This scheme is accurate for dynamic simulations that represent waves with frequencies well below the natural frequencies of individual elements. The condition is usually guaranteed by the wavelength criterion described by [Eq. \(1.29\)](#). For higher frequencies, it is believed that inaccuracies arise from the fact that velocities used in computing strain increments are not defined (in time) at the center of the time interval, Δt , for the case of a zone multiplier being unequal to the gridpoint multiplier. This represents a departure from the second-order accuracy of the central difference scheme used in *FLAC*. However, it is always possible to assess the accuracy of the scheme for any part of the simulation by running a short period of the simulation with and without dynamic multi-stepping. The results may be directly compared.

Dynamic multi-stepping is invoked with the command **SET multi on**. The effect of dynamic multi-stepping on calculation speed is model-dependent (i.e., the more zones that have a high multiplier, the greater the increase in speed). Although multi-stepping is not implemented within structural elements, substantial savings can still be obtained by using multi-stepping for a system in which stiff structures are connected to soft continuum elements. In a typical system, only a small proportion of computer time is spent in structural calculations, so there is only a small penalty for performing these calculations at every timestep, compared to the savings obtained by performing infrequent grid calculations.

[Example 1.1](#) illustrates the effect of dynamic multi-stepping. The model consists of a “wall” of material with a modulus 20 times greater than the surrounding soil material. A shear wave is applied at the base of the model for a 1 second time period. With **SET multi on**, the wall zones have a multiplier of 1 and the soil zones have a multiplier of 4. (The gridpoint and zone multipliers are stored in separate *FISH* extra variables for monitoring.) The calculation is 2.25 times faster with dynamic multi-stepping. Velocity histories monitored at the base of the model and top of the wall

are identical with and without multi-stepping. [Figure 1.2](#) plots the histories for the multi-stepping run.

There is no direct printout of the multi-stepping multipliers, but *FISH* intrinsic **zmsmul** and **gmsmul** (see [Section 2.5.3](#) in the *FISH* volume) may be used to determine the multipliers used during cycling.

Dynamic multi-stepping can be used with structural elements. The grid timestep multipliers are set to 1 for all gridpoints connected to structural nodes. Multipliers are not used in structures; their natural timestep is used. This timestep may be small, but if the grid not attached to the structure does have a large natural timestep, these gridpoints will have large multipliers, thus saving execution time.

A user-defined integer multiplier can be specified with the optional **max** keyword.

For additional information and example applications of dynamic multi-stepping, see Unterberger, Cundall and Zettler (1997). The application of dynamic multi-stepping in numerical predictions of vibrations caused by rail traffic in tunnels is presented in Unterberger, Hochgatterer and Poisel (1996) and Daller, Unterberger and Hochgatterer (1996).

Example 1.1 *Shear wave applied to a stiff wall in a soft soil – with dynamic multi-stepping*

```

;--- Test multistepping option ---
; ... model has a stiff retaining wall
conf dyn ext=5
grid 40 20
mod elas
prop dens 2000 bulk 2e8 shea 1e8
model null i=1,10 j=11,20
prop bulk 4e9 shear 2e9 i=11,12 j=11,20 ; 20 times stiffness
fix y i=1
fix y i=41
def setup
  freq = 1.0
  omega = 2.0 * pi * freq
end
setup
def wave
  wave = sin(omega*dytime)
end
apply xvel=1 hist=wave j=1
apply yvel=0          j=1
hist xvel i 11 j 21
hist yvel i 11 j 21
hist xvel i 11 j 1
hist dytime
set ncw=50
set multi=on ; Comment out this line, and compare times & histories

```

```

def tim
  tim = 0.01 * (clock - old_time)
end
cyc 1
def qqq ; Save multipliers in ex_1 and ex_2 - for interest
  loop i (1,izones)
    loop j (1,jzones)
      ex_1(i,j) = zmsmul(i,j)
    endLoop
  endLoop
  loop i (1,igp)
    loop j (1,jgp)
      ex_2(i,j) = gmsmul(i,j)
    endLoop
  endLoop
  old_time = clock
end
qqq
solve dytime 1.0
; pri ex_1 zon ; (look at multipliers)
; pri ex_2
save dyn_ms.sav

```

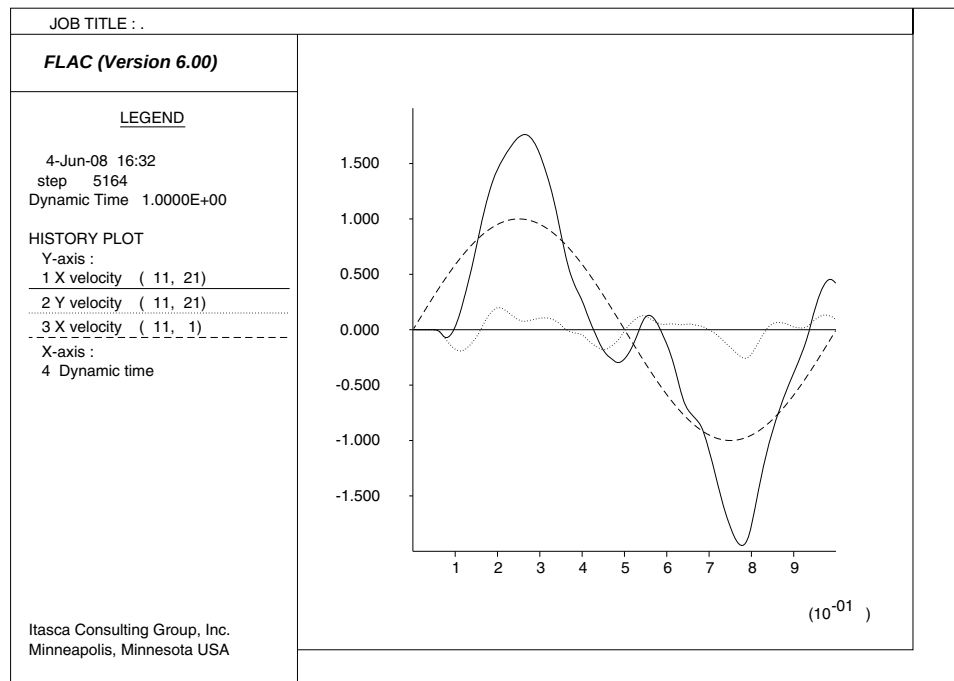


Figure 1.2 Velocities at model base ($i = 11, j = 1$), and top of wall ($i = 11, j = 21$)

1.4 Dynamic Modeling Considerations

There are three aspects that the user should consider when preparing a *FLAC* model for a dynamic analysis: (1) dynamic loading and boundary conditions; (2) mechanical damping; and (3) wave transmission through the model. This section provides guidance on addressing each aspect when preparing a *FLAC* data file for dynamic analysis. [Sections 1.5](#) and [1.6](#) illustrate the use of most of the features discussed here.

1.4.1 Dynamic Loading and Boundary Conditions

FLAC models a region of material subjected to external and/or internal dynamic loading by applying a dynamic input boundary condition at either the model boundary or at internal gridpoints. Wave reflections at model boundaries are minimized by specifying either quiet (viscous), free-field or three-dimensional radiation-damping boundary conditions. The types of dynamic loading and boundary conditions are shown schematically in [Figure 1.3](#); each condition is discussed in the following sections.

1.4.1.1 Application of Dynamic Input

In *FLAC*, the dynamic input can be applied in one of the following ways:

- (a) an acceleration history;
- (b) a velocity history;
- (c) a stress (or pressure) history; or
- (d) a force history.

Dynamic input is usually applied to the model boundaries with the **APPLY** command. Accelerations, velocities and forces can also be applied to interior gridpoints by using the **INTERIOR** command. Note that the free-field boundary, shown in [Figure 1.3](#), is not required if the only dynamic source is within the model (see [Section 1.4.1.4](#)).

The history function for the input is treated as a multiplier on the value specified with the **APPLY** or **INTERIOR** command. The history multiplier is assigned with the **hist** keyword and can be in one of three forms:

- (1) a table defined by the **TABLE** command;
- (2) a history defined by the **HISTORY** command; or
- (3) a *FISH* function.

With **TABLE** input, the multiplier values and corresponding time values are entered as individual pairs of numbers in the specified table; the first number of each pair is assumed to be a value of dynamic time. The time intervals between successive table entries need not be the same for all entries. Note that the use of tables to provide dynamic multipliers can be quite inefficient compared

to the other two options. When using the **HISTORY** command to derive the history multiplier, the values stored in the specified history are assumed to be spaced at constant intervals of dynamic time. The interval is contained in the data file that is input with the **HISTORY read** command and associated with a particular history number. If a *FISH* function is used to provide the multiplier, the function must access dynamic time within the function, using the *FLAC* scalar variable **dytime**, and compute a multiplier value that corresponds to this time. [Example 1.9](#) provides an example of dynamic loading derived from a *FISH* function.

Dynamic input can be applied either in the x - or y -direction corresponding to the xy -axes for the model, or in the normal and shear directions to the model boundary. Certain boundary conditions cannot be mixed at the same boundary segment (see [Table 1.3](#) in the **Command Reference** for a summary of the compatibility of boundary conditions).

One restriction when applying velocity or acceleration input to model boundaries is that these boundary conditions cannot be applied along the same boundary as a quiet (viscous) boundary condition (compare [Figure 1.3\(a\)](#) to [Figure 1.3\(b\)](#)), because the effect of the quiet boundary would be nullified. See [Section 1.4.1.3](#) for a description of quiet boundaries. To input seismic motion at a quiet boundary, a stress boundary condition is used (i.e., a velocity record is transformed into a stress record and applied to a quiet boundary).

A velocity wave may be converted to a stress wave using the formula

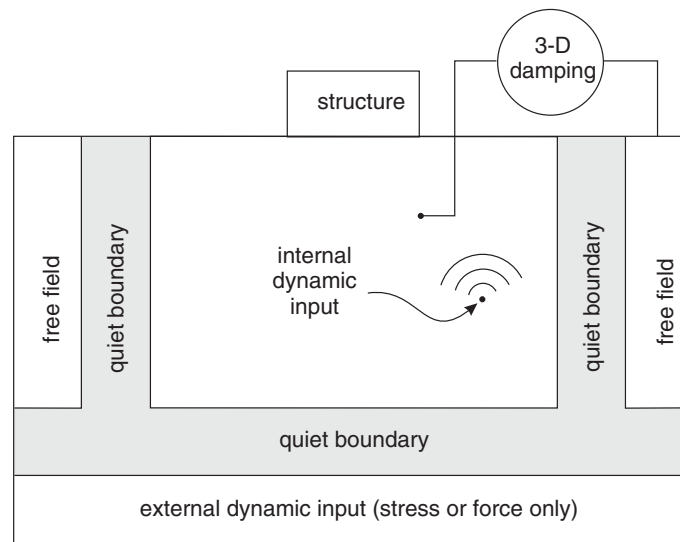
$$\sigma_n = 2(\rho C_p) v_n \quad (1.14)$$

or

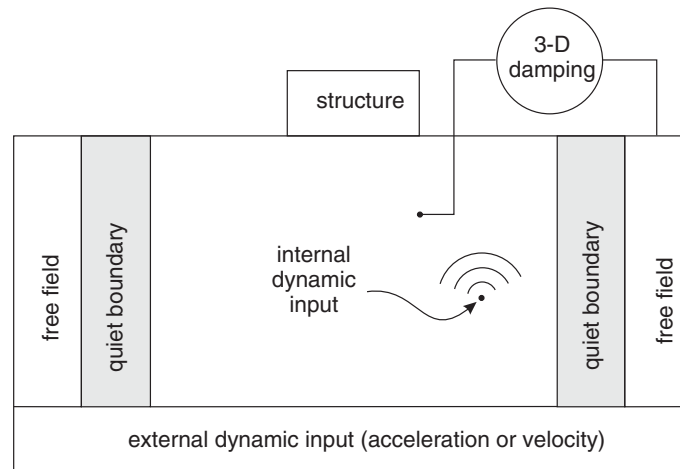
$$\sigma_s = 2(\rho C_s) v_s \quad (1.15)$$

where:

- σ_n = applied normal stress;
- σ_s = applied shear stress;
- ρ = mass density;
- C_p = speed of p -wave propagation through medium;
- C_s = speed of s -wave propagation through medium;
- v_n = input normal particle velocity; and
- v_s = input shear particle velocity.



(a) Flexible base



(a) Rigid base

Figure 1.3 Types of dynamic loading boundary conditions available in FLAC

C_p is given by

$$C_p = \sqrt{\frac{K + 4G/3}{\rho}} \quad (1.16)$$

and C_s is given by

$$C_s = \sqrt{G/\rho} \quad (1.17)$$

The formulae assume plane-wave conditions. The factor of two in Eqs. (1.14) and (1.15) accounts for the fact that the applied stress must be double that observed in an infinite medium, since half the input energy is absorbed by the viscous boundary. The formulation is similar to that of Joyner and Chen (1975). To illustrate wave input at a quiet boundary, consider Example 1.2, in which a pulse is applied as a stress history to the bottom of a vertical, 50 m high column. The bottom of the column is declared “quiet” in both horizontal directions, and the top is free. The properties are chosen such that the shear wave speed is 100 m/sec, and the product, ρC_s , is 10^5 . The amplitude of the stress pulse is set, therefore, to 2×10^5 , according to Eq. (1.14), in order to generate a velocity amplitude of 1 m/sec in the column. Figure 1.4 shows time histories of x -velocity at the base, middle and top of the column; the amplitude of the outgoing wave is seen to be 1 m/sec, as expected. The first three pulses in Figure 1.4 correspond, in order, to the outgoing waves at the base, middle and top. The final two pulses correspond to waves reflected from the free surface, measured at the middle and base, respectively. The velocity-doubling effect of a free surface, as well as the lack of waves after a time of about 1.3 seconds, can be seen, which confirms that the quiet base is working correctly. The doubling effect associated with a free surface is described in texts on elastodynamics (e.g., Graff 1991).

Example 1.2 *Shear wave propagation in a vertical column*

```

config dyn
grid 1,50
model elas
prop dens 1000 shear 1e7 bulk 2e7
def wave
  if dytime > 1.0 / freq
    wave = 0.0
  else
    wave = 0.5 * (1.0 - cos(2.0*pi*freq * dytime))
  endif
end
set freq=4.0
fix y
apply xquiet j=1
apply sxy -2e5 hist wave j=1

```

```

hist xvel i 1 j 1
hist xvel i 1 j 26
hist xvel i 1 j 51
hist dytime
solve dytime 1.8
save dyn_02.sav

```

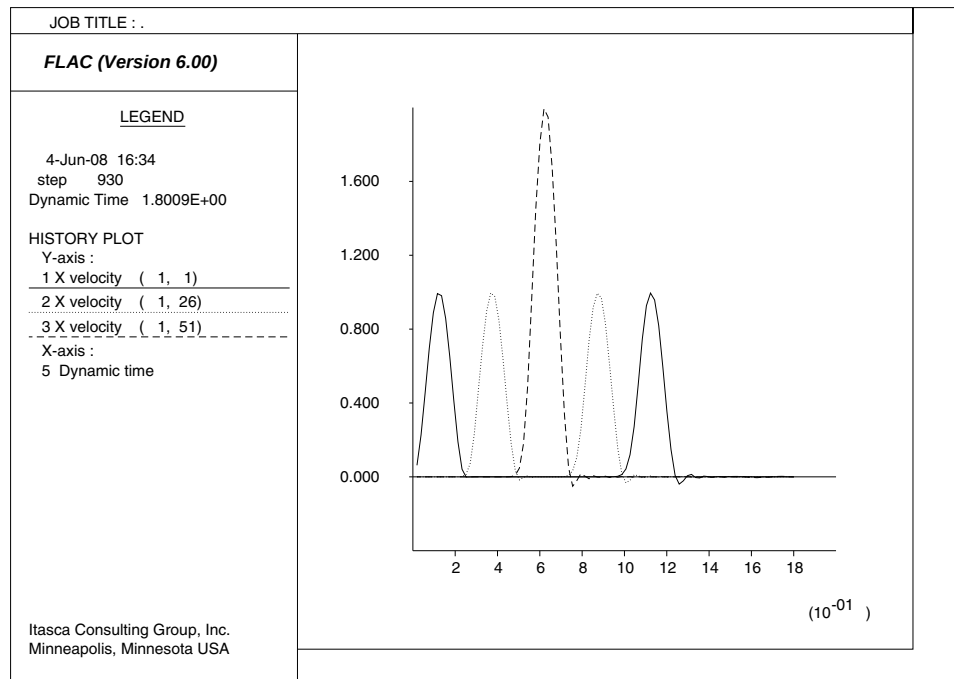


Figure 1.4 Primary and reflected waves in a bar: stress input through a quiet boundary

1.4.1.2 Baseline Correction

If a “raw” acceleration or velocity record from a site is used as a time history, the *FLAC* model may exhibit continuing velocity or residual displacements after the motion has finished. This arises from the fact that the integral of the complete time history may not be zero. For example, the idealized velocity wave form in [Figure 1.5\(a\)](#) may produce the displacement wave form in [Figure 1.5\(b\)](#) when integrated. The process of “baseline correction” should be performed, although the physics of the *FLAC* simulation usually will not be affected if it is not done. It is possible to determine a low frequency wave (for example, [Figure 1.5\(c\)](#)) which, when added to the original history, produces a final displacement which is zero ([Figure 1.5\(d\)](#)). The low frequency wave in [Figure 1.5\(c\)](#) can be a polynomial or periodic function, with free parameters that are adjusted to give the desired results. An example baseline correction function of this type is given as a *FISH* function in [Example 1.25](#) (see [Section 1.6.1.4](#)).

Baseline correction usually applies only to complex wave forms derived, for example, from field measurements. When using a simple, synthetic wave form, it is easy to arrange the process of generating the synthetic wave form to ensure that the final displacement is zero. Normally, in seismic analysis, the input wave is an acceleration record. A baseline-correction procedure can be used to force both the final velocity and displacement to be zero. Earthquake engineering texts should be consulted for standard baseline correction procedures.

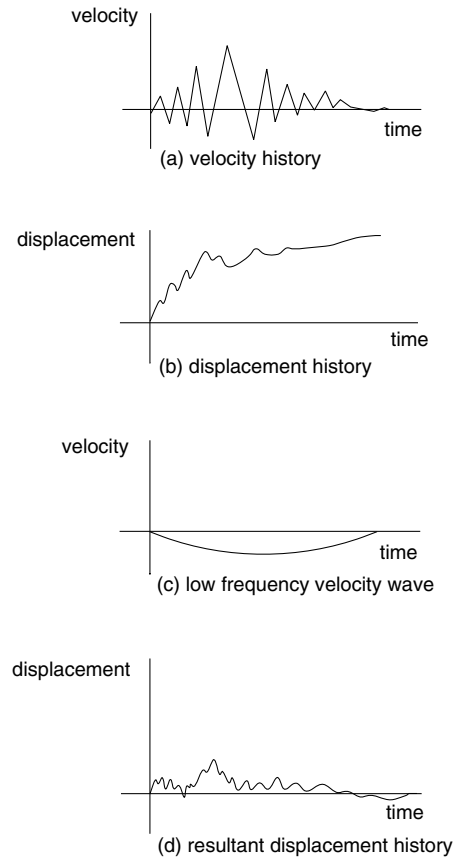


Figure 1.5 *The baseline correction process*

An alternative to baseline correction of the input record is to apply a displacement shift at the end of the calculation if there is a residual displacement of the entire model. This can be done by applying a fixed velocity to the mesh to reduce the residual displacement to zero. This action will not affect the mechanics of the deformation of the model. Computer codes to perform baseline corrections are available from several Internet site (e.g., <http://nsmp.wr.usgs.gov/processing.html>).

1.4.1.3 Quiet Boundaries

The modeling of geomechanics problems involves media which, at the scale of the analysis, are better represented as unbounded. Deep underground excavations are normally assumed to be surrounded by an infinite medium, while surface and near-surface structures are assumed to lie on a half-space. Numerical methods relying on the discretization of a finite region of space require that appropriate conditions be enforced at the artificial numerical boundaries. In static analyses, fixed or elastic boundaries (e.g., represented by boundary-element techniques) can be realistically placed at some distance from the region of interest. In dynamic problems, however, such boundary conditions cause the reflection of outward propagating waves back into the model and do not allow the necessary energy radiation. The use of a larger model can minimize the problem, since material damping will absorb most of the energy in the waves reflected from distant boundaries. However, this solution leads to a large computational burden. The alternative is to use quiet (or absorbing) boundaries. Several formulations have been proposed. The viscous boundary developed by Lysmer and Kuhlemeyer (1969) is used in *FLAC*. It is based on the use of independent dashpots in the normal and shear directions at the model boundaries. The method is almost completely effective at absorbing body waves approaching the boundary at angles of incidence greater than 30°. For lower angles of incidence, or for surface waves, there is still energy absorption, but it is not perfect. However, the scheme has the advantage that it operates in the time domain. Its effectiveness has been demonstrated in both finite-element and finite-difference models (Kunar et al. 1977). A variation of the technique proposed by White et al. (1977) is also widely used.

More efficient energy absorption (particularly in the case of Rayleigh waves) requires the use of frequency-dependent elements, which can only be used in frequency-domain analyses (e.g., Lysmer and Waas 1972). These are usually termed “consistent boundaries,” and involve the calculation of dynamic stiffness matrices coupling all of the boundary degrees-of-freedom. Boundary element methods may be used to derive these matrices (e.g., Wolf 1985). A comparative study of the performance of different types of elementary, viscous and consistent boundaries was documented by Roesset and Ettouney (1977).

The quiet-boundary scheme proposed by Lysmer and Kuhlemeyer (1969) involves dashpots attached independently to the boundary in the normal and shear directions. The dashpots provide viscous normal and shear tractions given by

$$t_n = -\rho C_p v_n \quad (1.18)$$

$$t_s = -\rho C_s v_s$$

where: v_n and v_s are the normal and shear components of the velocity at the boundary;

ρ is the mass density; and

C_p and C_s are the p - and s -wave velocities.

These viscous terms can be introduced directly into the equations of motion of the gridpoints lying on the boundary. A different approach, however, was implemented in *FLAC*, whereby the tractions

t_n and t_s are calculated and applied at every timestep in the same way that boundary loads are applied. This is more convenient than the former approach, and tests have shown that the implementation is equally effective. The only potential problem concerns numerical stability, because the viscous forces are calculated from velocities lagging by half a timestep. In practical analyses to date, no reduction of timestep has been required by the use of the non-reflecting boundaries. Timestep restrictions demanded by small zones are usually more important.

Dynamic analysis starts from some in-situ condition. If a velocity boundary is used to provide the static stress state, this boundary condition can be replaced by a quiet boundary; the boundary reaction forces will be automatically calculated and maintained throughout the dynamic loading phase. Note that the boundaries must not be freed before applying the quiet boundary condition; otherwise, the reaction forces will be lost.

Care should be taken to avoid changes in static loading during the dynamic phase. For example, if a tunnel is excavated after quiet boundaries have been specified on the bottom boundary, the whole model will start to move upward. This is because the total gravity force no longer balances the total reaction force at the bottom that was calculated when the boundary was changed to a quiet one. If a *stress* boundary condition is applied for the static solution, a stress boundary condition of opposite sign must also be applied over the same boundary when the quiet boundary is applied for the dynamic phase. This will allow the correct reaction forces to be in place at the boundary for the dynamic calculation.

Quiet boundary conditions can be applied in the x - and y -directions, or along inclined boundaries, in the normal and shear directions, using the **APPLY** command with appropriate keywords (**xquiet**, **yquiet**, **nquiet** or **squiet**). When applying quiet boundary conditions in the normal and shear directions, **nquiet** and **squiet** should always be specified together. These conditions, individually, do not account for the coupling between x - and y -directions for inclined boundaries. When using the **APPLY** command to install a quiet boundary condition, it must be appreciated that the material properties used in Eq. (1.18) are obtained from the zones immediately adjacent to the boundary. Thus, appropriate material properties for boundary zones must be in place at the time the **APPLY** command is given, in order for the correct properties of the quiet boundary to be stored.

Quiet boundaries are best-suited when the dynamic source is within a grid. Quiet boundaries should not be used alongside boundaries of a grid when the dynamic source is applied as a boundary condition at the top or base, because the wave energy will “leak out” of the sides. In this situation, free-field boundaries (described below) should be applied to the sides.

1.4.1.4 Free-Field Boundaries

Numerical analysis of the seismic response of surface structures such as dams requires the discretization of a region of the material adjacent to the foundation. The seismic input is normally represented by plane waves propagating upward through the underlying material. The boundary conditions at the sides of the model must account for the free-field motion which would exist in the absence of the structure. In some cases, elementary lateral boundaries may be sufficient. For example, if only a shear wave were applied on the horizontal boundary AC, shown in Figure 1.6, it would be possible to fix the boundary along AB and CD in the vertical direction only (see the example in Section 1.7.4). These boundaries should be placed at distances sufficient to minimize wave

reflections and achieve free-field conditions. For soils with high material damping, this condition can be obtained with a relatively small distance (Seed et al. 1975). However, when the material damping is low, the required distance may lead to an impractical model. An alternative procedure is to “enforce” the free-field motion in such a way that boundaries retain their non-reflecting properties (i.e., outward waves originating from the structure are properly absorbed). This approach was used in the continuum finite-difference code NESSI (Cundall et al. 1980). A technique of this type was developed for *FLAC*, involving the execution of a one-dimensional free-field calculation in parallel with the main-grid analysis.

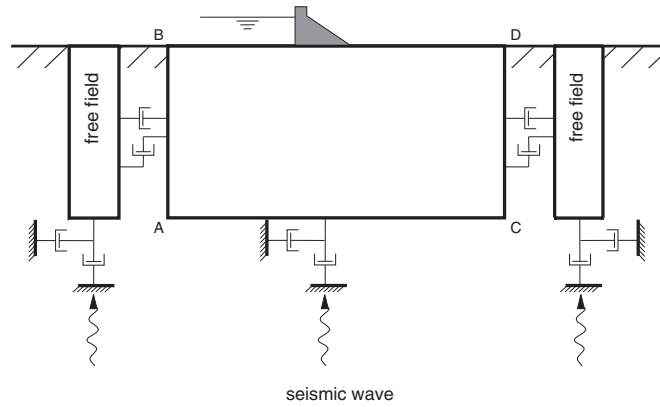


Figure 1.6 *Model for seismic analysis of surface structures and free-field mesh*

The lateral boundaries of the main grid are coupled to the free-field grid by viscous dashpots to simulate a quiet boundary (see Figure 1.6), and the unbalanced forces from the free-field grid are applied to the main-grid boundary. Both conditions are expressed in Eqs. (1.19) and (1.20), which apply to the left-hand boundary. Similar expressions may be written for the right-hand boundary:

$$F_x = -[\rho C_p(v_x^m - v_x^{\text{ff}}) - \sigma_{xx}^{\text{ff}}]\Delta S_y \quad (1.19)$$

$$F_y = -[\rho C_s(v_y^m - v_y^{\text{ff}}) - \sigma_{xy}^{\text{ff}}]\Delta S_y \quad (1.20)$$

where:

- ρ = density of material along vertical model boundary;
- C_p = p -wave speed at left-hand boundary;
- C_s = s -wave speed at left-hand boundary;
- ΔS_y = mean vertical zone size at boundary gridpoint;
- v_x^m = x -velocity of gridpoint in main grid at left boundary;
- v_y^m = y -velocity of gridpoint in main grid at left boundary;
- v_x^{ff} = x -velocity of gridpoint in left free field;
- v_y^{ff} = y -velocity of gridpoint in left free field;
- σ_{xx}^{ff} = mean horizontal free-field stress at gridpoint; and
- σ_{xy}^{ff} = mean free-field shear stress at gridpoint.

In this way, plane waves propagating upward suffer no distortion at the boundary because the free-field grid supplies conditions that are identical to those in an infinite model. If the main grid is uniform, and there is no surface structure, the lateral dashpots are not exercised because the free-field grid executes the same motion as the main grid. However, if the main-grid motion differs from that of the free field (due, say, to a surface structure that radiates secondary waves), then the dashpots act to absorb energy in a manner similar to the action of quiet boundaries.

The free-field model consists of a one-dimensional “column” of unit width, simulating the behavior of the extended medium. An explicit finite-difference method was selected for the model. The height of the free field equals the length of the lateral boundaries. It is discretized into n elements corresponding to the zones along the lateral boundaries of the *FLAC* mesh. Element masses are lumped at the $n + 1$ gridpoints. A linear variation of the displacement field is assumed within each element; the elements are, therefore, in a state of uniform strain (and stress).

The following conditions are required in order to apply the free-field boundary condition:

1. The lateral boundaries of the grid *must* be vertical and straight.
2. The free-field boundaries may be applied to the whole grid or to a sub-grid, starting at (1,1), with the left-hand boundary being $i = 1$. The right-hand boundary corresponds to the last-encountered non-null zone, scanning along $j = 1$ with increasing i numbers. Any other disconnected sub-grids are not considered when the free-field boundaries are created. Therefore, if sub-grids are used in a simulation that requires free-field boundaries to the main grid, this grid must be the “first” one (i.e., its left and bottom sides must be lines $i = 1$ and $j = 1$, respectively). The optional keyword **ilimits** forces the free field to be applied on the outer i limits of the grid (as specified in the **GRID** command). This keyword should be used if null zones exist on the $j = 1$ row of zones. It is advisable to perform **PLOT apply** to verify that the free field is applied to the correct boundary before starting a dynamic simulation.

3. The bottom zones ($j = 1$) at $i = 1$ and $i = imax$ must not be null.
4. The model should be in static equilibrium before the free-field boundary is applied.
5. The free-field condition must be applied *before* changing other boundary conditions for the dynamic stage of an analysis.
6. The free-field condition can only be applied for a plane-strain or plane-stress analysis. It is not applicable for axisymmetric geometry.
7. Both lateral boundaries of the grid must be included in the free field because the free field is automatically applied to both boundaries when the **APPLY ff** command is given.
8. The free field can be specified for a groundwater flow analysis (**CONFIG gw**). A one-dimensional fluid-flow model will also be created when **APPLY ff** is issued, and pore pressures will be calculated in the free field. Note that only vertical flow is modeled in the free field.
9. Interfaces and attach-lines do not get transferred to the free-field grid. Thus, an **INTERFACE** or **ATTACH** condition should not extend to the free-field boundary. The effect of an interface can be reproduced with a layer of zones having the same properties of the interface. Alternatively, a continuous grid can be wrapped around the grid containing the internal interfaces or attached lines.
10. Initialization of mechanical damping in the grid should be done *before* the **APPLY ff** command is given.
11. The use of 3D damping when the free field is derived from the sides of a sub-grid may not work correctly; 3D damping should only be used when the free field is applied to the whole grid.

The static equilibrium conditions prior to the dynamic analysis are transferred to the free field automatically when the command **APPLY ff** is invoked. All zone data (including model types and current state variables) in the first and last columns of model zones is copied to the free-field region. Free-field information can be viewed by specifying the **PRINT** command with the range **$imax + 1$** for the left-hand side free field, and **$imax + 2$** for the right-hand side free field, where **$imax$** is the highest gridpoint index in the i -direction. Note that stresses are referred to by the name of the first subzone (e.g., σ_{xx}^{ff} is printed with the command **PRINT asxx**, for the range corresponding to free-field zones). Free-field loads, applied velocities and quiet boundaries are updated automatically using the current values of the first and last columns of the grid.

Any model or nonlinear behavior, as well as fluid coupling and vertical flow, may exist in the free field. However, the free field performs a small-strain calculation, even if the main grid is executing in large-strain mode. In this case, the results will be approximately correct, provided the deformations near the free-field boundaries are relatively small (e.g., compared to grid dimensions).

The application of the free-field boundary is illustrated in [Example 1.3](#). A shear-stress wave is applied to the base of the model. [Figure 1.7](#) shows the resulting x -velocity at the top of the model at different locations in the free field and the main grid.

Example 1.3 Shear wave loading of a model with free-field boundaries

```
; --- Free-field test ---
config dyn
def wave
  wave = 0.5 * (1.0 - cos(2*pi*dytime/period))
end
set period 0.015
grid 16 10
mod elas
gen line 6 10 8 6
gen line 8 6 10 10
mod null reg 7 10
prop bulk 66667 shear 40000 den 0.0025
set grav 10
fix x i=1
fix x i=17
fix y j=1
set dyn off
hist unbal
hist ydis i 5 j 5
hist ydis i 5 j 11
solve
save ff0.sav
set dyn on
apply ff
apply xquiet j=1
apply yquiet j=1
apply sxy -1.0 hist wave j 1
set dytime 0
hist reset
hist dytime
hist xvel i 5 j 11
hist xvel i 18 j 11
hist xvel i 19 j 11
hist wave
solve dytime 0.02
ret
```

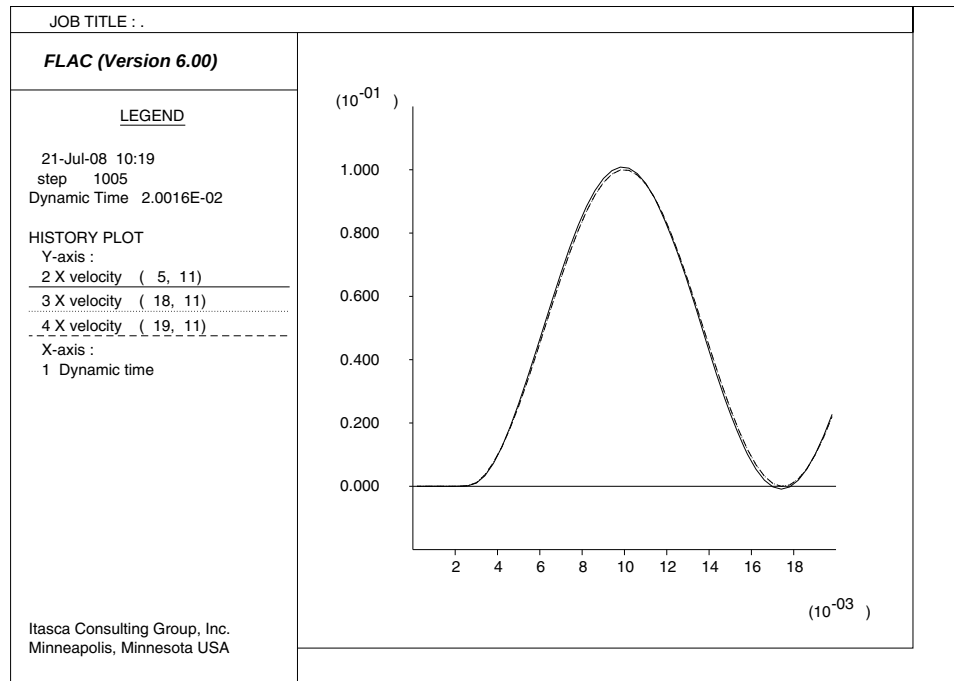


Figure 1.7 *x-velocity histories at top of model with free-field boundaries*

1.4.1.5 Three-Dimensional Radiation Damping

A vibrating structure located on the surface of the modeled region creates a disturbance both in the plane of analysis and in the out-of-plane direction. The energy radiated in-plane is reasonably absorbed by the quiet boundary condition. However, in a three-dimensional system, energy would be radiated in the out-of-plane direction. To represent this effect approximately, dashpots are connected from all gridpoints in the main grid to corresponding gridpoints in the free field (although the force is not applied to the free-field grid). This mechanism is termed three-dimensional radiation damping and is invoked by the **SET 3d_damp** command. The 3D damper acts on the difference between the actual particle velocity under the structure and the free-field velocity around the model region. The scheme is identical to that described by Lysmer et al. (1975). The dashpot constant, c , has the value

$$c = \frac{2 \rho C_s^{\text{ff}}}{W} \quad (1.21)$$

where: c = coefficient of 3D damping;

C_s^{ff} = free-field shear wave velocity; and

W = out-of-plane width of structure.

The free-field boundaries (i.e., **APPLY ff**) must be specified when using 3D damping. The dashpot can be connected to either the left-hand side or the right-hand side of the free field (see [Figure 1.3](#)).

1.4.1.6 Deconvolution and Selection of Dynamic Boundary Conditions

Design earthquake ground motions developed for seismic analyses are usually provided as outcrop motions, often rock outcrop motions.* However, for *FLAC* analyses, seismic input must be applied at the base of the model rather than at the ground surface as illustrated in Figure 1.8. The question then arises, “What input motion should be applied at the base of a *FLAC* model in order to properly simulate the design motion?”

The appropriate input motion at depth can be computed through a “deconvolution” analysis using a 1D wave propagation code such as the equivalent-linear program *SHAKE*. This seemingly simple analysis is often the subject of considerable confusion resulting in improper ground motion input for *FLAC* models. The application of *SHAKE* for adapting design earthquake motions for *FLAC* input is described. Two typical cases are:

- 1 A rigid base, where an acceleration-time history is specified at the base of the *FLAC* mesh.
- 2 A compliant base, where a quiet (absorbing) boundary is used at the base of the *FLAC* mesh.

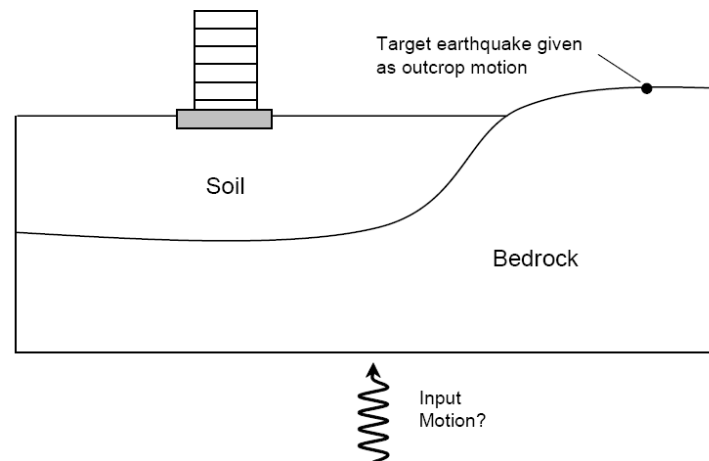


Figure 1.8 Seismic input to *FLAC*

Input of an earthquake motion into *FLAC* is typically done using either a “rigid base” or a “compliant base.” For a rigid base, a time history of acceleration (or velocity or displacement) is specified for gridpoints along the base of the mesh. While simple to use, a potential drawback of a rigid base is that the motion at the base of the model is completely prescribed. Hence, the base acts as if it were a fixed displacement boundary reflecting downward-propagating waves back into the model. Thus, a rigid base is not an appropriate boundary for general application unless a large dynamic

* This section is abstracted with permission from the publication by Mejia and Dawson (2006).

impedance contrast is meant to be simulated at the base (e.g. low velocity sediments over high velocity bedrock).

For a compliant base simulation, a quiet boundary is specified along the base of the *FLAC* mesh. See [Section 1.4.1.3](#) for a description of quiet boundaries. Note that if a history of acceleration is recorded at a gridpoint on the quiet base, it will not necessarily match the input history. The input stress-time history specifies the upward-propagating wave motion into the *FLAC* model, but the actual motion at the base will be the superposition of the upward motion and the downward motion reflected back from the *FLAC* model.

SHAKE (Schnabel et al. 1972) is a widely used 1D wave propagation code for site response analysis. *SHAKE* computes the vertical propagation of shear waves through a profile of horizontal visco-elastic layers. Within each layer, the solution to the wave equation can be expressed as the sum of an upward-propagating wave train and a downward-propagating wave train. The *SHAKE* solution is formulated in terms of these upward- and downward-propagating motions within each layer as illustrated in [Figure 1.9](#):

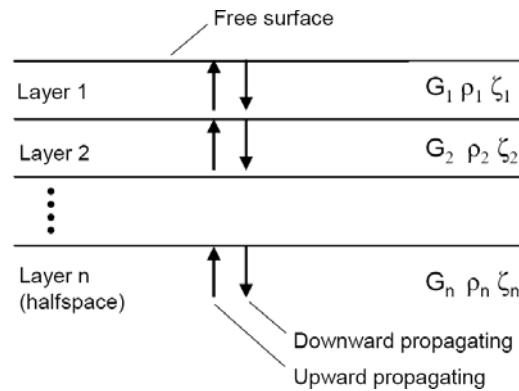


Figure 1.9 *Layered system analyzed by SHAKE (layer properties are shear modulus, G , density, ρ and damping fraction, ζ)*

The relation between waves in one layer and waves in an adjacent layer can be solved by enforcing the continuity of stresses and displacements at the interface between the layers. These well-known relations for reflected and transmitted waves at the interface between two elastic materials (Kolsky 1963) can be expressed in terms of recursion formulas. In this way, the upward- and downward-propagating motions in one layer can be computed from the upward and downward motions in a neighboring layer.

To satisfy the zero shear stress condition at the free surface, the upward- and downward-propagating motions in the top layer must be equal. Starting at the top layer, repeated use of the recursion formulas allows the determination of a transfer function between the motions in any two layers of the system. Thus, if the motion is specified at one layer in the system, the motion at any other layer can be computed.

SHAKE input and output is not in terms of the upward-and downward-propagating wave trains, but in terms of the motions at: a) the boundary between two layers, referred to as a “within” motion; or

b) at a free surface, referred to as an “outcrop” motion. The within motion is the superposition of the upward- and downward-propagating wave trains. The outcrop motion is the motion that would occur at a free surface at that location. Hence the outcrop motion is simply twice the upward-propagating wave-train motion. If needed, the upward-propagating motion can be computed by taking half the outcrop motion. At any point, the downward-propagating motion can then be computed by subtracting the upward-propagating motion from the within motion.

The *SHAKE* solution is in the frequency domain, with conversion to and from the time-domain performed with a Fourier transform. The deconvolution analysis discussed below illustrates the application of *SHAKE* for a linear, elastic case. [Section 1.7.2](#) describes a comparison between *FLAC* and *SHAKE* for a layered, linear-elastic soil deposit. *SHAKE* can also address nonlinear soil behavior approximately, through the equivalent linear approach. Analyses are run iteratively to obtain shear modulus and damping values for each layer that are compatible with the computed effective strain for the layer. See [Section 1.7.3](#) for a comparison of *FLAC* to *SHAKE* for a layered, nonlinear elastic soil.

Deconvolution for a Rigid Base – The deconvolution procedure for a rigid base is illustrated in [Figure 1.10](#). The goal is to determine the appropriate base input motion to *FLAC* such that the target design motion is recovered at the top surface of the *FLAC* model. The profile modeled consists of three 20-m thick elastic layers with shear wave velocities and densities as shown in the figure. The *SHAKE* model includes the three elastic layers and an elastic half-space with the same properties as the bottom layer. The *FLAC* model consists of a column of linear elastic elements. The target earthquake is input at the top of the *SHAKE* column as an outcrop motion. Then, the motion at the top of the half-space is extracted as a within motion and is applied as an acceleration-time history to the base of the *FLAC* model. Mejia and Dawson (2006) show that the resulting acceleration at the surface of the *FLAC* model is virtually identical to the target motion. The *SHAKE* within motion is appropriate for rigid-base input because, as described above, the within motion is the actual motion at that location, the superposition of the upward- and downward-propagating waves.

Deconvolution for a Compliant Base – The deconvolution procedure for a compliant base is illustrated in [Figure 1.11](#). The *SHAKE* and *FLAC* models are identical to those for the rigid body exercise, except that a quiet boundary is applied to the base of the *FLAC* mesh. For application through a quiet base, the upward-propagating wave motion (1/2 the outcrop motion) is extracted from *SHAKE* at the top of the half-space. This acceleration-time history is integrated to obtain a velocity, which is then converted to a stress history using [Eq. \(1.15\)](#). Again, the resulting acceleration at the surface of the *FLAC* model is shown by Mejia and Dawson (2006) to be virtually identical to the target motion. As an additional check of the computed accelerations, they also show that the response spectra for both the compliant-base and rigid-base cases closely match the response spectra of the target motion.

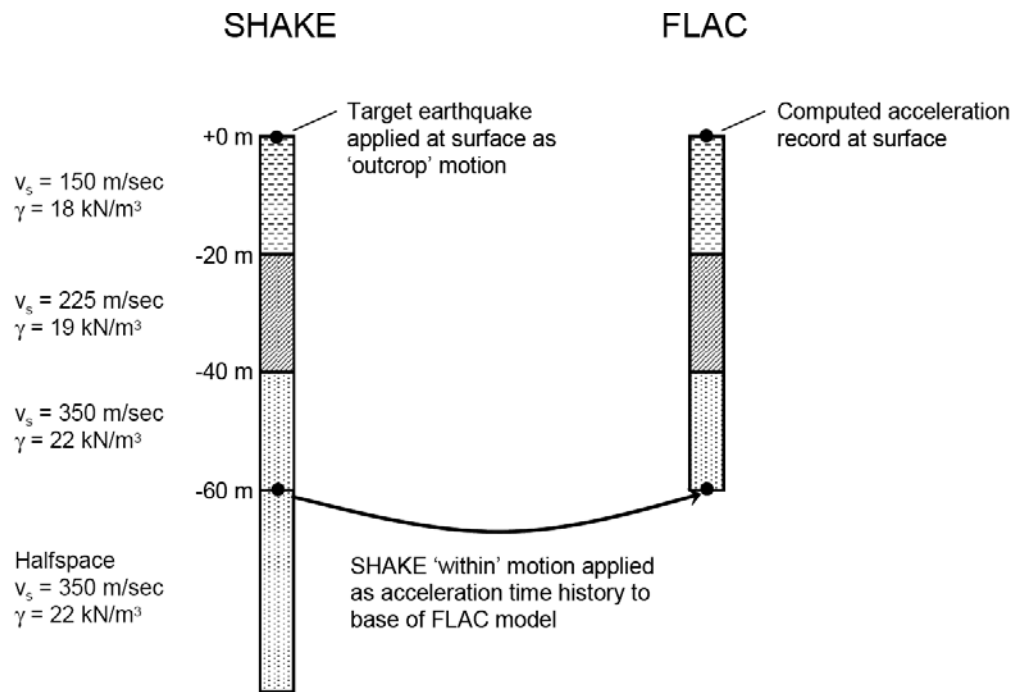


Figure 1.10 Deconvolution procedure for a rigid base
(after Mejia and Dawson 2006)

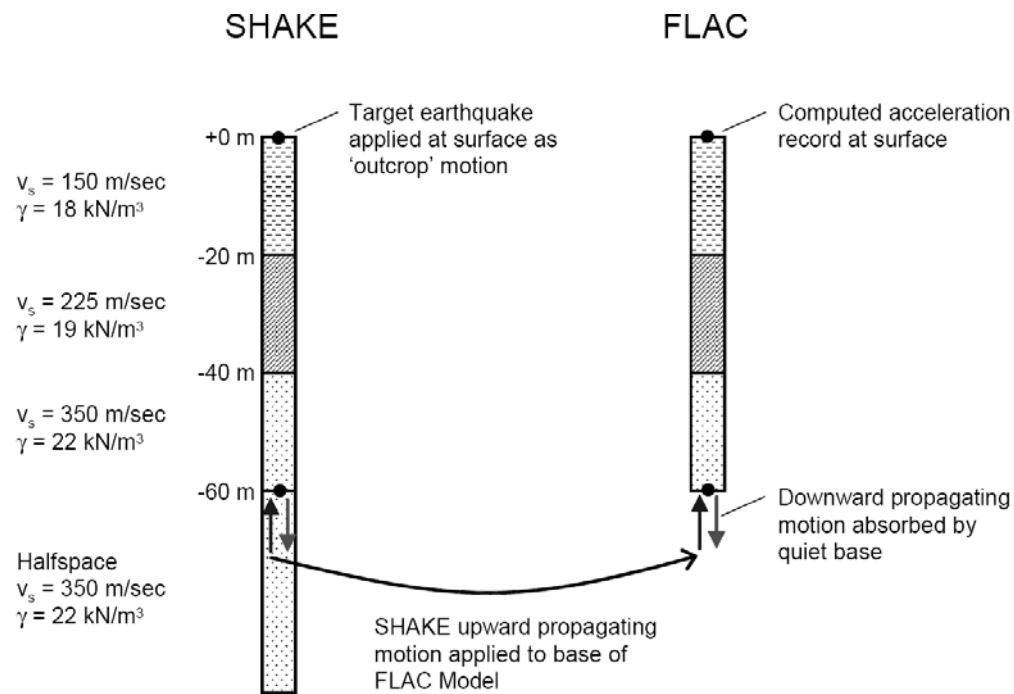


Figure 1.11 Deconvolution procedure for a compliant base
(after Mejia and Dawson 2006)

Although useful for illustrating the basic ideas behind deconvolution, the previous example is not the typical case encountered in practice. The situation shown in [Figure 1.12](#), where one or more soil layers (expected to behave nonlinearly) overlay bedrock (assumed to behave linearly), is more common. A *FLAC* model for this case will usually include the soil layers and an elastic base of bedrock. To compute the correct *FLAC* compliant base input, a *SHAKE* model is constructed as shown in the figure. The *SHAKE* model includes a bedrock layer equal in thickness to the elastic base of the *FLAC* mesh, and an underlying elastic half-space with bedrock properties. The target motion is input to the *SHAKE* model as an outcrop motion at the top of the bedrock (point A). Designating this motion as outcrop means that the upward-propagating wave motion in the layer directly below point A will be set equal to $1/2$ the target motion. The upward-propagating motion for input to *FLAC* is extracted at Point B as $1/2$ the outcrop motion.

For the compliant-base case there is actually no need to include the soil layers in the *SHAKE* model, as these will have no effect on the upward-propagating wave train between points A and B. In fact, for this simple case, it is not really necessary to perform a formal deconvolution analysis, as the upward-propagating motion at point B will be almost identical to that at point A. Apart from an offset in time, the only differences will be due to material damping between the two points, which will generally be small for bedrock. Thus, for this very common situation, the correct input motion for *FLAC* is simply $1/2$ of the target motion. (Note that the upward-propagating wave motion must be converted to a stress-time history using [Eq. \(1.15\)](#), which includes a factor of 2 to account for the stress absorbed by the viscous dashpots.)

For a rigid-base analysis, the within motion at point B is required. Since this within motion incorporates downward-propagating waves reflected off the ground surface, the nonlinear soil layers must be included in the *SHAKE* model. However, soil nonlinearity will be modeled quite differently in *FLAC* and *SHAKE*. Thus, it is difficult to compute the appropriate *FLAC* input motion for a rigid-base analysis.

Another typical case encountered in practice is illustrated in [Figure 1.13](#). Here, the soil profile is deep, and rather than extending the *FLAC* mesh all the way down to bedrock, the base of the model ends within the soil profile. Note that the mesh must be extended to a depth below which the soil response is essentially linear. Again, the design motion is input at the top of the bedrock (point A) as an outcrop motion, and the upward-propagating motion for input to *FLAC* is extracted at point B. As in the previous example, for a compliant-base analysis there is no need to include the soil layers above point B in the *SHAKE* model. These layers have no effect on the upward-propagating motion between points A and B. Unlike the previous case, the upward-propagating motion can be quite different at points A and B, depending on the impedance contrast between the bedrock and linear soil layer. Thus, it is not appropriate to skip the deconvolution analysis and use the target motion directly.

A rigid base is only appropriate for cases with a large impedance contrast at the base of the model. However, the use of *SHAKE* to compute the required input motion for a rigid base of a *FLAC* model leads to a good match between the target surface motion and the surface motion computed by *FLAC*, *only* for a model that exhibits a low level of nonlinearity. The input motion already contains the effect of all layers above the base, because it contains the downward-propagating wave.

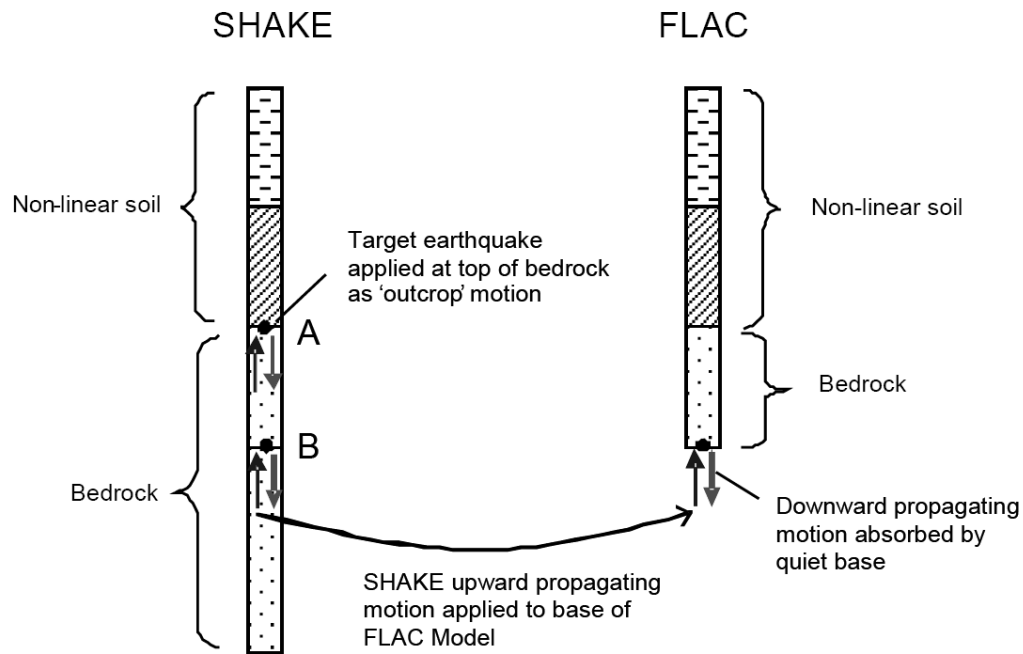


Figure 1.12 *Compliant-base deconvolution procedure for a typical case (after Mejia and Dawson 2006)*

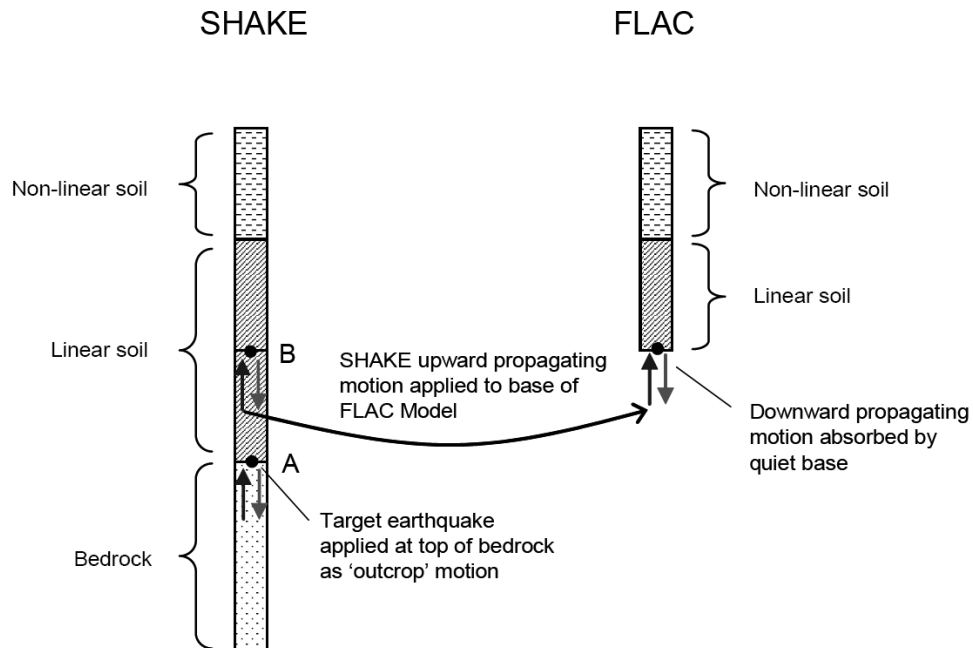


Figure 1.13 *Compliant-base deconvolution procedure for another typical case (after Mejia and Dawson 2006)*

A different approach must be taken if a *FLAC* model with a rigid base is used to simulate more realistic systems (e.g., sites that exhibit strong nonlinearity, or the effect of a surface or embedded structure). In the first case, the real nonlinear response is not accounted for by *SHAKE* in its estimate of base motion. In the second case, secondary waves from the structure will be reflected from the rigid base, causing artificial resonance effects.

A compliant base is almost always the preferred option because downward-propagating waves are absorbed. In this case, the quiet-base condition is selected, and only the upward-propagating wave from *SHAKE* is used to compute the input stress history. By using the upward-propagating wave only at a quiet *FLAC* base, no assumptions need to be made about secondary waves generated by internal nonlinearities or structures within the grid, because the incoming wave is unaffected by these; the outgoing wave is absorbed by the compliant base.

Although the presence of reflections from a rigid base is not always obvious in complex nonlinear *FLAC* analyses, they can have a major impact on analysis results, especially when cyclic-degradation or liquefaction-soil models are employed. Mejia and Dawson (2006) present examples that illustrate the nonphysical wave reflections calculated in models with a rigid base. One example, shown in Figure 1.14, demonstrates the difficulty with a rigid boundary. The nonphysical oscillations that result from a rigid base are shown by comparison to results for a compliant base in Figure 1.15. The inputs in both cases (rigid and compliant) were derived by deconvoluting the same surface motion.

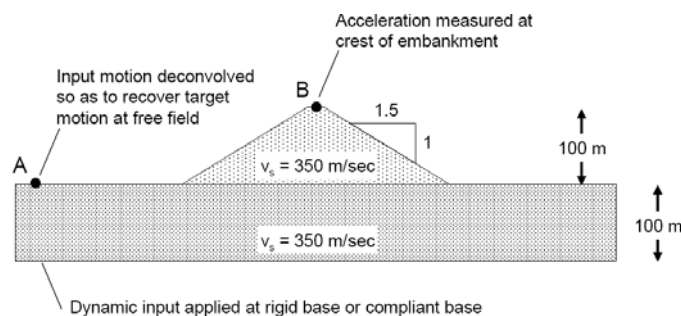


Figure 1.14 Embankment analyzed with a rigid and compliant base (after Mejia and Dawson 2006)

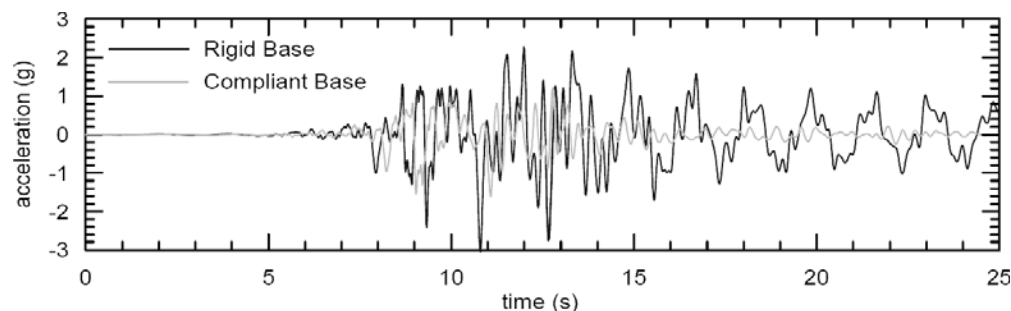


Figure 1.15 Computed accelerations at crest of embankment (after Mejia and Dawson 2006)

1.4.1.7 Hydrodynamic Pressures

The dynamic interaction between water in a reservoir and a concrete dam can have a significant influence on the performance of the dam during an earthquake. Westergaard (1933) established a mathematical basis for procedures to represent this interaction, and this approach is commonly used in engineering practice. Although the advent of computers has enabled numerical solution of coupled differential equations of fluid-structure systems, the formula proposed by Westergaard is widely used for stability analysis of smaller dams, and preliminary calculations in the design of large dams.

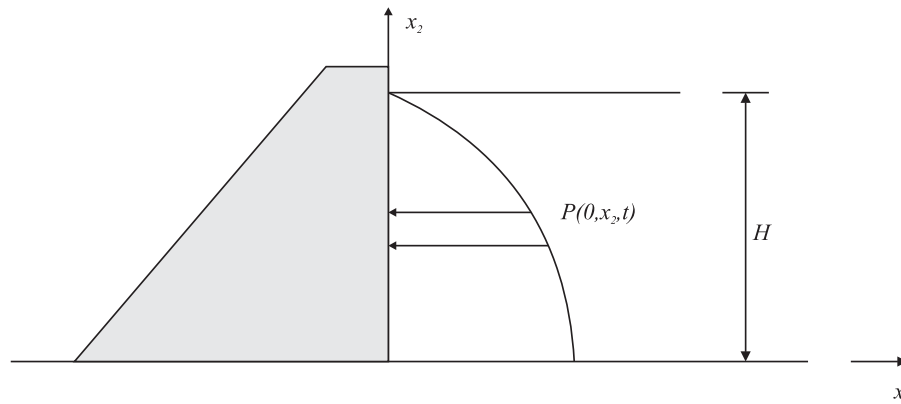


Figure 1.16 *Hydrodynamic pressure acting on a rigid dam with a vertical upstream face*

The hydrodynamic pressure acting on a rigid concrete dam over a reservoir height, H , is depicted in [Figure 1.16](#). The pressure can be derived from the equation of motion for a fluid. The equation of motion for a fluid with small Reynold's number can be written as

$$c^2 \left[\frac{\partial^2 \Phi}{\partial x_1^2} + \frac{\partial^2 \Phi}{\partial x_2^2} \right] = \frac{\partial^2 \Phi}{\partial t^2} \quad (1.22)$$

where c is the speed of sound in water, and Φ is the velocity potential. The water pressure can be written as a function of the velocity potential:

$$p = \rho_w \frac{\partial \Phi}{\partial t} \quad (1.23)$$

where ρ_w is the density of water.

Additional assumptions are made, to simplify the loading condition:

1. The water is assumed to be incompressible, which reduces Eq. (1.22) to the Laplace equation: $\frac{\partial^2 \Phi}{\partial x_1^2} + \frac{\partial^2 \Phi}{\partial x_2^2} = 0$.
2. The free surface of the reservoir is assumed to be at rest. Thus, $\frac{\partial \Phi}{\partial t} = 0$ at $x_2 = H$.
3. The reservoir is assumed to be infinitely long. Therefore, as $x_1 \rightarrow \infty$, $\Phi \rightarrow 0$.
4. Hydrodynamic motion is assumed to be horizontal only: $\frac{\partial \Phi}{\partial x_2} = 0$ at $x_2 = 0$.
5. The upstream face of the dam is vertical and the dam is rigid: $\frac{\partial \Phi}{\partial x_1} = f(t)$ at $x_1 = 0$.

The solution of Eq. (1.22) with the above assumptions can be obtained for an arbitrary acceleration, $\ddot{x}_0(t)$, in the form of an infinite Fourier series:

$$p(0, x_2, t) = 8\ddot{x}_0(t)\rho_w H \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{((2n-1)\pi)^2} e^{\frac{-(2n-1)x_1}{4H}} \cos\left(\frac{(2n-1)x_2}{4H}\right) \quad (1.24)$$

Eq. (1.24) can be approximated as

$$p(0, x_2, t) = \rho_w \ddot{x}_0(t) H \frac{C_m}{2} \left[1 - \frac{x_2^2}{H^2} + \sqrt{1 - \frac{x_2^2}{H^2}} \right] \quad (1.25)$$

where $C_m = 0.743$ and $\ddot{x}$ is the horizontal acceleration at the dam face.

Eq. (1.25) is implemented in *FLAC* by adjusting the gridpoint mass on the upstream face of the dam to account for the hydrodynamic pressure. The equivalent pressure, $\bar{p}$, resulting from the inertial forces associated with the gridpoint and the hydrodynamic pressure of the water in the reservoir, averaged over the area associated with the gridpoint, can be written as

$$\bar{p}(0, x_2, t) = \rho_{ec} \ddot{x}_0(t) \frac{A_g}{\Delta x_2} \quad (1.26)$$

where A_g is the area associated with the gridpoint, and Δx_2 is the contact length on the upstream face of the dam through which the water load is applied for the gridpoint.

ρ_{ec} is the equivalent density of the gridpoint and is given by

$$\rho_{ec} = \rho_c + \rho_{sc} \quad (1.27)$$

where

$$\rho_{sc} = \rho_w \frac{H \Delta x_2}{A_g} \frac{C_m}{2} \left[1 - \frac{x_2^2}{H^2} + \sqrt{1 - \frac{x_2^2}{H^2}} \right] \quad (1.28)$$

ρ_c is the density of concrete such that the gridpoint mass is given by $m_g = A_g \rho_c$. The scaled gridpoint mass $m_{sg} = A_g \rho_{ec}$ is used only for the motion calculation in the horizontal direction; the effect of the increased mass does not influence the vertical forces.

The gridpoint mass is adjusted by adding the term (as determined from [Eq. \(1.28\)](#)) to account for the hydrodynamic pressure. The *FISH* gridpoint variable **gmscl** is available to store the gridpoint mass adjustment. The *FISH* function **_westergaard** is provided to apply the hydrodynamic pressures to a vertical dam face. The *FISH* function requires the following input:

_dx	x_1 component of the unit vector pointing in the direction of the water
_dy	x_2 component of the unit vector pointing in the direction of the water
height	height of the water in the reservoir
yb	x_2 coordinate of the base of the reservoir
den_w	density of water

A simple example is presented to illustrate the effect of hydrodynamic pressures on a concrete dam. The dynamic loading is applied in three different ways. First, the dam is subjected to a dynamic loading without taking into account the hydrodynamic pressure. Second, the hydrodynamic pressure is applied as a boundary condition by means of the Westergaard scaling of the gridpoint mass, as described above. Third, the hydrodynamic pressure is simulated by modeling the water directly as zones adjacent to the dam zones. [Figure 1.17](#) shows the model for the first two loading cases, and [Figure 1.18](#) shows the model for the third case. The dynamic loading is a velocity sine wave applied to the base of the model. The models are first brought to a static equilibrium state with the reservoir loading applied along the upstream vertical face of the dam. The dynamic loading is then applied for a period of 10 seconds. The horizontal displacement at the top of the dam at the upstream face is monitored for all three cases. The results are plotted for comparison in [Figure 1.19](#). These results illustrate the effect on displacement of the hydrodynamic pressures. The case using the Westergaard adjustment is in good agreement with the case modeling the water explicitly.

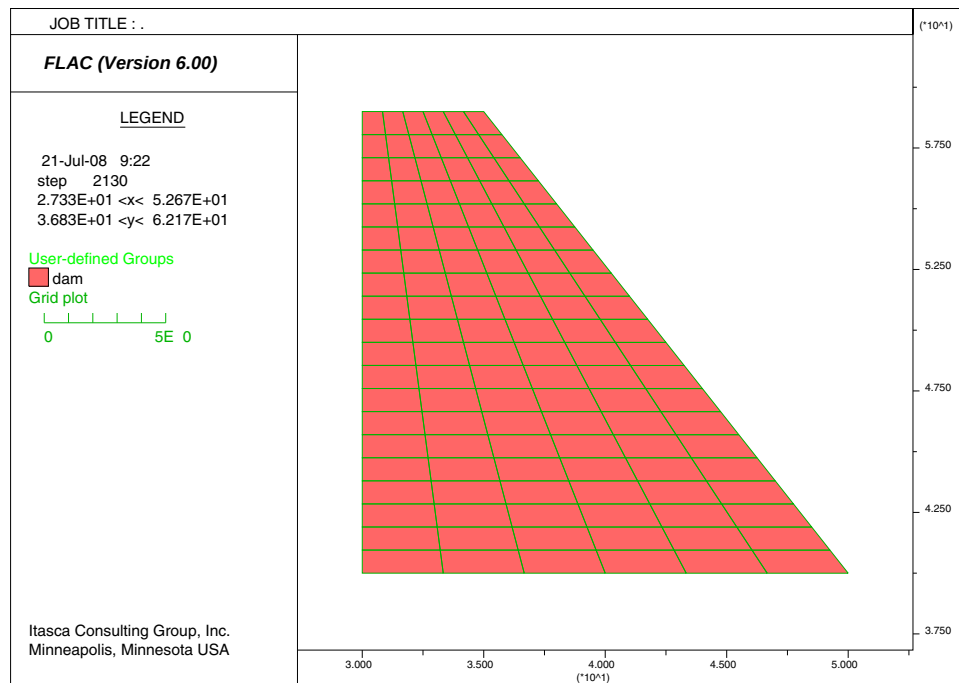


Figure 1.17 Dam model with hydrodynamic pressure boundary on upstream face

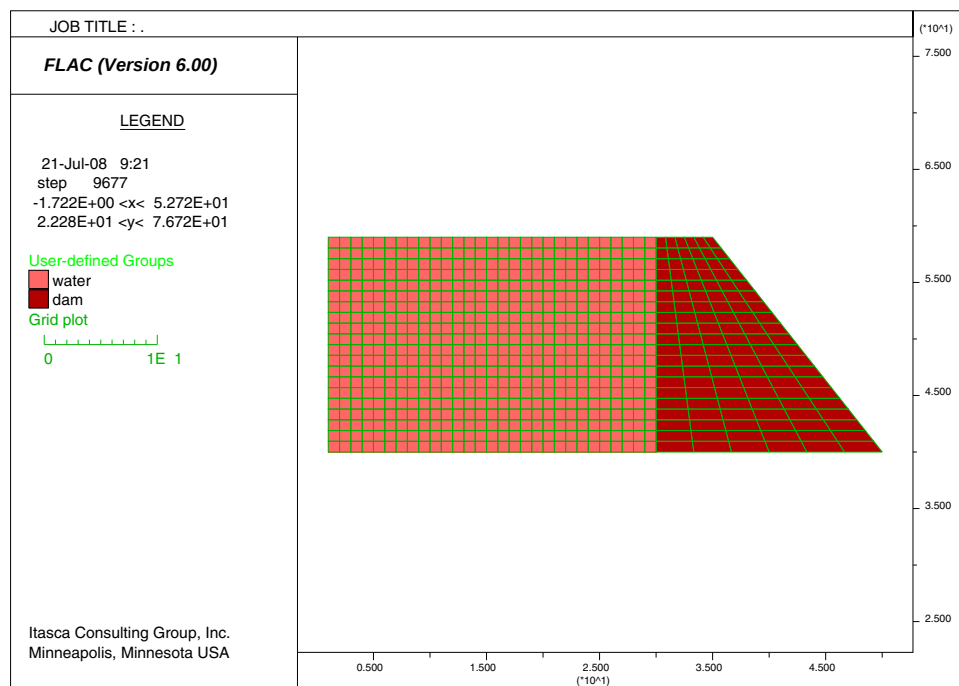


Figure 1.18 Dam model with water zones on upstream face

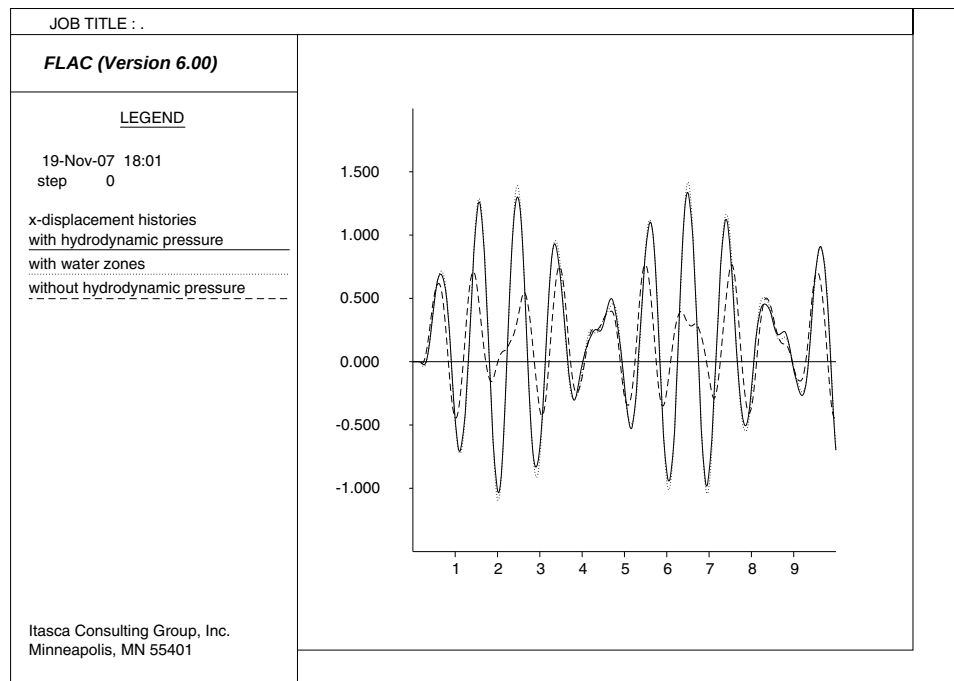


Figure 1.19 Comparison of x -displacement at top of dam

Example 1.4 Hydrodynamic pressure acting on a dam

```

config dyn ex 10
grid 80,60
model elastic
prop dens 2000 bulk 1e8 shear 3e7
model null j = 40
model null j = 41 60 i = 1 30
model null j = 41 60 i = 37 80
gen 30.0,40.0 30.0,59.0 35.0,59.0 50.0,40.0 i 31 37 j 41 61
group dam i 31 37 j 41 61
set grav 10
model null j 1 40
fix x y j 41
set dyn off
solve
save grid.sav
initial xdisp 0 ydisp 0
initial xvel 0 yvel 0
apply pressure 0.0 var 0.0 190000.0 from 31,61 to 31,41
solve
save pressure.sav

```

```
ini xdisp 0 ydisp 0 xvel 0 yvel 0
set dyn on
call westergaard.fis
set _dx=-1 _dy=0. yb = 40. height = 19. den_w = 1e3
_westergaard
set step 100000000
def sine_wave
    sine_wave = sin(2*pi*freq*dytime)
end
hist sine_wave
hist xdisp i 31 j 61
hist xdisp i 31 j 54
hist xdisp i 31 j 50
hist xdisp i 31 j 41
set dytime 0.0 freq 1
apply yvelo 0.0 xvel 1.0 hist sine_wave i 31 37 j 41
hist dytime
solve dyt 10
save wester.sav
hist write 2 vs 6 table 1
call tabtofile.fis
set filename='table1.dat'
set tabin 1
tabtofile
save wester_result.sav
;
restore pressure.sav
ini xdisp 0 ydisp 0 xvel 0 yvel 0
set dyn on
set step 100000000
def sine_wave
    sine_wave = sin(2*pi*freq*dytime)
end
hist sine_wave
hist xdisp i 31 j 61
hist xdisp i 31 j 54
hist xdisp i 31 j 50
hist xdisp i 31 j 41
set dytime 0.0 freq 1
apply yvelo 0.0 xvel 1.0 hist sine_wave i 31 37 j 41
hist dytime
solve dyt 10
save small_NO_wester.sav
hist write 2 vs 6 table 2
call tabtofile.fis
set filename='table2.dat'
```

```

set tabin 2
tabtofile
save small_NO_wester_result.sav
;
new
config dyn ex 10
grid 80,60
model elastic
prop dens 2000 bulk 1e8 shear 3e7
model null j = 40
model null j = 41 60 i = 37 80
gen 0,40 0 59 30 59 30 40 i 1 31 j 41 61
gen 30.0,40.0 30.0,59.0 35.0,59.0 50.0,40.0 i 31 37 j 41 61
group 'dam' i 31 37 j 41 61
model null i 1 30 j 41 60
set grav 10
model null j 1 40
fix x y j 41
set dyn off
solve
save water_grid.sav
initial xdisp 0 ydisp 0
initial xvel 0 yvel 0
group 'water' i 1 29 j 41 60
model elastic group 'water'
prop density=1000.0 bulk=2E9 shear=0 group 'water'
fix x i 1
ini x add 1.0 y add 0.0 nmregion 29 41
interface 1 aside from 30,41 to 30,61 bside from 31,41 to 31,61
interface 1 unglued kn=1.4E9 ks=1.4E9 cohesion=0.0 dilation=0.0 &
  friction=0.0 tbond=1e10 bslip=Off
history 999 unbalanced
solve
save water_pressure.sav
ini xdisp 0 ydisp 0 xvel 0 yvel 0
set dyn on
set step 100000000
def sine_wave
  sine_wave = sin(2*pi*freq*dytime)
end
hist sine_wave
hist xdisp i 31 j 61
hist xdisp i 31 j 54
hist xdisp i 31 j 50
hist xdisp i 31 j 41
set dytime 0.0 freq 1

```

```
apply yvelo 0.0 xvel 1.0 hist sine_wave i 1 30 j 41
apply yvelo 0.0 xvel 1.0 hist sine_wave i 31 37 j 41
hist dytime
solve dyt 10
save water.sav
hist write 2 vs 6 table 3
call tabtofile.fis
set filename='table3.dat'
set tabin 3
tabtofile
save water_result.sav
;
new
;... STATE: TABLES ....
call table1.dat
call table2.dat
call table3.dat
save tables.sav
;*** plot commands ****
;plot name: syy
plot hold grid syy fill min -500000.0 max 0.0 int 50000.0 grid magnify 1.2
;plot name: disp
plot hold grid apply iwhite displacement max 10.97 grid magnify 2.5
;plot name: sxx
plot hold sxx fill min -1.5E7 max 1.0E7 int 5000000.0
;plot name: xdisp
plot hold xdisp fill
;plot name: hist
plot hold history 2 line vs 6
;plot name: xdisp
label table 1
with hydrodynamic pressure
label table 2
with water zones
label table 3
without hydrodynamic pressure
plot hold table 1 line 2 line 3 line alias 'x-displacement histories'
```

1.4.2 Wave Transmission

1.4.2.1 Accurate Wave Propagation

Numerical distortion of the propagating wave can occur in a dynamic analysis as a function of the modeling conditions. Both the frequency content of the input wave and the wave-speed characteristics of the system will affect the numerical accuracy of wave transmission. Kuhlemeyer and Lysmer (1973) show that for accurate representation of wave transmission through a model, the spatial element size, Δl , must be smaller than approximately one-tenth to one-eighth of the wavelength associated with the highest frequency component of the input wave – i.e.,

$$\Delta l \leq \frac{\lambda}{10} \quad (1.29)$$

where λ is the wavelength associated with the highest frequency component that contains appreciable energy.

1.4.2.2 Filtering

For dynamic input with a high peak velocity and short rise-time, the Kuhlemeyer and Lysmer requirement may necessitate a very fine spatial mesh and a corresponding small timestep. The consequence is that reasonable analyses may be prohibitively time- and memory-consuming. In such cases, it may be possible to adjust the input by recognizing that most of the power for the input history is contained in lower-frequency components (e.g., use “FFT.FIS” in [Section 3](#) in the *FISH* volume). By filtering the history and removing high-frequency components, a coarser mesh may be used without significantly affecting the results.

The filtering procedure can be accomplished with a low-pass filter routine such as the Fast Fourier Transform technique (e.g., “FILTER.FIS” in [Section 3](#) in the *FISH* volume). The unfiltered velocity record shown in [Figure 1.20](#) represents a typical wave form containing a very high frequency spike. The highest frequency of this input exceeds 50 Hz but, as shown by the power spectral density plot of Fourier amplitude versus frequency ([Figure 1.21](#)), most of the power (approximately 99%) is made up of components of frequency 15 Hz or lower. It can be inferred, therefore, that by filtering this velocity history with a 15 Hz low-pass filter, less than 1% of the power is lost. The input filtered at 15 Hz is shown in [Figure 1.22](#), and the Fourier amplitudes are plotted in [Figure 1.23](#). The difference in power between unfiltered and filtered input is less than 1%, while the peak velocity is reduced 38%, and the rise time is shifted from 0.035 to 0.09 second. Analyses should be performed with input at different levels of filtering to evaluate the influence of the filter on model results.

If a simulation is run with an input history that violates [Eq. \(1.29\)](#), the output will contain spurious “ringing” (superimposed oscillations) that is nonphysical. The input spectrum *must* be filtered before being applied to a *FLAC* grid. This limitation applies to all numerical models in which a continuum is discretized; it is not just a characteristic of *FLAC*. Any discretized medium has an upper limit to the frequencies that it can transmit, and this limit must be respected if the results are to be meaningful. Users of *FLAC* commonly apply sharp pulses or step wave forms to a *FLAC* grid; this is *not* acceptable under most circumstances, because these wave forms have spectra that

extend to infinity. It is a simple matter to apply, instead, a smooth pulse that has a limited spectrum, as discussed above. Alternatively, artificial viscosity may be used to spread sharp wave fronts over several zones (see [Section 1.4.3.10](#)), but this method strictly only applies to isotropic strain components.

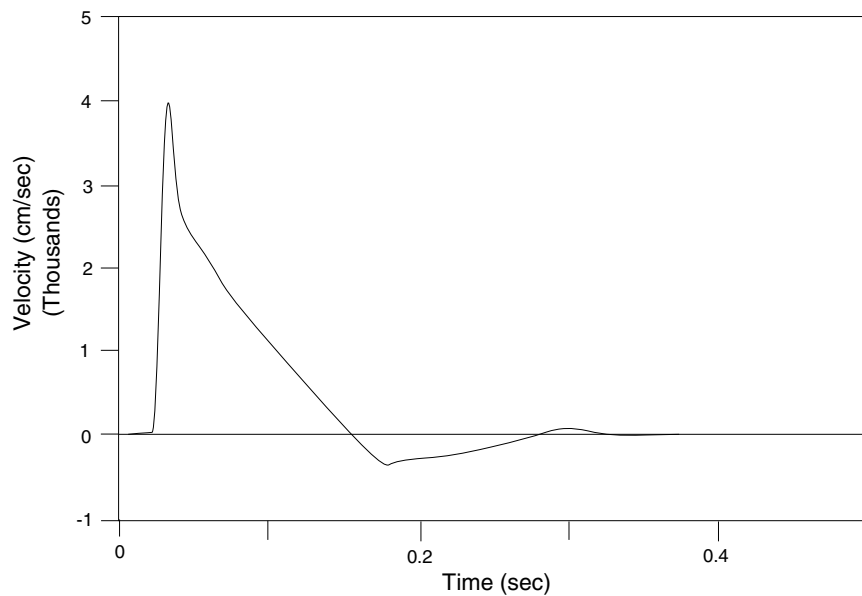


Figure 1.20 *Unfiltered velocity history*

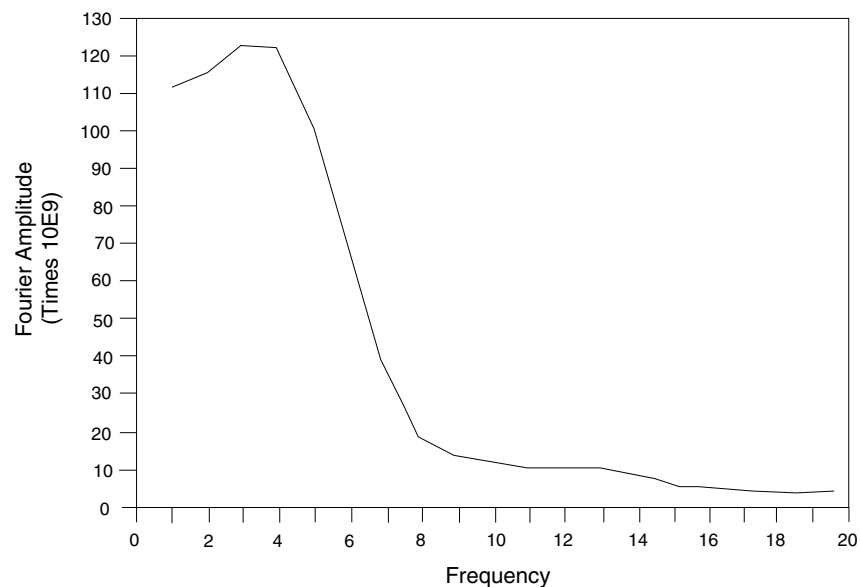


Figure 1.21 *Unfiltered power spectral density plot*

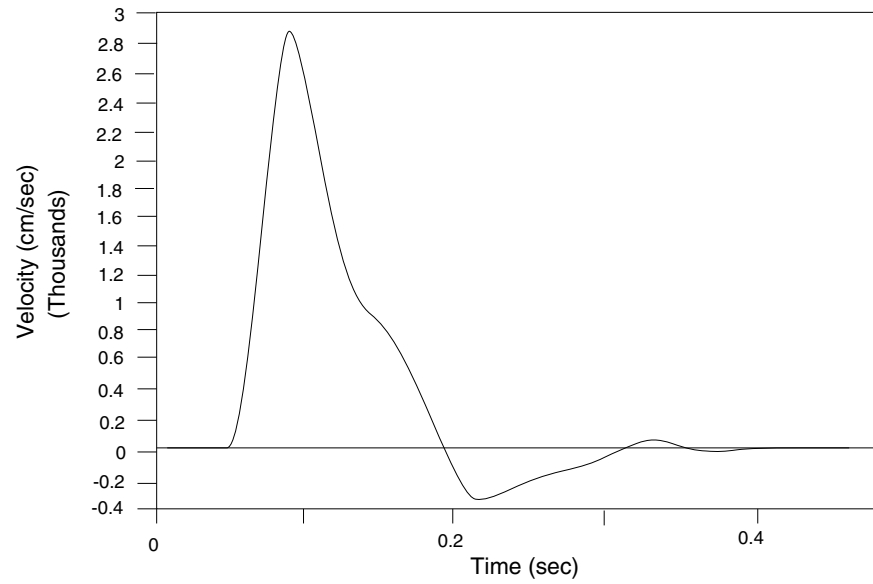


Figure 1.22 *Filtered velocity history at 15 Hz*

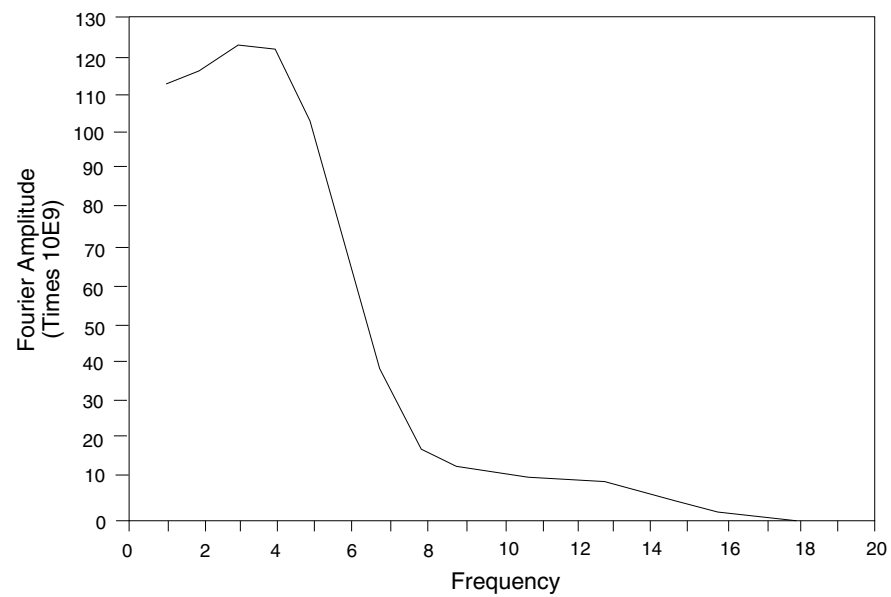


Figure 1.23 *Results of filtering at 15 Hz*

1.4.3 Mechanical Damping and Material Response

Natural dynamic systems contain some degree of damping of the vibration energy within the system; otherwise, the system would oscillate indefinitely when subjected to driving forces. Damping is due, in part, to energy loss as a result of internal friction in the intact material and slippage along interfaces, if these are present.

FLAC uses a dynamic algorithm for solution of two general classes of mechanical problems: quasi-static and dynamic. Damping is used in the solution of both classes of problems, but quasi-static problems require more damping for rapid convergence to equilibrium. The damping for static solutions is discussed in [Section 1.3.4](#) in **Theory and Background**.

For a dynamic analysis, the damping in the numerical simulation should reproduce in magnitude and form the energy losses in the natural system when subjected to a dynamic loading. In soil and rock, natural damping is mainly hysteretic (i.e., independent of frequency – see Gemant and Jackson 1937, and Wegel and Walther 1935). It is difficult to reproduce this type of damping numerically because of at least two problems (see Cundall 1976, and comments in [Section 1.2.2](#)). First, many simple hysteretic functions do not damp all components equally when several wave forms are superimposed. Second, hysteretic functions lead to path-dependence, which makes results difficult to interpret. However, if a constitutive model that contains an adequate representation of the hysteresis that occurs in a real material is found, then *no additional damping* would be necessary. This comment is addressed to users who program their own constitutive models in the *FISH* language or in C++; the built-in models are not considered to model dynamic hysteresis well enough to omit additional damping completely.

For several reasons, it is impractical to use the “real” stress/strain response of the material in numerical simulations. For example: (a) there are no laws that describe the complete material response; and (b) existing laws that capture many important aspects have many material parameters, requiring extensive calibration.

In time-domain programs, *Rayleigh damping* is commonly used to provide damping that is approximately frequency-independent over a restricted range of frequencies. Although Rayleigh damping embodies two viscous elements (in which the absorbed energy is dependent on frequency), the frequency-dependent effects are arranged to cancel out at the frequencies of interest. Rayleigh damping is described in [Sections 1.4.3.1](#) through [1.4.3.3](#).

An alternative damping algorithm, *hysteretic damping*, is described in [Sections 1.4.3.4](#) through [1.4.3.6](#). This form of damping allows strain-dependent modulus and damping functions to be incorporated directly into the *FLAC* simulation. This makes it possible to make direct comparisons between calculations made with the equivalent-linear method and a fully nonlinear method, without making any compromises in the choice of constitutive model.

For routine engineering design, we must use an approximate representation of cyclic energy dissipation. In *FLAC*, when using simple plasticity models such as Mohr-Coulomb, the choice is between Rayleigh damping and hysteretic damping. Here, we make some general comparisons between the two approaches, to enable a choice to be made. In general, hysteretic damping is the more realistic of the two, and it entails no reduction in timestep. For further discussion, see [Section 1.4.3.11](#).

For low levels of cyclic strain and fairly uniform conditions, Rayleigh damping and hysteretic damping give similar results, provided that the levels of damping set for both are consistent with the levels of cyclic strain experienced. The results will differ in the following two circumstances:

1. When the system is nonuniform (e.g., layers of quite different properties), then cyclic strain levels may be different in different locations and at different times. Using hysteretic damping, these different strain levels produce realistically different damping levels in time and space, while constant and uniform Rayleigh damping parameters can only reproduce the average response. It would be possible to adjust the Rayleigh damping parameters to account for spatial variations in damping using an iterative (strain-compatible) scheme, as used in the equivalent linear method (see [Section 1.2.1](#)). It may also be possible to adjust the Rayleigh damping parameters in time, although some practical difficulties may be encountered.
2. As yield is approached, neither Rayleigh damping nor hysteretic damping account for the energy dissipation of extensive yielding. Thus, irreversible strain occurs externally to both schemes, and dissipation is represented by the yield model (e.g., Mohr-Coulomb). Under this condition, the mass-proportional term of Rayleigh damping may inhibit yielding because rigid-body motions that occur during failure modes are erroneously resisted. Hysteretic damping may give rise to larger permanent strains in such a situation, but this condition is usually believed to be more realistic compared to one using Rayleigh damping.

We note that hysteretic damping provides almost no energy dissipation at very low cyclic strain levels, which may be unrealistic. To avoid low-level oscillation, it is recommended that a small amount (e.g., 0.2%) of stiffness-proportional Rayleigh damping be added when hysteretic damping is used in a dynamic simulation.

Another form of damping in *FLAC*, the *local damping* embodied in *FLAC*'s static solution scheme, may be used dynamically, but with a damping coefficient appropriate to wave propagation. Local damping in dynamic problems is useful as an approximate way to include hysteretic damping. However, it becomes increasingly unrealistic as the complexity of the wave forms increases (i.e., as the number of frequency components increases). Local damping cannot properly capture the energy loss of multiple frequency cyclic loading. Local damping is described in more detail in [Section 1.4.3.7](#).

A fourth form of damping, *artificial viscosity*, is also provided in *FLAC*. This damping may be used for analyses involving sharp dynamic fronts – it is described in [Section 1.4.3.10](#).

1.4.3.1 Rayleigh Damping

Rayleigh damping was originally used in the analysis of structures and elastic continua, to damp the natural oscillation modes of the system. The equations, therefore, are expressed in matrix form.

A damping matrix, C , is used, with components proportional to the mass (M) and stiffness (K) matrices:

$$C = \alpha M + \beta K \quad (1.30)$$

where: α = the mass-proportional damping constant; and

β = the stiffness-proportional damping constant.

The mass-proportional term is analogous to a dashpot connecting each *FLAC* gridpoint to “ground.” The stiffness-proportional term is analogous to a dashpot connected across each *FLAC* zone (responding to the strain rate). Although both terms are frequency-dependent, an approximately frequency-independent response can be obtained over a limited frequency range, with the appropriate choice of parameters, as discussed below.

For a multiple degree-of-freedom system, the critical damping ratio, ξ_i , at any angular frequency of the system, ω_i , can be found from (Bathe and Wilson 1976)

$$\alpha + \beta \omega_i^2 = 2 \omega_i \xi_i \quad (1.31)$$

or

$$\xi_i = \frac{1}{2} \left(\frac{\alpha}{\omega_i} + \beta \omega_i \right) \quad (1.32)$$

The critical damping ratio, ξ_i , is also known as the fraction of critical damping for mode i with angular frequency ω_i .

Figure 1.24 shows the variation of the normalized critical damping ratio with angular frequency ω_i . Three curves are given: mass components only; stiffness components only; and the sum of both components. As shown, mass-proportional damping is dominant at lower angular-frequency ranges, while stiffness-proportional damping dominates at higher angular frequencies. The curve representing the sum of both components reaches a minimum at

$$\xi_{\min} = (\alpha \beta)^{1/2} \quad (1.33)$$

$$\omega_{\min} = (\alpha / \beta)^{1/2}$$

or

$$\alpha = \xi_{\min} \omega_{\min} \quad (1.34)$$

$$\beta = \xi_{\min} / \omega_{\min}$$

The center frequency is then defined as

$$f_{\min} = \omega_{\min} / 2\pi \quad (1.35)$$

Note that at frequency $\omega_{\min}$ (or $f_{\min}$) (and *only* at that frequency), mass damping and stiffness damping each supply half of the total damping force.

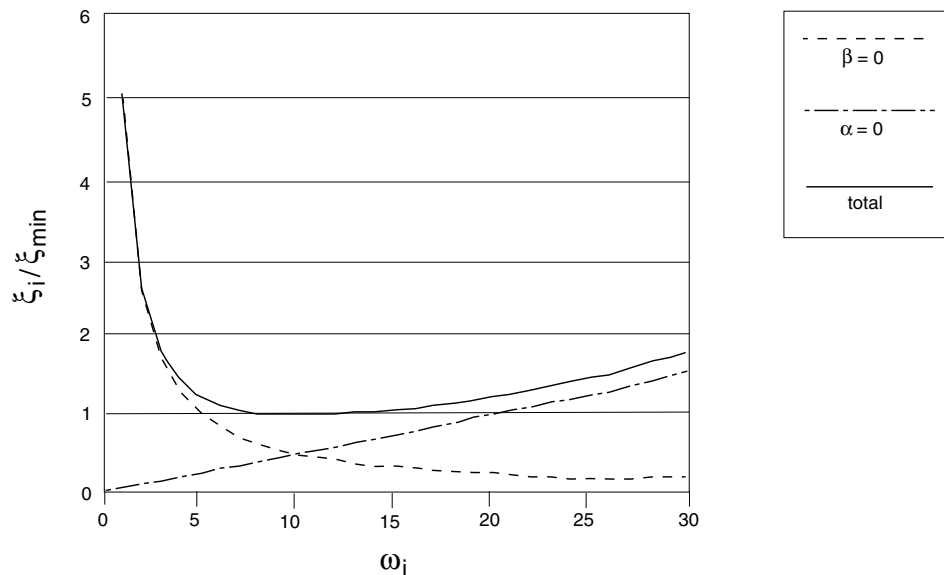


Figure 1.24 Variation of normalized critical damping ratio with angular frequency

Rayleigh damping is specified in *FLAC* with the parameters $f_{\min}$ in Hertz (cycles per second) and $\xi_{\min}$, both specified with the command **SET dy_damp rayleigh** or **INITIAL dy_damp rayleigh**.

Stiffness-proportional damping causes a reduction in the critical timestep for the explicit solution scheme (see Belytschko 1983). As the damping ratio corresponding to the highest natural frequency is increased, the timestep is reduced (see Eq. (1.10)). This can result in a substantial increase in runtimes for dynamic simulations.

In *FLAC*, the internal timestep calculation takes account of stiffness-proportional damping, but it is still possible for instability to occur if the large-strain calculation is in effect (**SET large**) and very

large mesh deformation occurs. If this happens, it is necessary to reduce the timestep manually (via the **SET dydt** command).

For the case shown in [Figure 1.24](#), $\omega_{\min} = 10$ radians per second. It is evident that the damping ratio is almost constant over at least a 3:1 frequency range (e.g., from 5 to 15). Since damping in geologic media is commonly independent of frequency (as discussed in [Section 1.4.3](#)), $\omega_{\min}$ is usually chosen to lie in the center of the range of frequencies present in the numerical simulation – either natural frequencies of the model or predominant input frequencies. Hysteretic damping is thereby simulated in an approximate fashion.

Viscous stress increments representing the stiffness-proportional component of Rayleigh damping are added to the total stress increments during a timestep in order to compute gridpoint forces. Thus,

$$\{\sigma_v\} = \{\sigma_n\} + \frac{\beta}{\Delta t} \{\sigma_n - \sigma_o\} \quad (1.36)$$

where σ_v is a component of the combined stress tensor used to derive gridpoint forces, σ_n is the corresponding component returned from the constitutive law, and σ_o is the value of the component prior to invoking the constitutive law. The notation $\{ \}$ denotes the vector of all components.

Viscous stresses are not included with the accumulated total stresses (σ_n in [Eq. \(1.36\)](#)). However, the total stresses including viscous stresses (σ_v in [Eq. \(1.36\)](#)) can be printed, plotted and recorded as histories by using the **vsxx**, **vsyy**, **vszz** and **vsxy** keywords.

A stiffness matrix is not needed in this formulation. Rayleigh damping operates directly on the tangent modulus for the constitutive model, whether it is linear or nonlinear.

Stiffness-proportional damping is turned off when plastic failure occurs within a *FLAC* zone. Mass-proportional damping, however, remains active. If excessive failure occurs in a model, the mass-proportional term may inhibit yielding. In this case, it may be advisable to exclude Rayleigh damping from regions of strong plastic flow (by using the **INITIAL dy_damp** command to set Rayleigh damping in selected regions, as described in [Section 1.4.3.8](#)).

1.4.3.2 Example Application of Rayleigh Damping

In order to demonstrate how Rayleigh damping works in *FLAC*, the results of the following four damping cases can be compared; the example consists of a square grid in which gravity is suddenly applied. The conditions are:

- (a) undamped;
- (b) Rayleigh damping (both mass and stiffness damping);
- (c) mass damping only; and
- (d) stiffness damping only.

Eq. (1.5) provides data corresponding to each case in turn. The Rayleigh parameters are adjusted to give critical damping in cases (b), (c) and (d).

Example 1.5 *Block under gravity – undamped and 3 critically damped cases*

```

conf dy
gr 3 3
m e
prop den 1000 bu 1e8 sh .3e8
fix y j=1
set grav 10.0
hist n 1
hist ydisp i=3 j=4
hist dytime
save damp.sav
restore damp.sav
step 200
save damp1.sav
restore damp.sav
set dy_damp=rayl 1 25.0
step 445
save damp2.sav
restore damp.sav
set dy_damp=rayl 2 25.0 mass
step 80
save damp3.sav
restore damp.sav
set dy_damp=rayl 2 25.0 stiff
step 870
save damp4.sav

```

In the first case, with no damping, a natural frequency of oscillation of approximately 25 Hertz is observed (see Figure 1.25). The problem should be critically damped if: (1) a fraction of critical damping, $\xi_{\min}$, of 1 is specified; (2) the natural frequency of oscillation, $f_{\min}$, of 25 Hertz is specified; and (3) both mass and stiffness damping are used.

The results in Figure 1.26 show that the problem is critically damped. If only mass or stiffness damping is used, then $\xi_{\min}$ must be doubled to obtain critical damping (since each component contributes one-half to the overall damping). Figures 1.27 and 1.28 again show that the system is critically damped.

Note that the timestep is different for the three damped simulations. This is a result of the influence of stiffness-proportional damping, as discussed above.

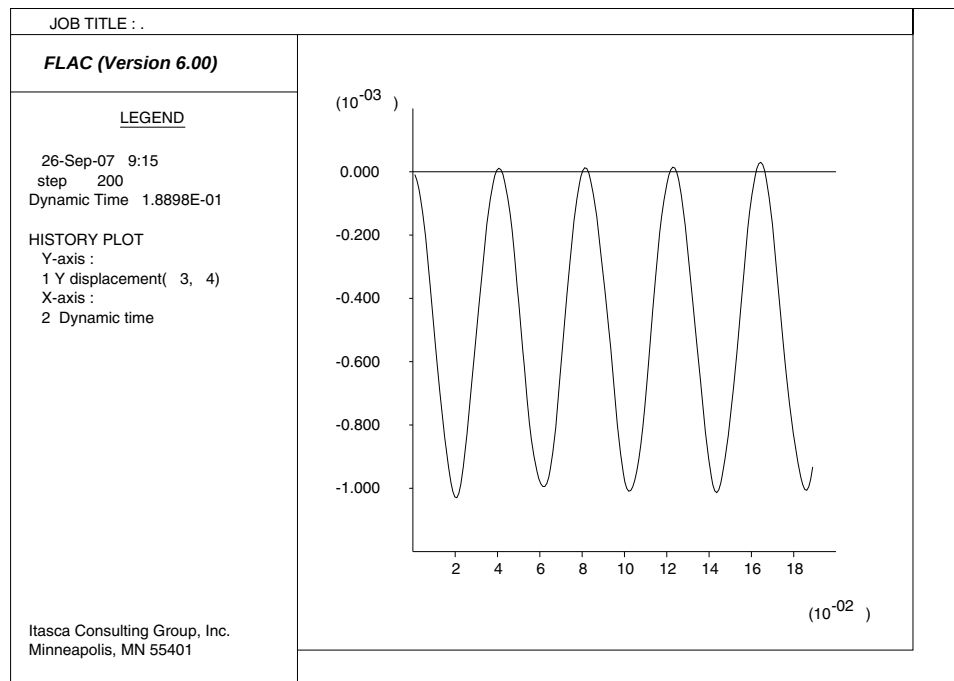


Figure 1.25 *Plot of vertical displacement versus time, for gravity suddenly applied to a square grid (no damping)*

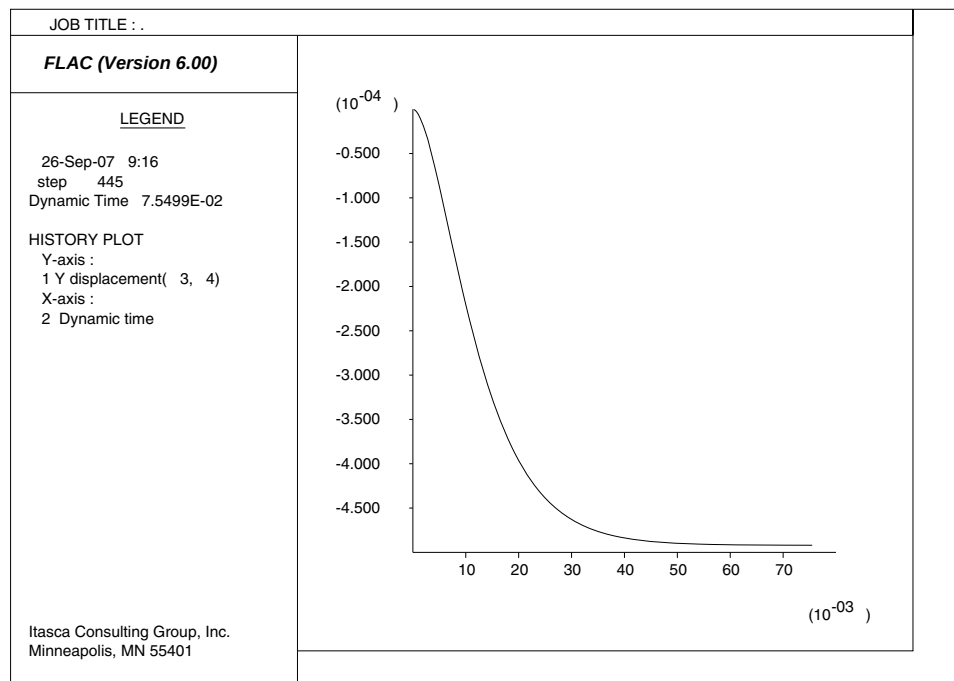


Figure 1.26 *Plot of vertical displacement versus time, for gravity suddenly applied to a square grid (mass and stiffness damping)*

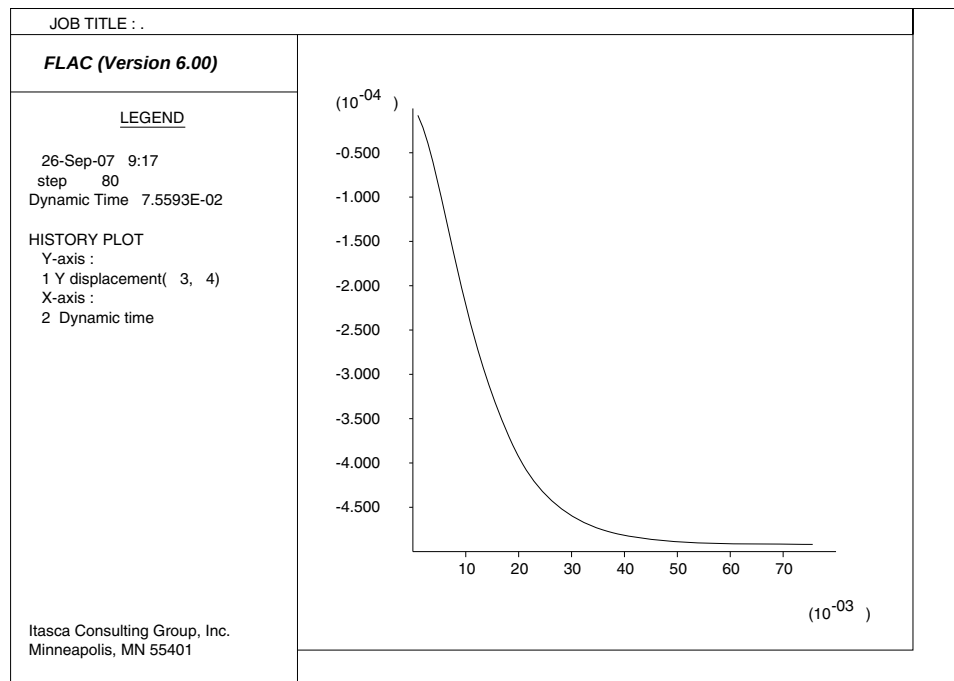


Figure 1.27 *Plot of vertical displacement versus time, for gravity suddenly applied to a square grid (mass damping only)*

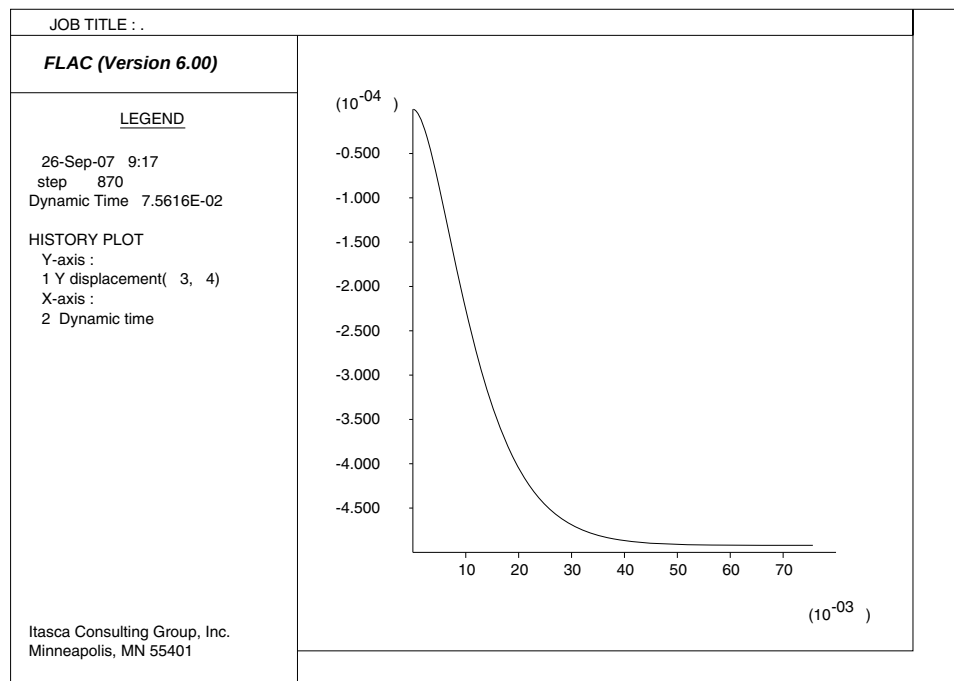


Figure 1.28 *Plot of vertical displacement versus time, for gravity suddenly applied to a square grid (stiffness damping only)*

1.4.3.3 Guidelines for Selecting Rayleigh Damping Parameters

Damping Ratio, $\xi_{\min}$

What is normally attempted in a dynamic analysis is the reproduction of the *frequency-independent* damping of materials at the correct level. For geological materials, damping commonly falls in the range of 2 to 5% of critical; for structural systems, 2 to 10% is representative (Biggs 1964). Also, see Newmark and Hall (1982) for recommended damping values for different materials. In analyses that use one of the plasticity constitutive models (e.g., Mohr-Coulomb), a considerable amount of energy dissipation can occur during plastic flow. Thus, for many dynamic analyses that involve large-strain, only a minimal percentage of damping (e.g., 0.5%) may be required. Further, dissipation will increase with amplitude for stress/strain cycles that involve plastic flow. $\xi_{\min}$ is adjusted to coincide with the correct physical damping ratio.

The equivalent-linear program *SHAKE* (Schnabel, Lysmer and Seed 1972) can be used to estimate material damping to represent the inelastic cyclic behavior of soils. An equivalent-linear analysis is performed for a layered soil deposit using the shear wave speeds and densities for the different layers. Strain-compatible values for the damping ratios and shear-modulus reduction factors are then determined. Average damping ratios and shear modulus reduction factors are estimated for each layer; the damping ratios are the estimates for input in the Rayleigh damping in *FLAC*. [Section 1.6.1](#) illustrates the use of *SHAKE* to estimate material damping parameters.

Center Frequency, $f_{\min}$

Rayleigh damping is frequency-dependent but has a “flat” region that spans about a 3:1 frequency range, as shown in [Figure 1.24](#). For any particular problem, a spectral analysis of typical velocity records might produce a response such as the one shown in [Figure 1.29](#).*

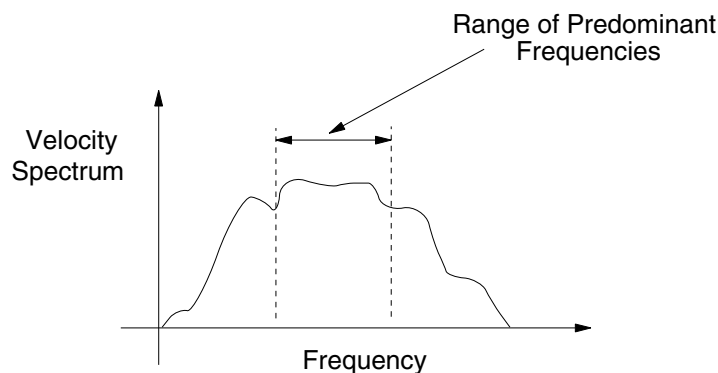


Figure 1.29 Plot of velocity spectrum versus frequency

* A spectral analysis based on a Fast Fourier Transform is supplied as a *FISH* function in the *FISH* library in [Section 3](#) in the *FISH* volume (see “FFT.FIS”). Application of “FFT.FIS” is shown in [Section 1.6.1](#).

If the highest predominant frequency is three times greater than the lowest predominant frequency, then there is a 3:1 span or range that contains most of the dynamic energy in the spectrum. The idea in dynamic analysis is to adjust $f_{\min}$ of the Rayleigh damping so that its 3:1 range coincides with the range of predominant frequencies in the problem. The predominant frequencies are neither the input frequencies nor the natural modes of the system, but a combination of both. The idea is to try to get the right damping for the *important* frequencies in the problem.

For many problems, the important frequencies are related to the natural mode of oscillation of the system. Examples of this type of problem include seismic analysis of surface structures such as dams, or dynamic analysis of underground excavations. The fundamental frequency, f , associated with the natural mode of oscillation of a system is

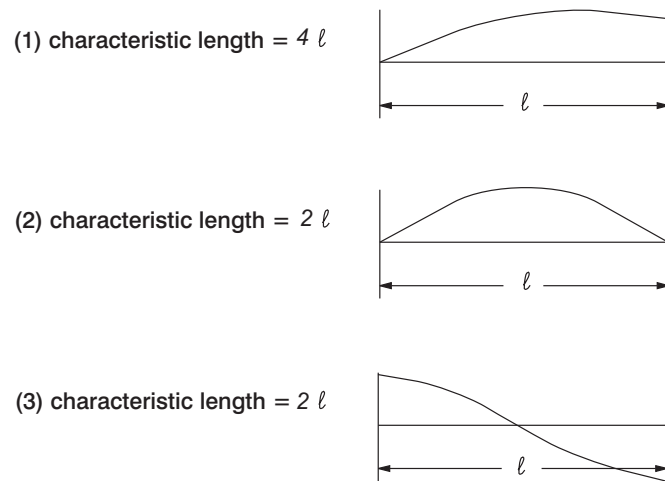
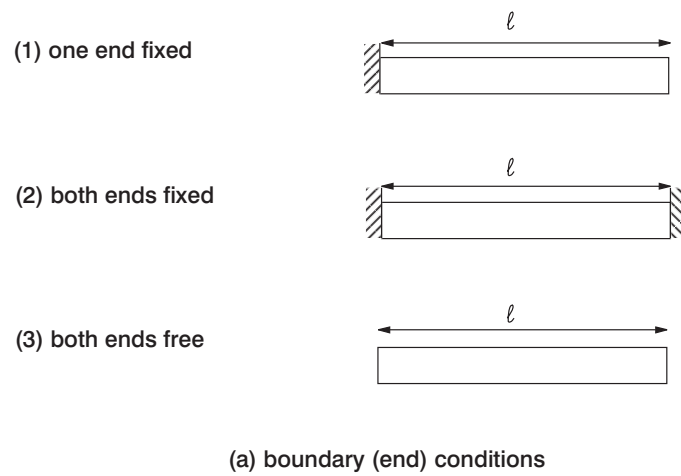
$$f = \frac{C}{\lambda} \quad (1.37)$$

where: C = speed of propagation associated with the mode of oscillation; and

λ = longest wavelength associated with the mode of oscillation.

For a continuous, elastic system (e.g., a one-dimensional elastic bar), the speed of propagation, C_p , for p -waves is given by Eq. (1.16), and for s -waves by Eq. (1.17). If shear motion of the bar gives rise to the lowest natural mode, then C_s is used in the above equation; otherwise, C_p is used if motion parallel to the axis of the bar gives rise to the lowest natural mode.

The longest wavelength (or characteristic length or fundamental wavelength) depends on boundary conditions. Consider a solid bar of unit length with boundary conditions as shown in Figure 1.30(a). The fundamental mode shapes for cases (1), (2) and (3) are as shown in Figure 1.30(b). If a wavelength for the fundamental mode of a particular system cannot be estimated in this way, then a preliminary run may be made with zero damping (for example, see Figure 1.25). A representative natural period may be estimated from time histories of velocity or displacement. Section 1.7.1 contains a simple example in which natural periods are estimated by undamped simulations. Section 1.6.1 describes seismic analyses in which predominant frequencies are estimated based on both the input frequencies and the natural modes of the system.



(b) characteristic lengths or fundamental wavelengths

Figure 1.30 *Comparison of fundamental wavelengths for bars with varying end conditions*

1.4.3.4 Hysteretic Damping

The equivalent-linear method (see [Section 1.2](#)) has been in use for many years to calculate the wave propagation (and response spectra) in soil and rock at sites subjected to seismic excitation. The method does not capture *directly* any nonlinear effects because it assumes linearity during the solution process; strain-dependent modulus and damping functions are only taken into account in an average sense, in order to approximate some *effects* of nonlinearity (damping and material softening). Although fully nonlinear codes such as *FLAC* are capable – in principle – of modeling the correct physics, it has been difficult to convince designers and licensing authorities to accept fully nonlinear simulations. One reason is that the constitutive models available to *FLAC* are either too simple (e.g., an elastic/plastic model, which does not reproduce the continuous yielding seen in soils), or too complicated (e.g., the Wang model [Wang et al. 2001], which needs many parameters and a lengthy calibration process). Further, there is a need to directly accept the same degradation curves used by equivalent-linear methods (see [Figures 1.31](#) and [1.32](#) for examples), to allow engineers to move easily from using these methods to using fully nonlinear methods.

A further motivation for incorporating such cyclic data into a hysteretic damping model for *FLAC* and *FLAC*^{3D} is that the need for additional damping such as Rayleigh damping would be eliminated. Rayleigh damping is unpopular with code users because it often involves a drastic reduction in timestep, and a consequent increase in solution time.

Optional hysteretic damping is described here; it may be used on its own, or in conjunction with the other damping schemes, such as Rayleigh damping or local damping. (It may also be used with any of the built-in constitutive models, except for the transversely isotropic-elastic, modified Cam-clay and creep material models.)

The hysteretic damping formulation is not intended to be a complete constitutive model: it should be used as a supplement to one of the built-in nonlinear models, and not as a primary way to simulate yielding. If models that embody both plastic yield and an adequate hysteretic response are used, then there is no reason to add further damping; the hysteretic damping option in *FLAC* is only intended to provide damping for those models lacking intrinsic damping when not yielding. Further, Rayleigh damping should be unnecessary when hysteretic damping is in operation, apart from its possible use (at low levels: e.g., 0.2%) to remove high frequency noise.

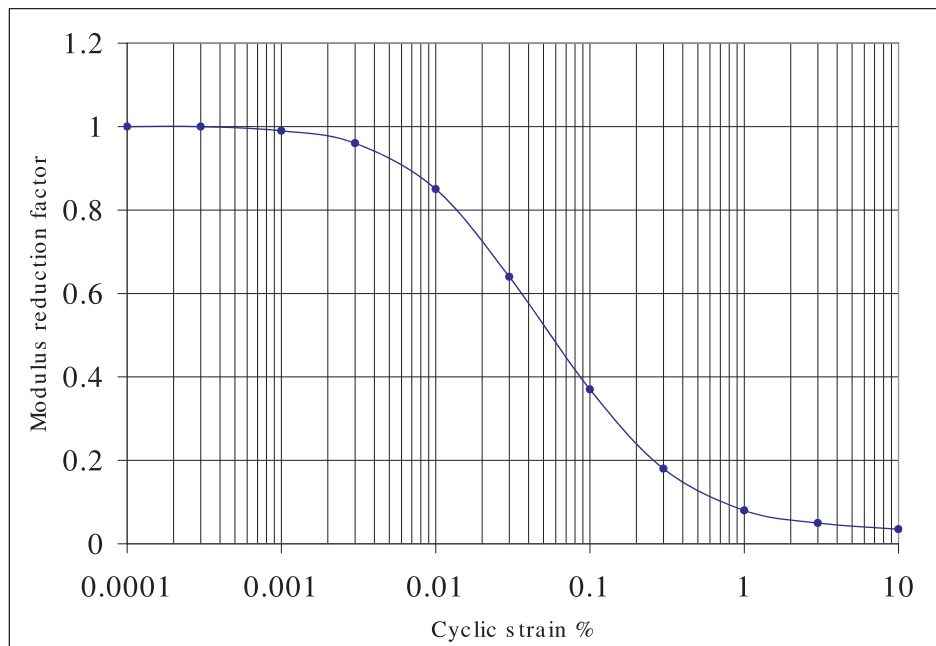


Figure 1.31 *Modulus reduction curve for sand (Seed & Idriss 1970 – “upper range”). The data set is from the file supplied with the SHAKE-91 code download. (<http://nisee.berkeley.edu/software/>)*

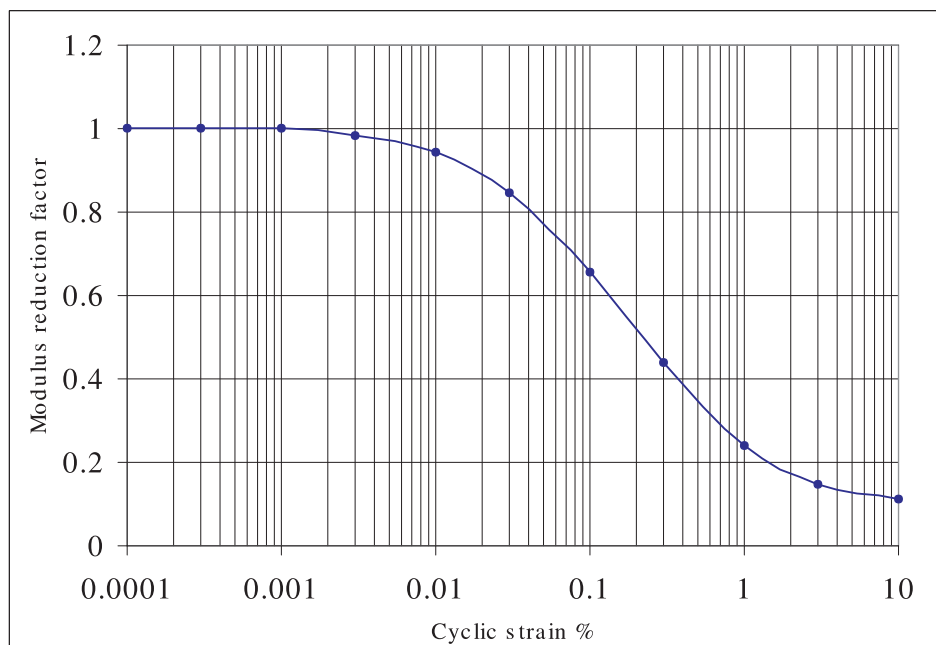


Figure 1.32 *Modulus reduction curve for clay (Seed & Sun 1989 – “upper range”). The data set is from the file supplied with the SHAKE-91 code download. (<http://nisee.berkeley.edu/software/>)*

1.4.3.5 Hysteretic Damping Formulation, Implementation and Calibration

Formulation

Modulus degradation curves, as illustrated in [Figures 1.31](#) and [1.32](#), imply a nonlinear stress/strain curve. If we assume an ideal soil in which the stress depends only on the strain (not on the number of cycles, or time), we can derive an incremental constitutive relation from the degradation curve, described by $\bar{\tau}/\gamma = M_s$, where $\bar{\tau}$ is the normalized shear stress ($= \tau/G_o$) (where G_o is the small-strain shear modulus of the material), γ is the shear strain and M_s is the strain-dependent normalized secant modulus.

$$\bar{\tau} = M_s \gamma \quad (1.38)$$

The tangent modulus, M_t , is then obtained as

$$M_t = \frac{d\bar{\tau}}{d\gamma} = M_s + \gamma \frac{dM_s}{d\gamma} \quad (1.39)$$

where M_t is the normalized tangent modulus. The incremental shear modulus in a nonlinear simulation is then given by $G_o M_t$. This is used in place of the given shear modulus, G_o (i.e., the property assigned with the name **shear**).

In order to handle two- and three-dimensional strain paths, an approach similar to that described for the “Finn model” (e.g., see [Section 1.4.4.2](#)) is used, whereby the shear strain is decomposed into components in strain space, and strain reversals are detected by changes in signs of the dot product of the current increment and the previous mean path. Following the formulation of the Finn model (replacing ε with γ , but otherwise using the same notation):

$$\gamma_1 := \gamma_1 + \Delta e_{11} - \Delta e_{22} \quad (1.40)$$

$$\gamma_2 := \gamma_2 + 2\Delta e_{12} \quad (1.41)$$

$$v_i = \gamma_i^o - \gamma_i^{oo} \quad (1.42)$$

$$z = \sqrt{v_i v_i} \quad (1.43)$$

$$n_i^o = \frac{v_i}{z} \quad (1.44)$$

$$d = (\gamma_i - \gamma_i^o)n_i \quad (1.45)$$

A reversal is detected when $|d|$ passes through a maximum, and the previous-reversal strain values are updated as given by Eqs. (1.46) and (1.47). Note that there is no “latency” period, as used in the Finn model (see Section 1.4.4.2); there is no minimum number of timesteps that must occur before a reversal is detected.

$$\gamma_i^{oo} = \gamma_i^o \quad (1.46)$$

$$\gamma_i^o = \gamma_i \quad (1.47)$$

Between reversals, the shear modulus is multiplied by M_t , using $\gamma = |d|$ in Eq. (1.39). The multiplier is applied to the shear modulus used in all built-in constitutive models, except for the transversely isotropic-elastic, modified Cam-clay and creep material models.

Implementation

The formulation described above is implemented in *FLAC* by modifying the strain-rate calculation so that the mean strain-rate tensor (averaged over all subzones) is calculated before any calls are made to constitutive model functions. At this stage the hysteretic logic is invoked, returning a modulus multiplier which is passed to any called constitutive model. The model then uses the multiplier M_t to adjust the apparent value of tangent shear modulus of the full zone being processed.

In addition to the “backbone curve,” provided by applying Eq. (1.39) to a modulus degradation curve (described below), two Masing (1926) rules are used to specify the behavior at reversal points. Essentially, these state that: (1) a new (but inverted) function is started upon reversal, implying that the initial unload modulus is G ; and (2) the first quarter-cycle of loading is scaled by one-half relative to all other cycles. Although Pyke (2004) concludes that “neither of the Masing rules is valid,” some simplifying assumption is necessary to ensure repeatable, closed loops. The fact that real soil departs from this ideal behavior is not believed to be too important in this context because the formulation is *not intended to be a complete constitutive model*, but simply to provide hysteretic damping as an alternative to Rayleigh damping.

An additional rule deals with sub-cycles: the hysteretic logic contains push-down FILO* stacks that record all state information (e.g., stress, strain, tangent modulus and previous reversal point) at the point of reversal, for both positive- and negative-going strain directions. If the strain level returns to – and exceeds – a previous value recorded in the stack (of the appropriate sign), the state information is popped from the stack, so that the behavior (and, hence, tangent multiplier) reverts back to that which applied at the time before the reversal.

* First In, Last Out

The operation of the various rules is illustrated by the example shown in [Figure 1.33](#). The initial loading is interrupted by a small unload/load cycle; after this there is a complete unloading (extending to negative strain) followed by a loading part that continues to a higher positive strain level than before. In particular, note: the half-scale initial loading curve; the slope of G_o at each reversal; and the restoration of the original loading path after execution of the small loop. In this case, both the positive and negative stacks are popped upon closure of the small loop (i.e., the entire loop is forgotten), but only the information from the positive stack is used to restore state information; the negative-stack information is discarded.

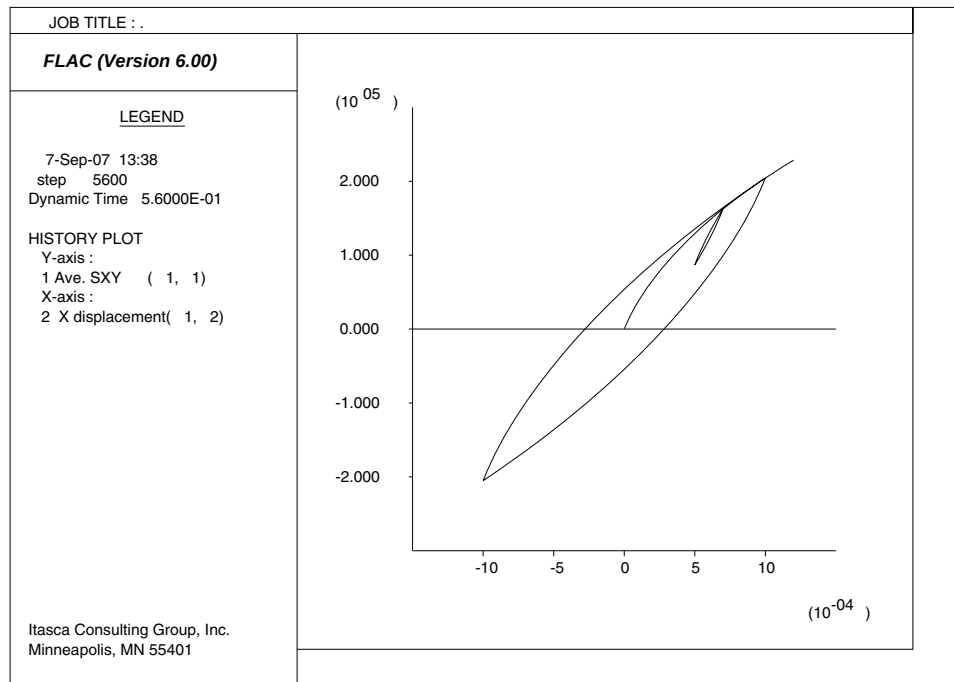


Figure 1.33 Various load/unload cycles, to illustrate rules used in the hysteretic damping formulation ([Example 1.6](#))

Example 1.6 One-zone sample loaded in shear with strain-rate reversal

```

conf dyn
grid 1 1
model elas
prop dens 1000 shear 5e8 bulk 10e8
fix x y
his sxy i 1 j 1
his xdis i=1 j=2
set dydt 1e-4
ini dy_damp hyst default -3.325 0.823
his sxy i 1 j 1

```

```

his xdis i 1 j 2
his nstep 1
ini xvel 1e-2 j=2
cyc 700
ini xvel mul -1
cyc 200
ini xvel mul -1
cyc 500
ini xvel mul -1
cyc 2000
ini xvel mul -1
cyc 2200
save revloop.sav

```

The degradation curves used in earthquake engineering are usually given as tables of values, with cyclic strains spaced logarithmically. Since the derivative of the modulus-reduction curve is required here (i.e., for [Eq. \(1.39\)](#)), the coarse spacing (e.g., 11 points in the curve shown in [Figure 1.31](#)) leads to unacceptable errors if numerical derivatives are calculated. Thus, the implemented hysteretic model uses only continuous functions to represent the modulus-reduction curve, so that analytical derivatives may be calculated. The various implemented functions are described below. If degradation curves are available only in table form, they must be fitted to one of the built-in functional forms before simulations can be performed.

The hysteretic damping feature is invoked with the command

```
initial dy_damp hyst name <v1 v2 v3 ...> <range>
```

Where **name** is the name of the fitting function (chosen from the list: **default**, **sig3**, **sig4** and **hardin** – see below), and **v1**, **v2**, **v3** ... are numerical values for function parameters. The optional range may be any acceptable range phrase for zones. Hysteretic damping may be removed from any range of zones with the command

```
initial dy_damp hyst off <range>
```

Note that the **INITIAL dy_damp hyst** command only applies where the **CONFIG dyn** mode of operation has been selected, and when **SET dyn=on** applies. Hysteretic damping operates independent of all other forms of damping, which may be also specified to operate “in parallel” with hysteretic damping.

Tangent-Modulus Functions

Various built-in functions are available to represent the variation of the shear modulus reduction factor, $G/G_{\max}$, with cyclic strain (given in *percent*), according to the keyword specified on the **INITIAL dy_damp hyst** command.

Default model – default

The default hysteresis model is developed by noting that the S-shaped curve of modulus versus logarithm of cyclic strain can be represented by a cubic equation, with zero slope at both low strain and high strain. Thus, the secant modulus, M_s , is

$$M_s = s^2(3 - 2s) \quad (1.48)$$

where

$$s = \frac{L_2 - L}{L_2 - L_1} \quad (1.49)$$

and L is the logarithmic strain,

$$L = \log_{10}(\gamma) \quad (1.50)$$

The parameters L_1 and L_2 are the extreme values of logarithmic strain (i.e., the values at which the tangent slope becomes zero). Thus, giving $L_1 = -3$ and $L_2 = 1$ means that the S-shaped curve will extend from a lower cyclic strain of 0.001% (10^{-3}) to an upper cyclic strain of 10% (10^1). Since the slopes are zero at these limits, it is not meaningful to operate the damping model with strains outside the limits. (Note that [Eq. \(1.48\)](#) is only assumed to apply for $0 \leq s \leq 1$, and that the tangent modulus will be set to zero otherwise.) The tangent modulus is given by

$$M_t = M_s + \gamma \frac{dM_s}{d\gamma} \quad (1.51)$$

Using the chain rule,

$$\frac{dM_s}{d\gamma} = \frac{dM_s}{ds} \cdot \frac{ds}{dL} \cdot \frac{dL}{d\gamma} \quad (1.52)$$

we obtain

$$M_t = s^2(3 - 2s) - \frac{6s(1 - s)}{L_2 - L_1} \log_{10} e \quad (1.53)$$

There is a further limit, $s > s_{\min}$, such that the tangent modulus is always positive (no strain softening). Thus,

$$s_{\min}^2(3 - 2s_{\min}) = \frac{6s_{\min}(1 - s_{\min})}{L_2 - L_1} \log_{10} e \quad (1.54)$$

or

$$2s_{\min}^2 - s_{\min}(A + 3) + A = 0 \quad (1.55)$$

where $A = 6\log_{10} e / (L_2 - L_1)$. The lowest positive root is

$$s_{\min} = \frac{A + 3 - \sqrt{(A + 3)^2 - 8A}}{4} \quad (1.56)$$

In applying the model, $M_t = 0$ if $s < s_{\min}$.

The default model function can be fit to degradation curves by using a curve-fitting graphing software (e.g., SigmaPlot, <http://www.systat.com/products/sigmaplot/>). For example, the numerical best-fit of the default model to the curves of Figures 1.31 and 1.32 are listed in Tables 1.1 and 1.2, respectively.

Sigmoidal models – sig3, sig4

Sigmoidal curves are monotonic within the defined range, and have the appropriate asymptotic behavior. Thus the functions are well-suited to the purpose of representing modulus degradation curves. The two types of sigmoidal model (3 and 4 parameters, respectively) are defined as follows:

sig3 model

$$M_s = \frac{a}{1 + \exp(-(L - x_o)/b)} \quad (1.57)$$

sig4 model

$$M_s = y_o + \frac{a}{1 + \exp(-(L - x_o)/b)} \quad (1.58)$$

The command line for invoking these models requires that 3 symbols (a , b and x_o) are defined by the parameters $\nu 1$, $\nu 2$, and $\nu 3$, respectively, for model **sig3** (Eq. (1.57)). For model **sig4**, the 4 symbols, a , b , x_o and y_o , are entered by means of the parameters $\nu 1$, $\nu 2$, $\nu 3$ and $\nu 4$, respectively. Numerical fits for the two models to the curve of Figures 1.31 and 1.32 are provided in Tables 1.1 and 1.2, respectively.

Hardin/Drnevich model – hardin

The following function was suggested by Hardin and Drnevich (1972):

$$M_s = \frac{1}{1 + \gamma/\gamma_{\text{ref}}} \quad (1.59)$$

It has the useful property that the modulus reduction factor is 0.5 when $\gamma = \gamma_{\text{ref}}$, so that the sole parameter, γ_{ref} , may be determined (by inspection) from the strain at which the modulus-reduction curve crosses the $G/G_{\text{max}} = 0.5$ line. Choosing a value of $\gamma_{\text{ref}} = 0.06$ produces a match to the curve of Figure 1.31, and a value of 0.234 produces a match to the curve of Figure 1.32.

Table 1.1 Numerical fits to Seed & Idriss data for sand

Data set	Default	Sig3	Sig4	Hardin
Sand – upper range (Seed & Idriss 1970)	$L_1 = -3.325$ $L_2 = 0.823$	$a = 1.014$ $b = -0.4792$ $x_o = -1.249$	$a = 0.9762$ $b = -0.4393$ $x_o = -1.285$ $y_o = 0.03154$	$\gamma_{\text{ref}} = 0.06$

Table 1.2 Numerical fits to Seed & Sun data for clay

Data set	Default	Sig3	Sig4	Hardin
Clay – upper range (Seed & Sun 1989)	$L_1 = -3.156$ $L_2 = 1.904$	$a = 1.017$ $b = -0.587$ $x_o = -0.633$	$a = 0.922$ $b = -0.481$ $x_o = -0.745$ $y_o = 0.0823$	$\gamma_{\text{ref}} = 0.234$

Calibration of Degradation Curves

Calibration of the tangent-modulus function involves both a comparison of the function results to the target shear-modulus reduction curve, and a comparison to the target damping-ratio curve. For example, by using the **sig3** model-fit mentioned above, the data file [Example 1.7](#) was used to exercise a one-zone *FLAC* model at several cyclic strain levels, to develop both a modulus reduction curve and a damping ratio curve.*

The following command was used to invoke hysteretic damping (see **sig3** parameters in [Table 1.1](#)):

```
ini dy_damp hyst sig3 1.014 -0.4792 -1.249
```

The results are summarized in [Figures 1.34](#) and [1.35](#), which present the tangent modulus results and damping ratio results from *FLAC* together with the Seed & Idriss results. Although the modulus results match the target data well over five orders of magnitude ([Figure 1.34](#)), the measured damping does not conform well with the published damping curves for the same material over the same range ([Figure 1.35](#)).

Example 1.7 One-zone sample exercised at several cyclic strain levels (using sig3 model)

```
conf dy
def setup
  givenShear = 1e8
  CycStrain = 0.1 ; (percent cyclic strain)
;---- derived ..
  setVel      = 0.01 * min(1.0,CycStrain/0.1)
  givenBulk   = 2.0 * givenShear
  timestep    = min(1e-4,1e-5 / CycStrain)
  nstep1      = int(0.5 + 1.0 / (timestep * 10.0))
  nstep2      = nstep1 * 2
  nstep3      = nstep1 + nstep2
  nstep5      = nstep1 + 2 * nstep2
end
setup
;
gri 1 1
m e
prop den 1000 sh givenShear bu givenBulk
fix x y
ini xvel setVel j=2
set dydt 1e-4
ini dy_damp hyst sig3 1.014 -0.4792 -1.249
```

* [Examples 1.7](#) and [1.8](#) illustrate the calculations for one cyclic shear-strain value. The data file to develop the complete curve is given in “MODRED.DAT” in the “\Dynamic” directory. For further explanation see “MODRED.FIS” in the *FISH* library in [Section 3](#) in the *FISH* volume.

```

his sxy i 1 j 1
his xdis i 1 j 2
his nstep 1
cyc nstep1
ini xv mul -1
cyc nstep2
ini xv mul -1
cyc nstep2
his write 1 vs 2 tab 1
def HLoop
  emax = 0.0
  emin = 0.0
  tmax = 0.0
  tmin = 0.0
  loop n (1,nstep5)
    emax = max(xtable(1,n),emax)
    emin = min(xtable(1,n),emin)
    tmax = max(ytable(1,n),tmax)
    tmin = min(ytable(1,n),tmin)
  endLoop
  slope = ((tmax - tmin) / (emax - emin)) / givenShear
  oo = out(' strain = '+string(emax*100.0)+'% G/Gmax = '+string(slope))
  Tbase = ytable(1,nstep3)
  Lsum = 0.0
  loop n (nstep1,nstep3-1)
    meanT = (ytable(1,n) + ytable(1,n+1)) / 2.0
    Lsum = Lsum + (xtable(1,n)-xtable(1,n+1)) * (meanT - Tbase)
  endLoop
  Usum = 0.0
  loop n (nstep3,nstep5-1)
    meanT = (ytable(1,n) + ytable(1,n+1)) / 2.0
    Usum = Usum + (xtable(1,n+1)-xtable(1,n)) * (meanT - Tbase)
  endLoop
  Wdiff = Usum - Lsum
  Senergy = 0.5 * xtable(1,nstep1) * yTable(1,nstep1)
  Drat = Wdiff / (Senergy * 4.0 * pi)
  oo = out(' damping ratio = '+string(Drat*100.0)+'%')
end
HLoop
save cyclic.sav

```

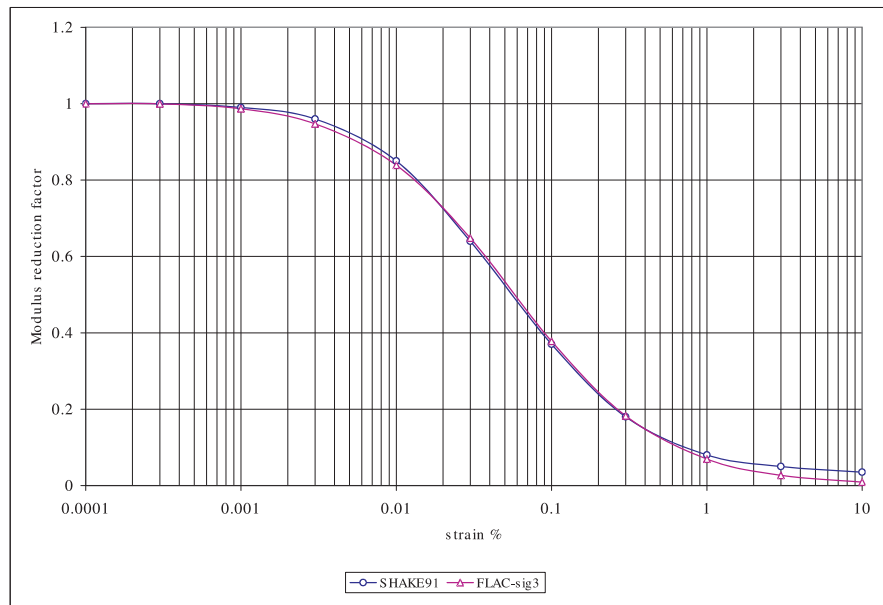


Figure 1.34 Results of several cyclic FLAC simulations for sig3 model—secant modulus values versus cyclic shear strain in %. Seed & Idriss data also shown.

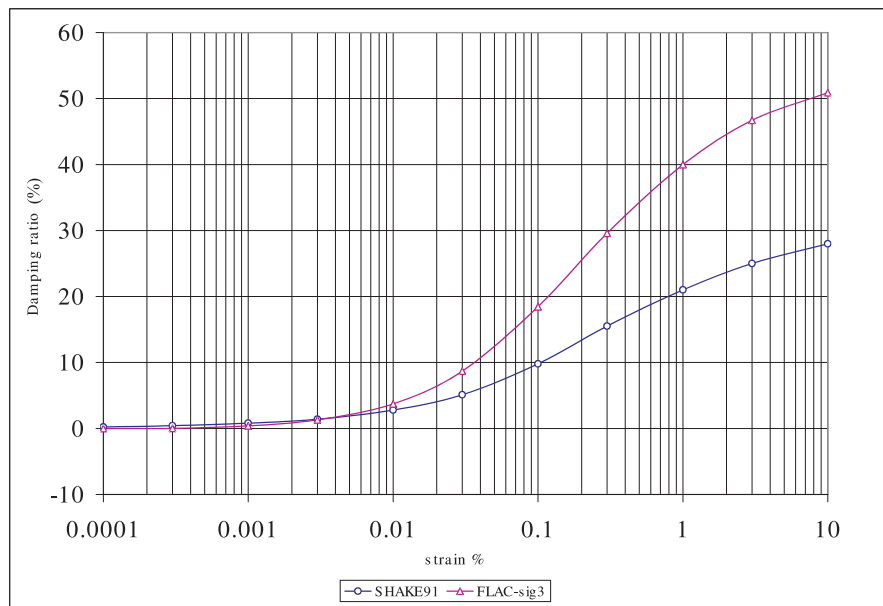


Figure 1.35 Results of several cyclic FLAC simulations for sig3 model—damping values versus cyclic shear strain in %. Seed & Idriss data also shown.

Clearly, the published data for modulus and damping is inconsistent with a conceptual model of strain- and time-independent material response. It is unclear whether the two sets of published data came from different tests, or if the nature of the test led to the inconsistencies. For example, the steady-state response (after many cycles of applied strain) may be different from the initial – single cycle – response. If this is true, then it is not evident that the steady-state response (presumably encompassed by the published results) is a better representation in typical earthquake simulations than the single-cycle response, because many earthquakes contain only one or two large-amplitude cycles. Thus, the single-cycle response may more correctly represent material behavior under earthquake loading. In this case, the damping and modulus curves are consistent.

Note that the numerical results are self-consistent because the two curves (normalized shear modulus and damping ratio) derive from the same basic stress-strain relation. The derivation of incremental stress/strain relations from modulus-reduction curves assumes that the hysteretic mechanism is stationary (i.e., that stress depends only on strain, and not on the number of cycles executed or on past history). In real soils there are often hardening or softening processes that cause successive cycles to be different, which may also explain why the modulus reduction is apparently inconsistent with the corresponding damping curve in a typical lab test.

Although the implemented hysteretic damping ignores these non-stationary effects, it is still possible to match both modulus and damping curves to a reasonable accuracy. In the absence of consistent laboratory data, it is suggested that a compromise approach be taken, in which both the damping and modulus curves are fitted over a reasonable range of strains (corresponding to the strains being modeled). As an example of this strategy, the default model is used (with data file [Example 1.8](#)), giving the *FLAC* results shown in [Figures 1.36](#) and [1.37](#). The hysteretic damping in this case was invoked with the following command (see **default** parameters in [Table 1.1](#)):

```
ini dy_damp hyst default -3.325 0.823
```

The results show that, over a middle range of strain (say, 0.001% to 0.3% strain), there is an approximate fit to both the modulus and damping curves of Seed & Idriss.

Example 1.8 One-zone sample exercised at several cyclic strain levels (using default model) with approximate fit over selected strain range

```
conf dy
def setup
  givenShear = 1e8
end
setup
gri 1 1
m e
prop den 1000 sh givenShear bu 2e8
fix x y
ini xvel 1e-2 j=2
set dydt 1e-4
ini dy_damp hyst default -3.5 1.3
his sxy i 1 j 1
```

```

his xdis i 1 j 2
his nstep 1
cyc 1000
ini xv mul -1
cyc 2000
ini xv mul -1
cyc 2000
his write 1 vs 2 tab 1
def HLoop
    emax = 0.0
    emin = 0.0
    tmax = 0.0
    tmin = 0.0
    loop n (1,5000)
        emax = max(xtable(1,n),emax)
        emin = min(xtable(1,n),emin)
        tmax = max(ytable(1,n),tmax)
        tmin = min(ytable(1,n),tmin)
    endLoop
    slope = ((tmax - tmin) / (emax - emin)) / givenShear
    oo = out(' strain = '+string(emax*100.0)+'% G/Gmax = '+string(slope))
    Tbase = ytable(1,3000)
    Lsum = 0.0
    loop n (1000,2999)
        meanT = (ytable(1,n) + ytable(1,n+1)) / 2.0
        Lsum = Lsum + (xtable(1,n)-xtable(1,n+1)) * (meanT - Tbase)
    endLoop
    Usum = 0.0
    loop n (3000,4999)
        meanT = (ytable(1,n) + ytable(1,n+1)) / 2.0
        Usum = Usum + (xtable(1,n+1)-xtable(1,n)) * (meanT - Tbase)
    endLoop
    Wdiff = Usum - Lsum
    Senergy = 0.5 * xtable(1,1000) * yTable(1,1000)
    Drat = Wdiff / (Senergy * 4.0 * pi)
    oo = out(' damping ratio = '+string(Drat*100.0)+'%')
end
HLoop
save cyclefit.sav

```

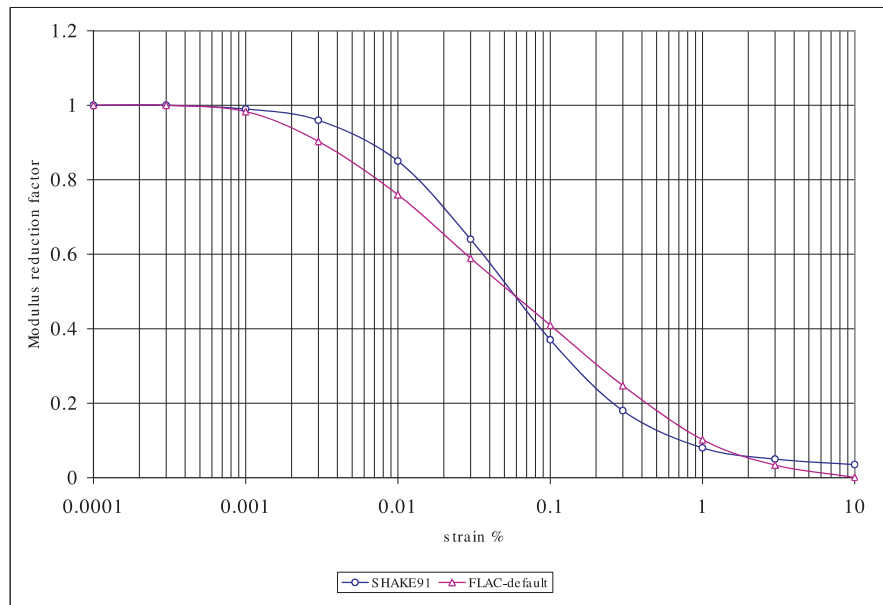


Figure 1.36 Results of several cyclic FLAC simulations for default model – secant modulus values versus cyclic shear strain in %. Seed & Idriss data also shown.

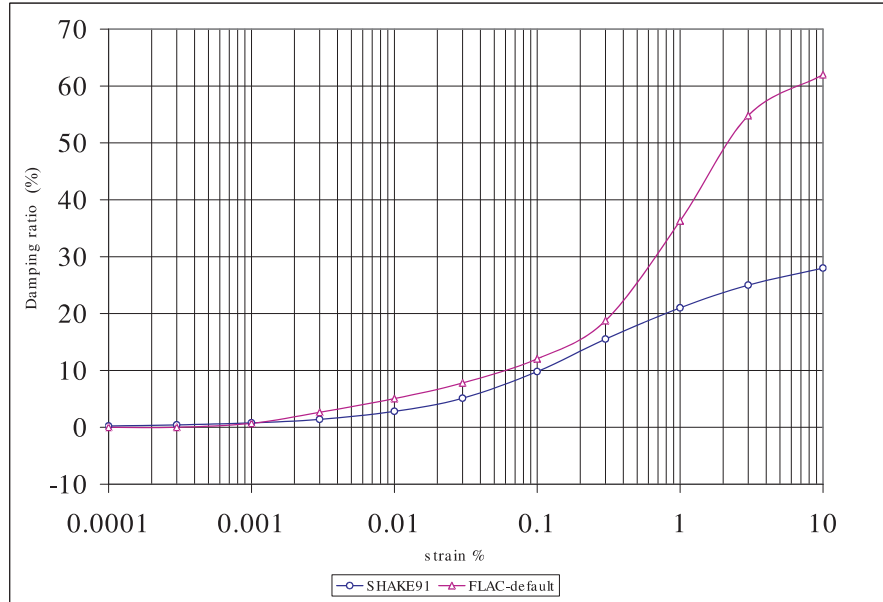


Figure 1.37 Results of several cyclic FLAC simulations for default model – damping values versus cyclic shear strain in %. Seed & Idriss data also shown.

In most attempts to match laboratory and numerical damping curves, it is noted that the damping provided by the hysteretic formulation at low cyclic strain levels is lower than that observed in the laboratory. This may lead to low-level noise, particularly at high frequencies. Although such noise hardly affects the essential response of the systems, for cosmetic reasons it may be removed by adding a small amount of Rayleigh damping. It is found that 0.2% Rayleigh damping (at an appropriate center frequency) is usually sufficient to remove residual oscillations without affecting the solution timestep.

Simple Application

An example of a 20 m layer excited by a digitized earthquake is provided to show that plausible behavior occurs for a case involving wave propagation, multiple and nested loops, and reasonably large cyclic strain. The data file [Example 1.9](#) is listed:

Example 1.9 One-dimensional earthquake excitation of uniform layer

```
new
conf dyn ext 5
grid 1 20
model elas
prop dens 1000 shear 5e8 bulk 10e8
fix y
his read 100 gilroy1.acc
apply xacc -0.02 his 100 yvel 0 j=1
def strain1
    strain1 = xdisp(1,2) - xdisp(1,1)
    strain10 = xdisp(1,11) - xdisp(1,10)
end
his dytime
his sxy i 1 j 1
his strain1
his sxy i 1 j 10
his strain10
his xacc i=1 j=1
his xacc i 1 j 11
his xacc i 1 j 21
ini dy_damp hyst default -3.325 0.823
solve dytime 25
save mdac.sav
```

The digitized earthquake record is described as “LOMA PRIETA GILROY.” The stress/strain loops for the bottom and middle of the layer are shown in [Figures 1.38](#) and [1.39](#), respectively, and the acceleration histories for 3 positions are shown in [Figure 1.40](#). The simulation is in one dimension, for excitation in the shear direction only. Note that for this example, the initial stresses are zero. If a non-zero initial vertical and horizontal stress state is specified, then the left and right boundaries should be attached to produce a one-dimensional simulation.

The hysteretic model seems to handle multiple nested loops in a reasonable manner. There is clearly more energy dissipation at the base of the model than at the middle. The maximum cyclic strain is about 0.15%. The magnitude of timestep is unaffected by the hysteretic damping.

Observations

A method has been developed to use cyclic modulus-degradation data directly in a *FLAC* simulation. The resulting model is able to reproduce the results of constant-amplitude cyclic tests, but it is also able to accommodate strain paths that are arbitrary in strain space and time. Thus, it should be possible to make direct comparisons between calculations made with an equivalent-linear method and a fully nonlinear method, without making any compromises in the choice of constitutive model. The developed method is not designed to be a plausible soil model; rather, its purpose is to allow current users of equivalent-linear methods a painless way to upgrade to a fully nonlinear method. Further, the hysteretic damping of the new formulation will enable users to avoid the use of Rayleigh damping and its unpopular timestep penalties. A comparison of a layered model, assuming nonlinear elastic material using *SHAKE*, to one using *FLAC* with hysteretic damping is provided in [Section 1.7.3](#).

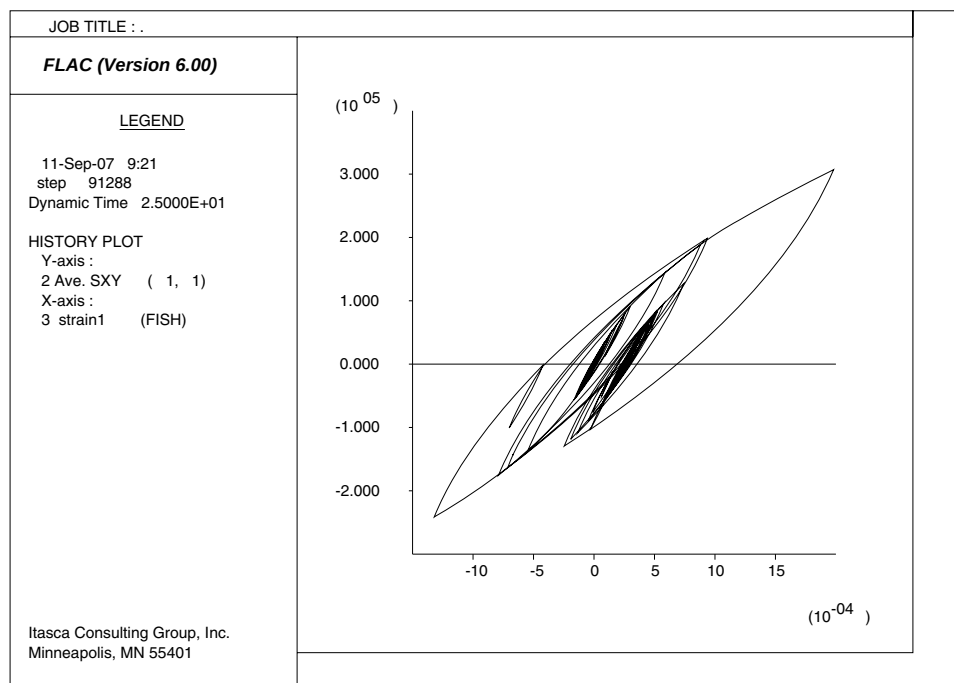


Figure 1.38 Shear stress vs shear strain for base of the layer; default *FLAC* hysteretic model

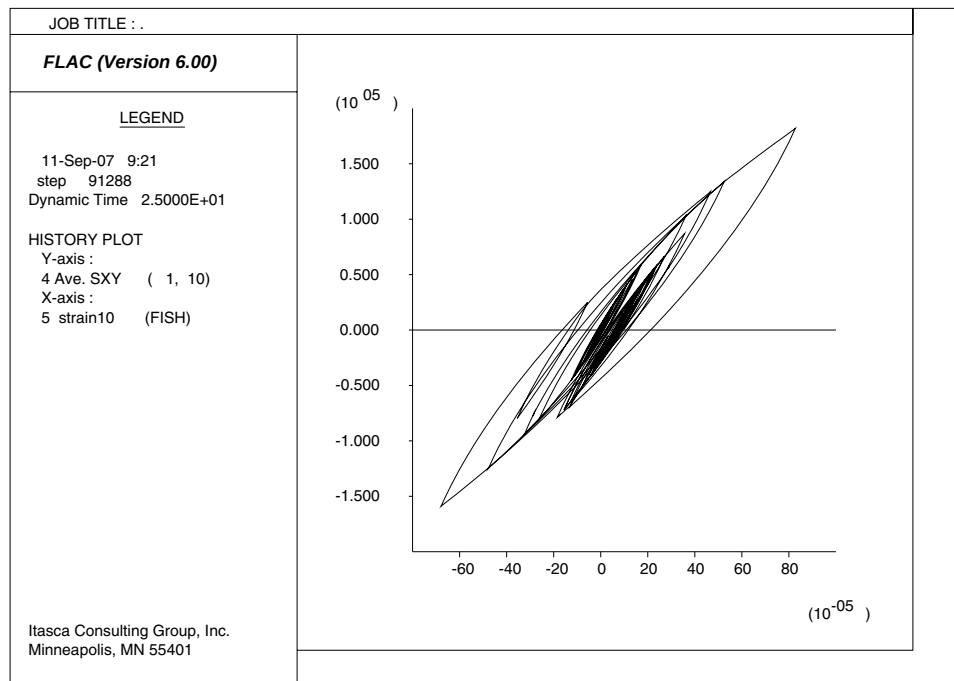


Figure 1.39 *Shear stress vs shear strain for middle of the layer; default FLAC hysteretic model*

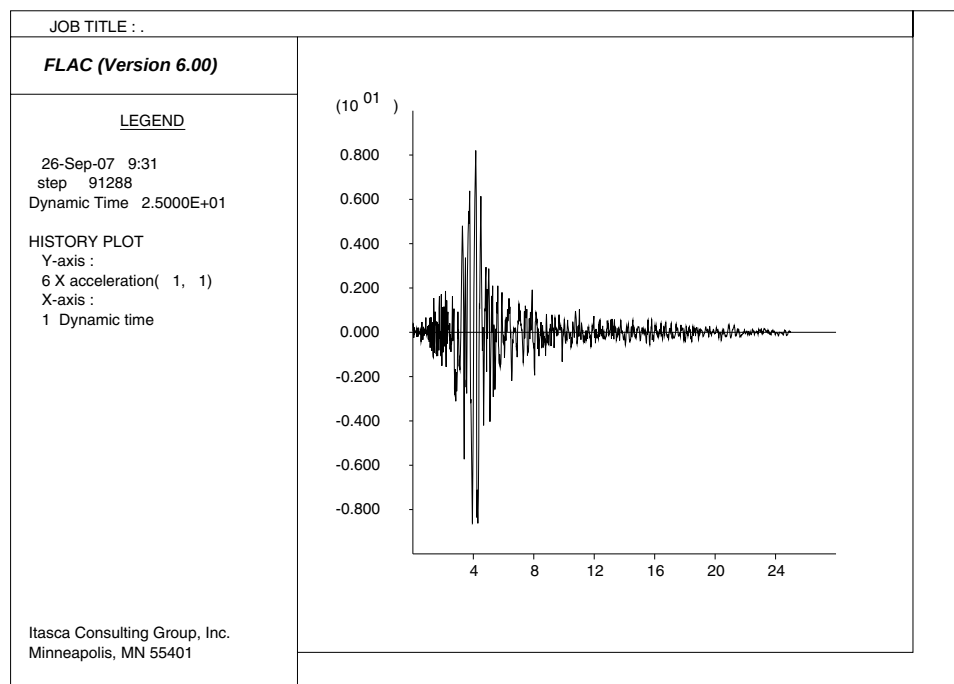


Figure 1.40 *Acceleration history for base of layer vs time (sec)*

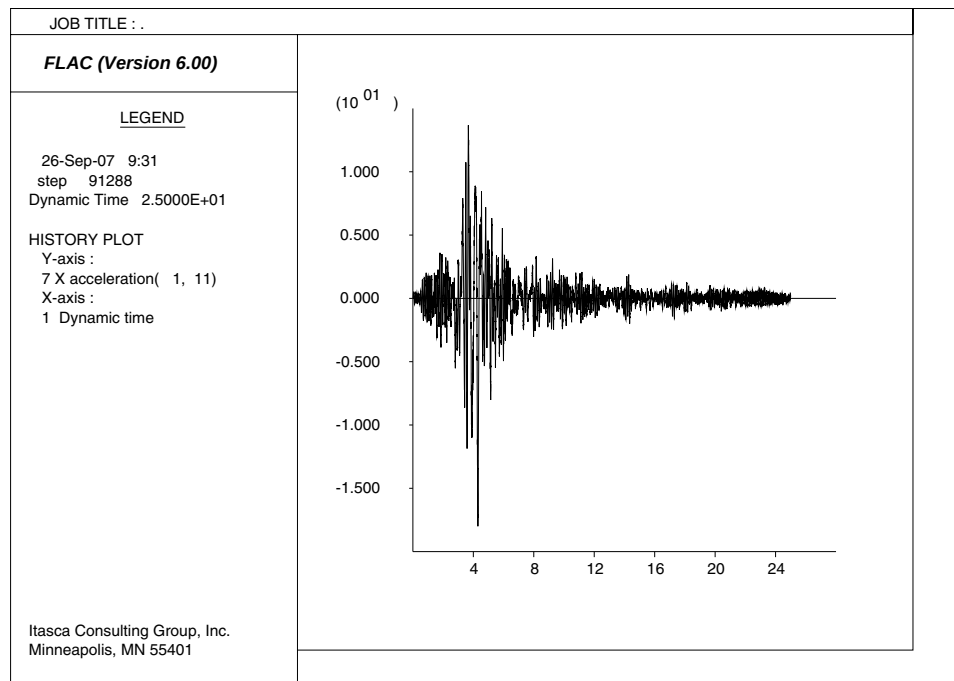


Figure 1.41 Acceleration history for middle of layer vs time (sec)

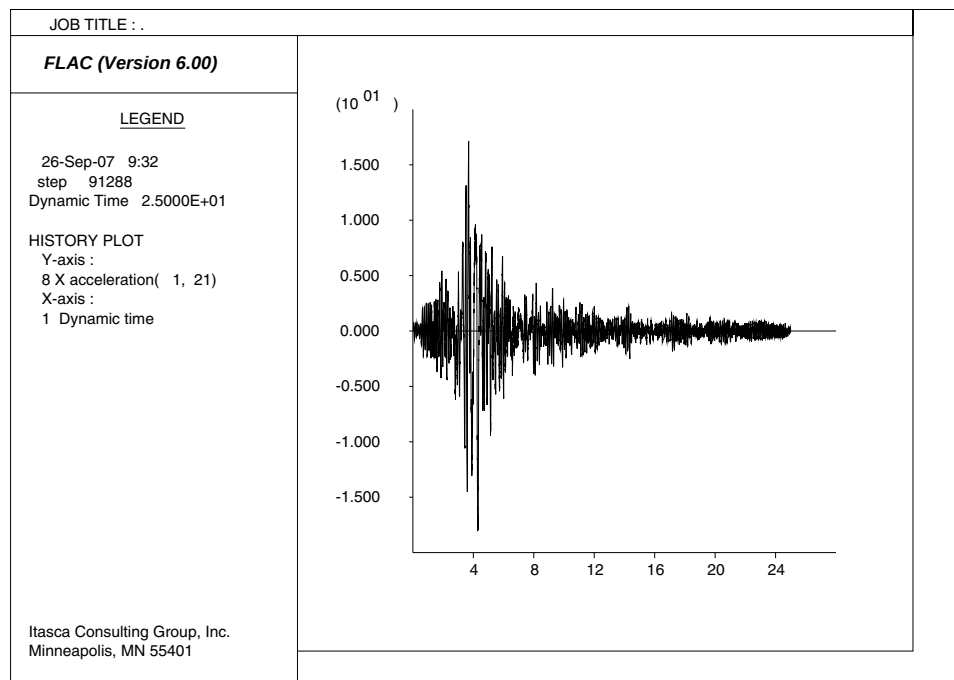


Figure 1.42 Acceleration history for surface of layer vs time (sec)

1.4.3.6 Practical Issues When Using Hysteretic Damping

The following conditions should be checked when hysteretic damping is applied in a dynamic simulation:

Compare hysteretic damping response to laboratory tests – It is generally not useful to compare apparent damping response from the hysteretic damping model used with a yield constitutive model to damping curves from laboratory results, because laboratory tests are likely to be unstable when shear failure occurs, and therefore unable to provide meaningful results. It is recommended that *FLAC*'s hysteretic response be matched to cyclic laboratory results obtained for a strain range that excludes failure, and that plasticity parameters (e.g., cohesion and friction angle) be matched to static laboratory strength tests. *FLAC* is able to combine the response in both regimes during a typical simulation of seismic response.

Check cyclic strain level – Hysteretic damping not only adds energy loss to dynamic straining, it also causes the *mean shear modulus to decrease* for large cyclic strains. This may lead to unexpected results (e.g., an *increased* response amplitude, due to the shift in resonance frequency closer to the dominant frequency of input waves).

Before running a dynamic model with hysteretic damping, an elastic simulation should be made without damping, to observe the maximum levels of cyclic strain that occur. If the cyclic strains are large enough to cause excessive reductions in shear modulus for a given modulus reduction curve, then the use of hysteretic damping is questionable – it will be performing outside of its intended range of application. The model properties and input should be checked. If the properties and input are reasonable, and the large cyclic strains are limited to small regions, then consider the possibility of excluding hysteretic damping from these regions and using only a yield model in these regions (because the large strains imply that yielding should occur).

Check modulus reduction factor – Even if cyclic strains under elastic conditions are small, the use of a yield model may increase the strains. The hysteretic damping formulation is *not intended to be a substitute for a yielding constitutive model*. It may be used in conjunction with a yield model (such as Mohr-Coulomb), but conflicts in the domain of application should be avoided, if meaningful results are to be expected.

Hysteretic damping is “switched off” for each zone while plastic flow is occurring (including the associated strain accumulation for the hysteretic damping calculation). Note that the stiffness-proportional component of Rayleigh damping is also switched off during plastic flow.

Even so, the apparent modulus used by the hysteretic damping logic may drop to low values, which can cause unrealistic response. Low (or zero) values of the modulus can be checked by monitoring **modfac**, which is the factor by which the small-strain modulus, G_o , is multiplied for hysteretic damping. (For example, use **HISTORY hyst modfac** for selected zones, or plot contours of **modfac** at critical times.)

In some simulations, large modulus reductions have been noticed in areas remote from regions of plastic flow (e.g., near the base of the model). It appears that the use of a single modulus-reduction curve is unrealistic in these cases. There is evidence (e.g., see Darendeli 2001) that degradation curves depend on the mean stress level: for example, at depth (high mean stress) there

is less damping and modulus reduction. By making the hysteretic damping depth-dependent, the simulation should be more realistic.

Check initial shear stress state – In laboratory tests, the initial shear stress is assumed to be zero, leading to hysteresis loops that are generally symmetrical. In practical applications, the initial shear stress is unlikely to be zero (for example, within soil elements near the surface of an embankment). The user of hysteretic damping must decide on the best estimate of the initial state of the material, because the hysteretic formulation in *FLAC* depends on the past history of shear strain. Two cases can be identified:

1. If hysteretic damping is activated *after* a set of equilibrium stresses has been installed, then the initial shear strain will be zero, and cyclic excursions of shear stress will tend to be symmetrical about the starting point.
2. If the initial stresses are built up by straining the model while hysteretic damping is active, then subsequent cyclic excursions of shear stress will tend to be asymmetrical because the initial bias in strain causes the slope of the stress/strain curve to be flatter on the side with higher stress.

These cases are illustrated by modifying the simple application given in [Example 1.9](#). With no initial shear stress, the cyclic response of the model is nearly symmetrical, as shown in [Figures 1.38](#) and [1.39](#).

The simulation is repeated with uniform shear stresses of 0.1 MPa initialized in the 20 m layer, and with an equal static shear stress applied at the boundary to maintain initial equilibrium. The following commands are added to [Example 1.9](#) after the **FIX y** command:

```
initial sxy 1e5
apply sxy 1e5 j 21
apply sxy 1e5 j 1
solve ;; to check equilibrium
```

The resulting dynamic response is identical to that of the original simulation, but the set of loops is shifted upward by 0.1 MPa. [Figure 1.43](#) shows the result at the base of the model; compare to [Figure 1.38](#). This corresponds to Case 1 above.

In order to make the initial strains compatible with the initial stresses, hysteretic damping is arranged to be active during the establishment of the initial stress state. For this case, the following commands are added to [Example 1.9](#) after the **FIX y** command:

```
apply sxy 1e5 j 21
apply sxy 1e5 j 1
ini dy_damp hyst default -3.325 0.823
ini dy_damp local 0.7
solve
initial xvel 0 yvel 0
set dytime 0
ini dy_damp local 0.0
```

Shear stresses are applied to the boundaries of the grid, but not initialized within the grid; thus, a static solution (using additional local damping with the hysteretic damping to speed convergence) is used to build up internal stresses. Because hysteretic damping is active during the static solution, strains will be compatible with stresses at the start of the dynamic simulation. The dynamic response of the system is indicated by the stress/strain loops plotted in [Figure 1.44](#). In this case there is a marked asymmetry. The soil is already partially yielded (as a result of the initial stress/strain state). Further straining in the same direction of loading produces more yielding, while straining in the opposite direction initially acts to reduce the yielding. This corresponds to Case 2 above.

The approach of using a full static solution while hysteretic damping is active, in order to obtain a compatible starting state for both stress and strain, may be used for more complicated models, such as embankments in which the slope area contains initial shear stresses. One drawback of the approach is that the static solution may involve many diminishing cycles of oscillation as the state of equilibrium is approached. Although these cycles tend to be quite small (and hardly affect the desired stress state), they cause many states to be stored on the memory stack (see [Section 1.4.3.3](#)). These stored states are deleted from the stack early in dynamic loading, but they occupy memory, and entail some initial overhead in computer time. It may be possible (in a future version of *FLAC*) to include logic to allow users to flush the stacks at the end of the static initialization, while retaining the latest state of stress-compatible strains for a dynamic simulation with hysteretic damping.

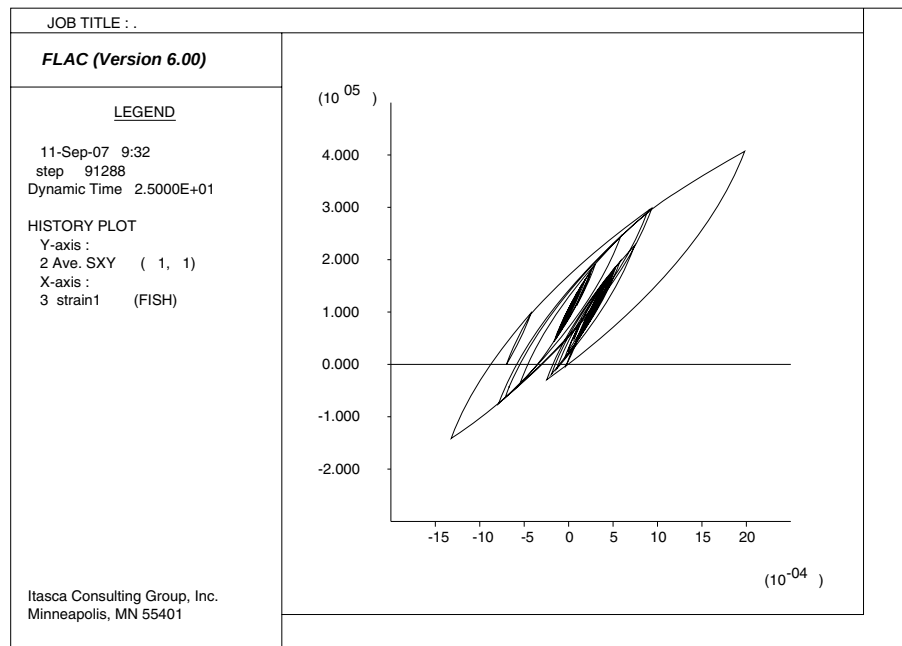


Figure 1.43 *Shear stress vs shear strain for base of the layer; with shear stress simply initialized to 0.1 MPa*

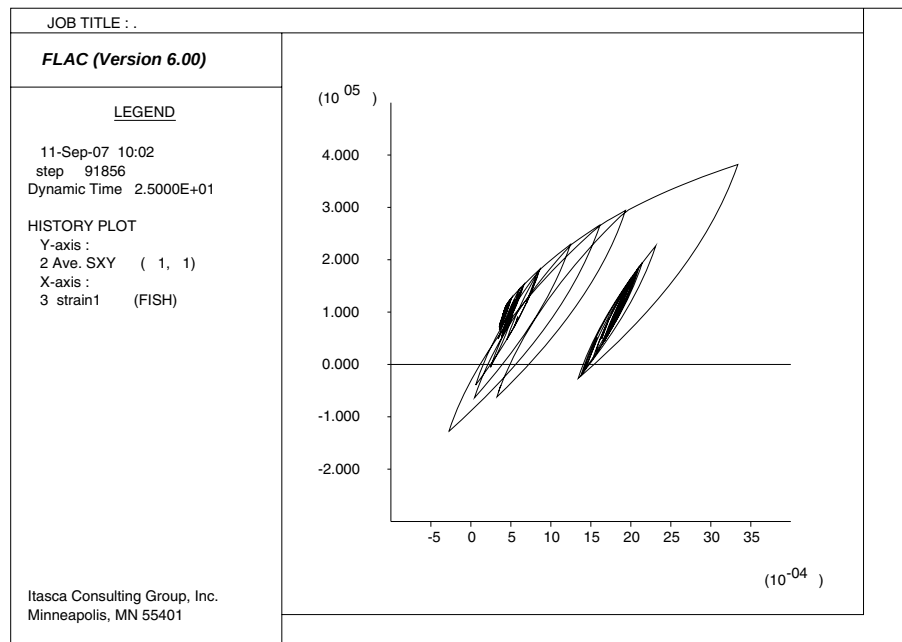


Figure 1.44 *Shear stress vs shear strain for base of the layer. The shear stress is 0.1 MPa and the initial strain is 0.041%, following the static solution.*

1.4.3.7 Local Damping for Dynamic Simulations

Local damping (see [Section 1.3.4](#) in **Theory and Background**) was originally designed as a means to equilibrate static simulations. However, it has some characteristics that make it attractive for dynamic simulations. It operates by adding or subtracting mass from a gridpoint or structural node at certain times during a cycle of oscillation; there is overall conservation of mass, because the amount added is equal to the amount subtracted. Mass is added when the velocity changes sign, and subtracted when it passes a maximum or minimum point. Hence, increments of kinetic energy are removed twice per oscillation cycle (at the velocity extremes). The amount of energy removed, ΔW , is proportional to the maximum, transient strain energy, W , and the ratio $\Delta W/W$ is independent of rate and frequency. Since $\Delta W/W$ may be related to fraction of critical damping, D (Kolsky 1963), we obtain the expression

$$\alpha_L = \pi D \quad (1.60)$$

where α_L is the local damping coefficient. Thus, the use of local damping is simpler than Rayleigh damping, because we do not need to specify a frequency. To compare the two types of damping, we repeat [Example 1.5](#) with 5% damping, which is a typical value used for dynamic analyses. [Example 1.10](#) provides the data file; we also set $f_{\min}$ to 24.1, which is a more accurate estimate of the natural frequency of the block. A similar run is done with local damping, with the coefficient set to $0.1571 (= 0.05\pi)$ – see [Example 1.11](#). In both runs, we specify the timestep at 5×10^{-4} , so that we can execute the same number of steps in each to obtain the same elapsed time. Displacement histories from the two runs are given in [Figures 1.45](#) and [1.46](#), respectively. The results are quite similar.

Example 1.10 Continuation of [Example 1.5](#) with 5% Rayleigh damping

```
conf dy
gr 3 3
m e
prop den 1000 bu 1e8 sh .3e8
fix y j=1
set grav 10.0
hist n 1
hist ydisp i=3 j=4
hist dytime
save damp.sav
set dydt=5e-4
set dy_damp=rayleigh 0.05 24.1
step 1000
save dyn_ray.sav
```

Example 1.11 Continuation of [Example 1.5](#) with 5% local damping

```

conf dy
gr 3 3
m e
prop den 1000 bu 1e8 sh .3e8
fix y j=1
set grav 10.0
hist n 1
hist ydisp i=3 j=4
hist dytime
save damp.sav
set dydt=5e-4
set dy_damp=local 0.1571 ; = pi * 0.05
step 1000
save dyn_loc.sav

```

A modified form of local damping – *combined damping* – may also be used in dynamic mode, but its performance is unknown. The formulation for combined damping is given in [Section 1.3.4](#) in **Theory and Background**, and the command to invoke it is **SET dy_damp combined value**.

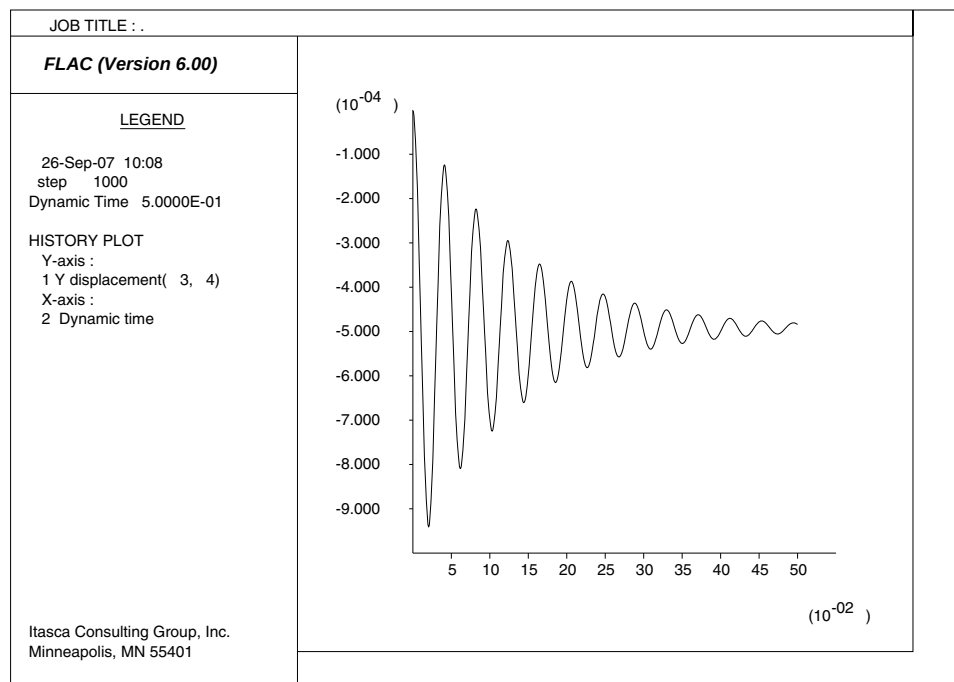


Figure 1.45 Displacement history – 5% Rayleigh damping

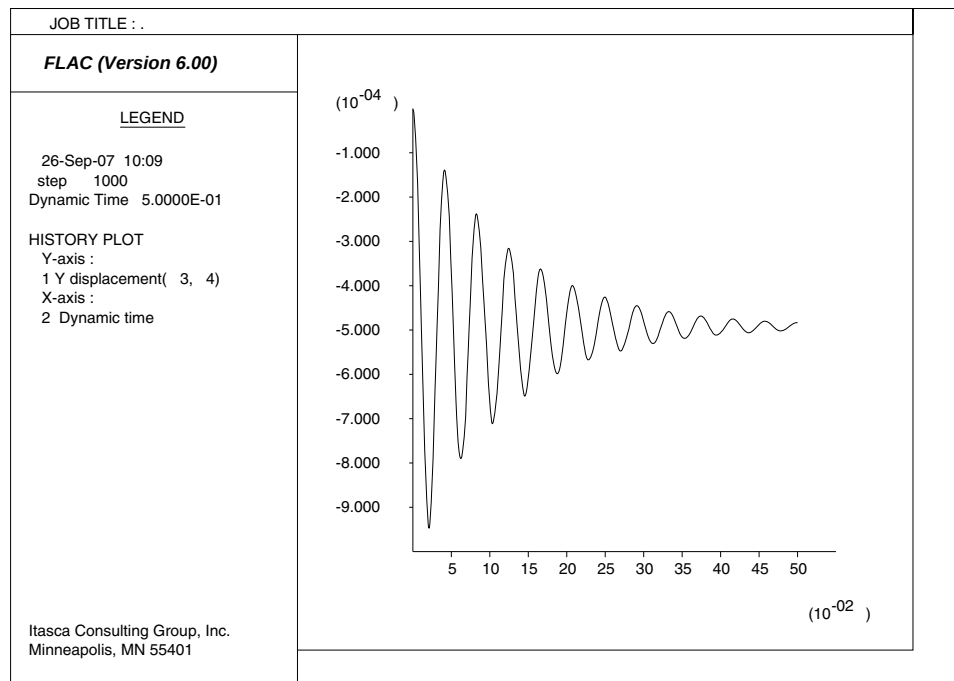


Figure 1.46 Displacement history – 5% local damping

CAUTION: Local damping appears to give good results for a simple case because it is frequency-independent and needs no estimate of the natural frequency of the system being modeled. However, this type of damping should be treated with caution, and the results compared to those with Rayleigh damping for each application. There is some evidence to suggest that, for complicated wave forms, local damping underdamps the high frequency components, and may introduce high frequency “noise.”

Local damping is *not* recommended for seismic simulations, because this type of damping cannot completely represent the energy loss of multiple cyclic loading properly.

1.4.3.8 Spatial Variation in Damping

Rayleigh damping and local damping are both assigned as global parameters by the **SET** command in *FLAC*. A spatial variation in the damping parameters (and the damping type) for Rayleigh, local and hysteretic damping can be prescribed via the **INITIAL dy_damp** command. For example, if different materials are known to have different fractions of critical damping, a different value for $\xi_{\min}$ can be assigned to each material. This can be demonstrated by modifying the example of a wave propagating in a column (Example 1.2). In Example 1.12, two separate identical grids are constructed, to enable a direct comparison to be made. Both grids contain two layers: a stiff layer in the lower half, and a soft layer in the upper half. The left-hand grid has uniform Rayleigh stiffness damping, while the right-hand grid has two values for the damping coefficient, corresponding to the two materials, although the *average* damping coefficient is the same as that of the left-hand grid.

The velocity histories at the free surface are plotted in [Figure 1.47](#) for both grids. Differences in response can be observed particularly in the second pulse (reflected from the material discontinuity).

Example 1.12 Spatial variation in damping

```

config dyn ext=5
grid 3,50
mod elas i=1 ; Create 2 grids, for comparison
mod elas i=3
prop dens 2500 bulk 2e7 shear 1e7 j=1,25      ; Two layers in
prop dens 2000 bulk 0.5e7 shear 0.25e7 j=26,50 ; each grid
def wave
  if dytime > 1.0/freq
    wave = 0.0
  else
    wave = (1.0 - cos(2.0*pi*freq*dytime)) / 2.0
  endif
end
set freq=2.0 ncw=50
ini dy_damp=rayl .1 freq stiff i=1,2          ; Uniform .. l.h. grid
ini dy_damp=rayl .02 freq stiff i=3,4 j=1,26   ; Non-uniform ..
ini dy_damp=rayl .18 freq stiff i=3,4 j=27,51 ; r.h. grid
fix y
apply xquiet j=1 i=1,2
apply xquiet j=1 i=3,4
apply sxy=-2e5 hist wave j=1 i=1,2
apply sxy=-2e5 hist wave j=1 i=3,4
hist xvel i=1 j=1 ; l.h. grid
hist xvel i=1 j=51
hist xvel i=3 j=1 ; r.h. grid
hist xvel i=3 j=51
hist dytime
solve dytime=3.5
save dyn_spac.sav

```

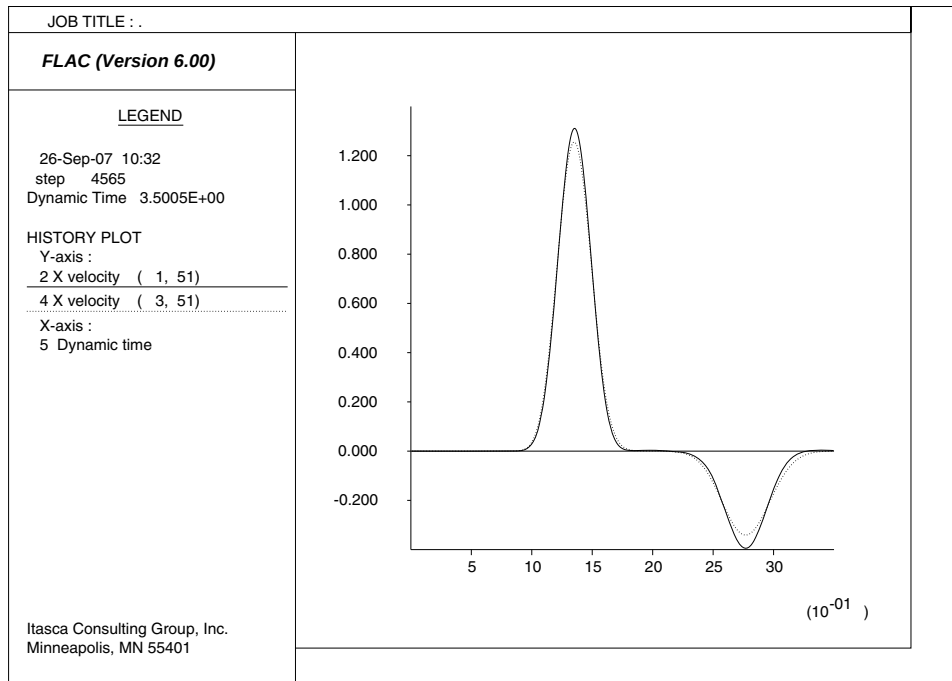


Figure 1.47 Velocity histories at a free surface for spatial variation in damping

The specification of nonuniform damping with the **INITIAL** command follows the syntax of both the **SET dy_damp** command and the **INITIAL** command. For example, variations, additions and multipliers can be prescribed for all parameters. In its simplest form, the **INITIAL dy_damp** command resembles the **SET dy_damp** command (e.g., the following two commands produce identical results):

```
set dy_damp rayl 0.05 25.0
ini dy_damp rayl 0.05 25.0
```

Note that a **SET dy_damp** command implicitly sets damping for all grid elements (and overrides any previous **INITIAL dy_damp** specifications). By using range parameters, several **INITIAL dy_damp** commands can be used to install different damping values (and even different damping types) in various locations. The **var** keyword can also be used. For example, we can modify the previous example of uniform damping:

```
ini dy_damp rayl 0.05 var 0.1,0.2 25.0 var -5,0
```

In this case, there are spatial variations in both the damping coefficient and the center frequency. The syntax follows the general rule for the **INITIAL** command, in that any parameter value may be followed by the keyword **var**, **add** or **mul**, with appropriate parameters for those keywords. Note that all damping parameters pertain to *gridpoints*. In particular, the Rayleigh stiffness-proportional term, which acts on *zone* strain rates, is derived by averaging, from values specified at the neighboring *gridpoints*.

The command **PRINT dy_damp** produces a normal grid printout, consisting of two or more blocks of data: the first block denotes the damping type (L, C or R, for local, combined or Rayleigh, respectively, with modifiers *m* and *s* for *mass* and *stiffness*); and the second block records the damping coefficient. In the case of Rayleigh damping, there is a third block of output that records the center frequency.

There is no direct plot of damping information, but the *FISH* grid intrinsic **damp** can be used to transfer appropriate data to the extra arrays for plotting. See [Section 2](#) in the *FISH* volume for a description of the intrinsic **damp**.

If damping parameters are modified with the *FISH* intrinsic **damp**, the change will not necessarily take effect immediately, because the code uses derived coefficients. In small-strain mode, derived coefficients are computed from user-given parameters when a **CYCLE** or **STEP** command is given; in large-strain mode, the derivation is done every 10 steps. A user-written *FISH* function may force the derived coefficients to be computed by executing the intrinsic **do_update**. Note that the timestep may change as a result (if the Rayleigh stiffness term is changed).

1.4.3.9 Structural Element Damping for Dynamic Simulations

Rayleigh or local damping can also be specified independently for structural elements by giving the **struct** keyword immediately following **SET dy_damp**. Damping is then applied specifically for all structural elements in the model. See, for example, [Example 1.17](#).

Structural damping operates in a way similar to damping in the grid. However, if a structural node is rigidly attached to a gridpoint, the gridpoint damping value is used rather than the structural node damping value. For the special case of a structural node attached to a null gridpoint (one surrounded by null zones), the damping for that gridpoint/node is zero.

Note that stiffness damping is included by default for pile coupling springs. This damping can be turned off by using the **SET dy_damp pile_sd off** command.

1.4.3.10 Artificial Viscosity

Von Neumann and Landshoff artificial viscosity terms are implemented in *FLAC* to control damping involving sharp fronts in dynamic analysis. These viscous damping terms are a generalization of the one-dimensional equations (1) and (3) in Wilkins (1980), and correspond to the original viscosity formulation of von Neumann and Richtmyer (see Wilkins 1980).

The artificial viscosity method was initially developed for numerical calculation of shock propagation in fluid dynamics. The method may not apply to elastic or plastic waves when shear stress components are significant when compared to mean pressure, because shear waves are not damped by the method. The purpose of the quadratic von Neumann term q_1 is to spread the shock over a number of grid spacings, and damp the oscillations behind the front. The effect of the linear Landshoff term q_2 is to diffuse the shock front over an increased number of zones as the shock progresses.

In the *FLAC* implementation, a linear combination, q , of the scalar viscosity terms q_1 and q_2 is used on a zone basis:

$$q = a_n q_1 + a_l q_2 \quad (1.61)$$

where a_n and a_l are two constants. The viscous terms have the form

$$q_1 = b c_0^2 \rho L^2 \dot{\epsilon}^2 \quad (1.62)$$

$$q_2 = b c_1 \rho L a \dot{\epsilon} \quad (1.63)$$

where: L is a characteristic zone dimension (square root of the zone area);

$\dot{\epsilon}$ is the zone volumetric rate;

ρ is the zone density;

a is the material p -wave speed: $a = \sqrt{\frac{(K + \frac{4}{3}G)}{\rho}}$

where K and G are bulk and shear moduli for the zone;

c_0 is a constant set = 2; and

c_1 is a constant set = 1.

and, to accommodate both compressive and dilatant shocks, we specify

$$b = -\text{sgn}(\dot{\epsilon})$$

The isotropic viscous stress contribution is added to the out-of-balance force for the nodes before resolution of the equations of motion.

The following command is provided to activate artificial damping for a *FLAC* model:

SET dy_damp avisc a_n a_l

where a_n and a_l are the two constants defined above, which should, in most instances, be assigned the value of 1.

Note that the presence of damping terms results in a slightly more stringent stability condition that has not been taken into consideration in the implementation. Hence, in some cases, it may be necessary to reduce the timestep to achieve satisfactory stability.

The data file in [Example 1.13](#) corresponds to a model with a sharp velocity wave (of the form shown in [Figure 1.48](#)) applied to the left boundary. The data file is run in both plane-strain and axisymmetry mode using the artificial viscosity model. (Replace the **CONFIG dyn** command with **CONFIG dyn axi** for the axisymmetry analysis.) The effect on wave transmission through the grid is illustrated by the x -velocity plots in [Figure 1.49](#) for the plane-strain model without artificial viscosity (**SET dy_damp avisc** command removed), compared to [Figure 1.50](#) for the model with artificial viscosity. [Figure 1.51](#) shows the results for the axisymmetry model with artificial viscosity.

Note that an alternative form of damping for shock waves is described in [Section 1.7.7](#). In this case, the scheme is coded in *FISH* (function **leak** in [Example 1.45](#)).

Example 1.13 Velocity wave with sharp front – artificial viscosity

```

config dyn
grid 150 300
model e
gen 0      0      0      20      0.019 20      0.019 0      i=1,2      j=1,301
gen 0.019 0      0.019 20      10      20      10      0      i=2,151 j=1,301
model null i=1      ; P-wave boundary
prop d 2.8 b 58.5e6 sh 34.3e6
apply nq i=151
apply sq i=151
fix y j=1
;--- do dynamic analysis ---
def wave
  wave = exp(-0.1842e06*(dytime-430.e-06))
  if dytime<430.e-06 then
    wave = 1.0
  end_if
  if dytime<1.0e-6 then
    wave = 1.0e-6 * dytime
  end_if
end
apply xvel=1.0 ,hist=wave i=2
hist nstep 1
hist dytime
hist wave
hist xvel i=2,j=150
hist xvel i=10,j=150
hist xvel i=20,j=150
hist xvel i=30,j=150
hist xvel i=50,j=150
hist xvel i=2,j=50
hist xvel i=10,j=50
hist xvel i=20,j=50
hist xvel i=30,j=50
hist xvel i=50,j=50
set large
save avisc_ini_ps.sav
solve dytime 10.0e-4
save avisc_ps.sav
restore avisc_ini_ps.sav
set dy_damp avisc 1 1
solve dytime 10.0e-4
save avisc_ps_damp.sav
new

```

```
config dyn axi
grid 150 300
model e
gen 0      0      0      20      0.019 20      0.019 0      i=1,2      j=1,301
gen 0.019 0      0.019 20      10      20      10      0      i=2,151 j=1,301
model null i=1      ; P-wave boundary
prop d 2.8 b 58.5e6 sh 34.3e6
apply nq i=151
apply sq i=151
fix y j=1
;--- do dynamic analysis ---
def wave
  wave = exp(-0.1842e06*(dytime-430.e-06))
  if dytime<430.e-06 then
    wave = 1.0
  end_if
  if dytime<1.0e-6 then
    wave = 1.0e-6 * dytime
  end_if
end
apply xvel=1.0 ,hist=wave i=2
hist nstep 1
hist dytime
hist wave
hist xvel i=2,j=150
hist xvel i=10,j=150
hist xvel i=20,j=150
hist xvel i=30,j=150
hist xvel i=50,j=150
hist xvel i=2,j=50
hist xvel i=10,j=50
hist xvel i=20,j=50
hist xvel i=30,j=50
hist xvel i=50,j=50
set large
set dy_damp avisc 1 1
solve dytime 10.0e-4
save avisc_axi_damp.sav
```

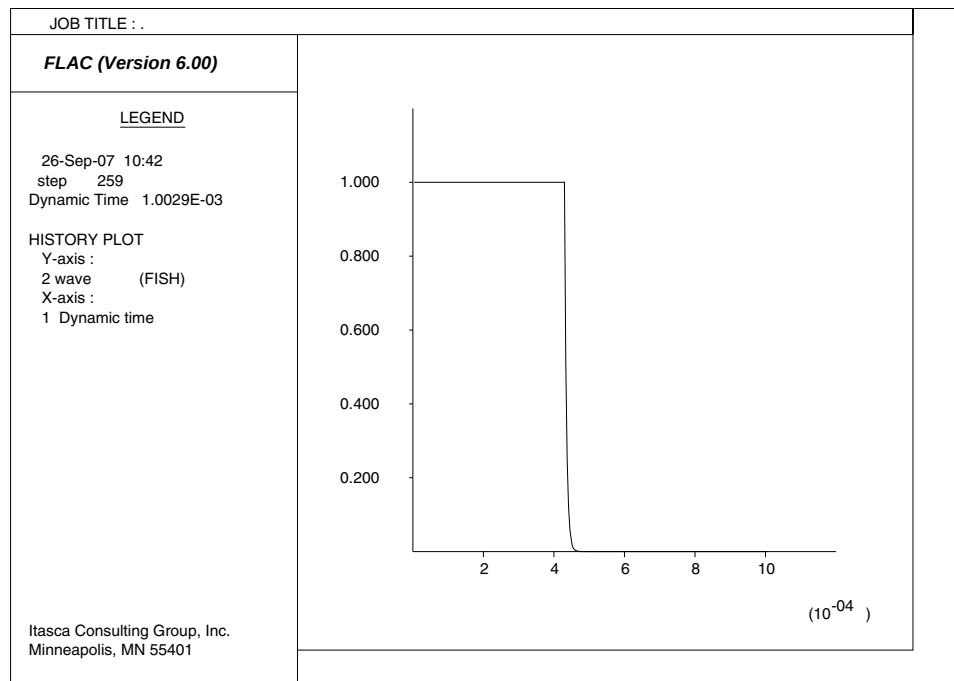


Figure 1.48 Velocity wave with sharp front

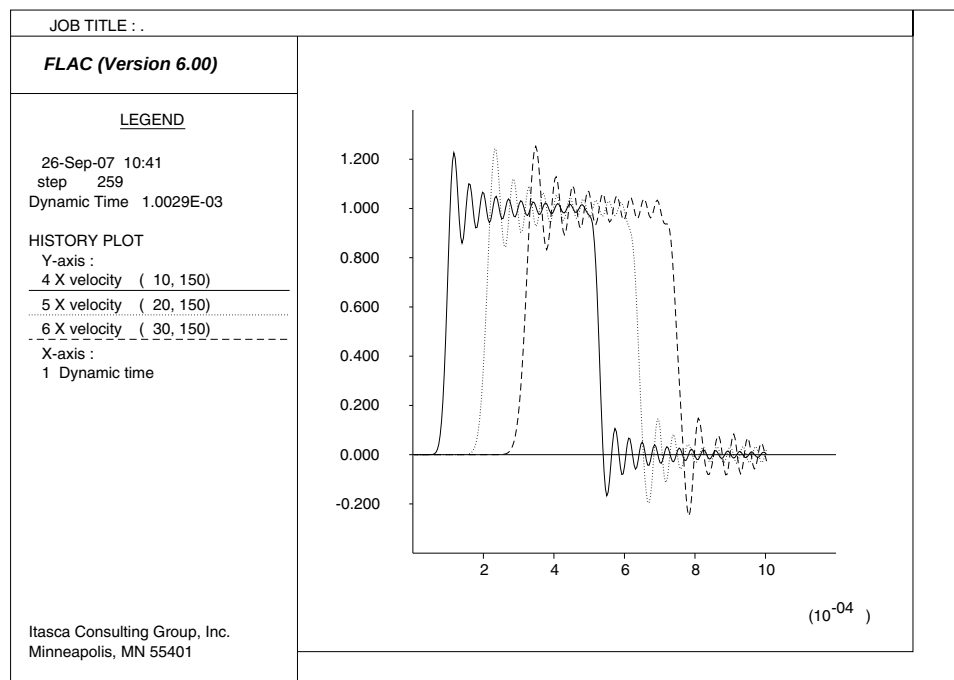


Figure 1.49 x-velocity histories for plane-strain model without artificial viscosity

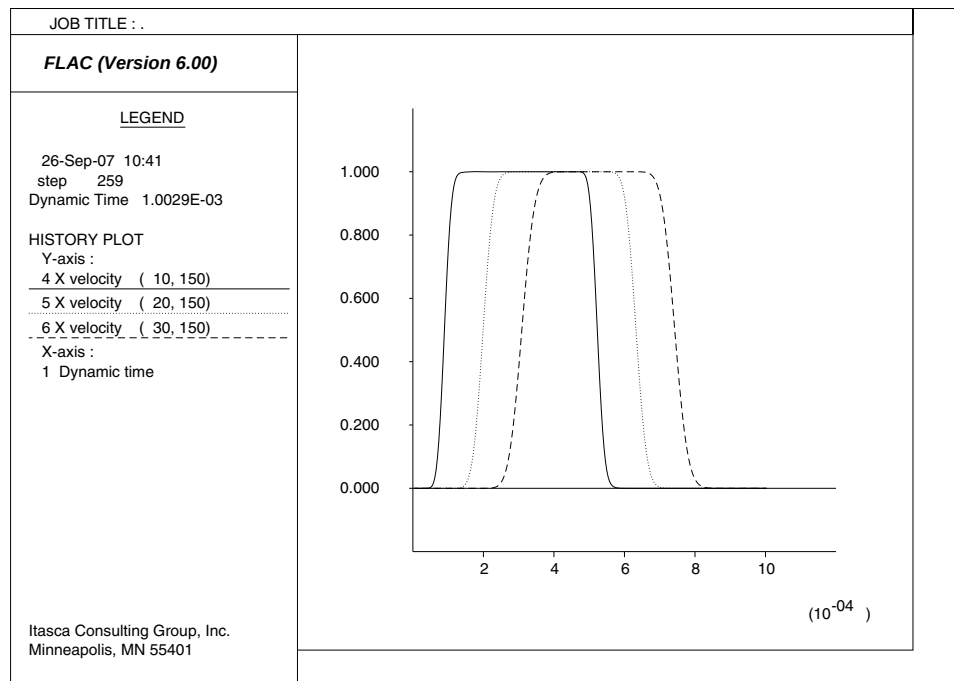


Figure 1.50 *x-velocity histories for plane-strain model with artificial viscosity*

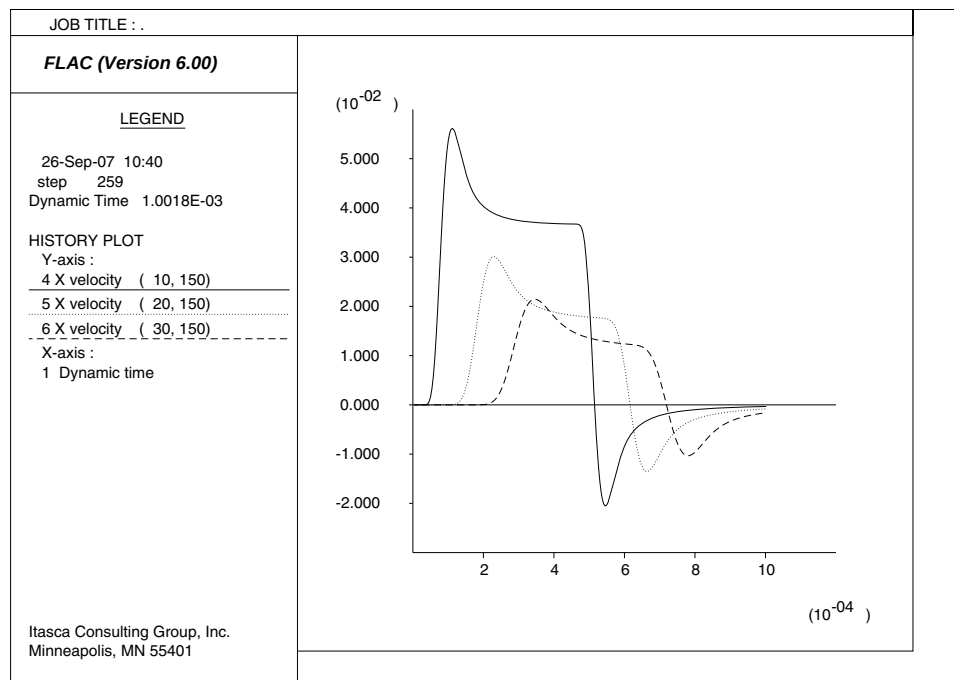


Figure 1.51 *x-velocity histories for axisymmetry model with artificial viscosity*

1.4.3.11 Integration of Damping Schemes and Nonlinear Material Models for Geo-materials

Energy dissipation in soil and rock is largely hysteretic in nature; the specific loss for each load/unload cycle of shear strain is independent of the rate at which the cycle is executed, but dependent on the amplitude of the cycle. Ideally, this behavior would be reproduced by an appropriate constitutive model, but adequate nonlinear models tend to be complicated, embodying many material parameters. Simpler models, such as Mohr Coulomb, are often used instead, in order to reproduce irreversible strain accumulation (e.g., slumping or slip on shear surfaces) that may occur during seismic loading. In such models, additional damping must be included to account for cyclic dissipation during the elastic part of the response, and during wave propagation through the site. Rayleigh damping is commonly used, but, as discussed in [Section 1.4.3.1](#), it only provides (approximately) rate-independent damping over a limited frequency range, and it entails a large reduction in critical timestep, and consequent long runtimes.

The following discussion illustrates the ways damping can be integrated with a simple material model. We consider the use of hysteretic damping plus damping arising from plastic flow. As discussed in [Section 1.4.3.4](#), hysteretic damping is based on a secant modulus-reduction curve for primary loading (the *backbone curve*) and a Masing rule assumption for unloading/reloading to provide energy dissipation. Hysteretic damping is applied in the elastic range only, and natural damping provided by the constitutive model operates in the plastic range. Three simple cases are presented. First, the energy dissipation provided by a standard elastic/plastic Mohr-Coulomb is shown. Second, damping is incorporated into a linearly elastic model using hysteretic (Hardin/Drnevich model) damping. And third, the Hardin/Drnevich model is combined with the Mohr-Coulomb model. The energy dissipation is compared for all three cases by evaluating the change in shear modulus and damping ratio for each case.

Natural damping with the Mohr-Coulomb model

Standard elastic/plastic models such as Mohr-Coulomb can produce shear-modulus reduction and damping ratio curves. Consider an elastic/plastic model with a constant shear modulus, G_{max} , and a constant yield stress, τ_m , subject to cyclic shear strain of amplitude $|\gamma|$. Below yield, the secant shear modulus, G , is simply equal to G_{max} . For cyclic excitation that involves yield, the secant modulus is

$$G = \tau_m / |\gamma| \quad (1.64)$$

The modulus-reduction curve relates the ratio G/G_{max} to the amplitude of shear strain, $|\gamma|$; it is simply obtained by dividing [Eq. \(1.64\)](#) by G_{max} , and using $\gamma_m = \tau_m/G_{max}$, we obtain, for $|\gamma| > \gamma_m$,

$$\frac{G}{G_{max}} = \frac{\gamma_m}{|\gamma|} \quad (1.65)$$

A stress-strain cycle of amplitude $\gamma_c > \gamma_m$, consisting of initial loading plus an unloading/reloading excursion, is sketched in [Figure 1.52](#).

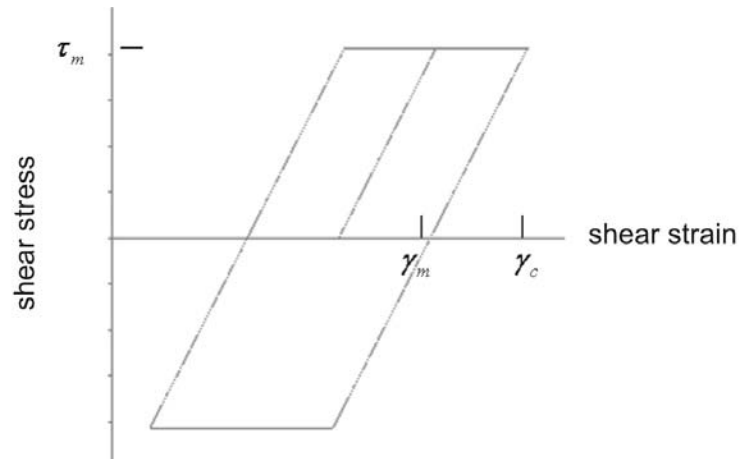


Figure 1.52 *Shear stress/strain cycle – Mohr-Coulomb model*

The maximum stored energy, W , during the cycle (assuming G represents an elastic modulus) is

$$W = \frac{1}{2} \tau_m \gamma_c \quad (1.66)$$

and the dissipated energy (corresponding to the area enclosed by the loop) is

$$\Delta W = 4 \tau_m (\gamma_c - \gamma_m) \quad (1.67)$$

Hence,

$$\frac{\Delta W}{W} = \frac{8(\gamma_c - \gamma_m)}{\gamma_c} \quad (1.68)$$

Denoting the damping ratio by D , and noting that for small D (Kolsky 1963),

$$D \approx \frac{1}{4\pi} \frac{\Delta W}{W} \quad (1.69)$$

or, substituting Eqs. (1.66) and (1.67) into Eq. (1.69),

$$D = \frac{2}{\pi} \frac{(\gamma_c - \gamma_m)}{\gamma_c} \quad (1.70)$$

We plot normalized modulus (G/G_{max}) from Eq. (1.65), and damping, D , from Eq. (1.70) against normalized cyclic strain, γ/γ_m , in Figure 1.53. It can be seen that even a simple model (where “simple” is taken in the context of dynamics) exhibits an evolution of modulus and damping that can be matched to experimental results over limited ranges of cyclic strain.

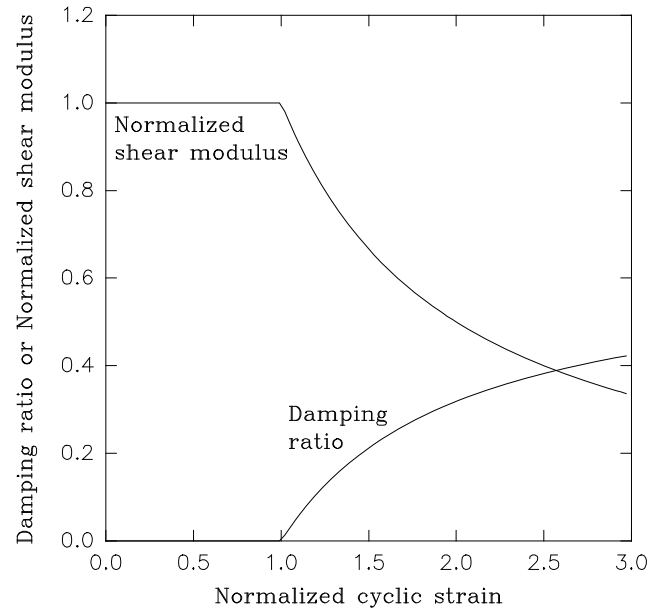


Figure 1.53 *Modulus and damping ratio versus cyclic strain for elastic/plastic Mohr-Coulomb model*

Hysteretic damping with the linear elastic model

For the hysteretic damping model using the Hardin/Drnevich function described in Section 1.4.3.5, the backbone curve is given by

$$\frac{\tau}{G_{max}} = \frac{\gamma}{1 + \frac{\gamma}{\gamma_{ref}}} \quad (1.71)$$

where γ_{ref} is the constant for the Hardin/Drnevich function. γ_{ref} is the ultimate value of τ/G_{max} :

$$\gamma_{ref} = \frac{\tau_m}{G_{max}} \quad (1.72)$$

This curve is followed for primary loading. For unloading/reloading, the Masing rule holds. For the case of cyclic shear loading at constant amplitude, γ_c , the Masing rule gives, for unloading,

$$\tau_{down} = -G_{max} \frac{\gamma_c - \gamma}{1 + \frac{\gamma_c - \gamma}{2\gamma_{ref}}} + \tau_c \quad (1.73)$$

and for reloading,

$$\tau_{up} = G_{max} \frac{\gamma_c + \gamma}{1 + \frac{\gamma_c + \gamma}{2\gamma_{ref}}} - \tau_c \quad (1.74)$$

where

$$\tau_c = G_{max} \frac{\gamma_c}{1 + \frac{\gamma_c}{\gamma_{ref}}} \quad (1.75)$$

The initial loading curve and loop traced in one cycle of unloading/reloading is sketched in [Figure 1.54](#):

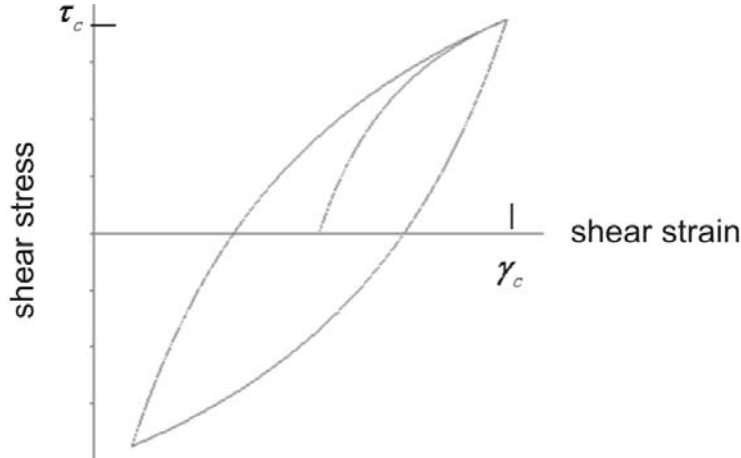


Figure 1.54 *Shear stress/strain cycle – elastic model with Hardin/Drnevich hysteretic damping*

The energy dissipated in one full unloading-reloading cycle is given by the integral

$$\Delta W = \int_{-\gamma_c}^{\gamma_c} (\tau_{up} - \tau_{down}) d\gamma \quad (1.76)$$

After introduction of Eqs. (1.73) and (1.74) in Eq. (1.76), and performing the integration, we obtain

$$\Delta W = 4G_{max}\gamma_{ref}^2 \left\{ 2 \left[\frac{\gamma_c}{\gamma_{ref}} - \ln \left(1 + \frac{\gamma_c}{\gamma_{ref}} \right) \right] - \frac{\left(\frac{\gamma_c}{\gamma_{ref}} \right)^2}{1 + \frac{\gamma_c}{\gamma_{ref}}} \right\} \quad (1.77)$$

The maximum stored energy in a cycle is

$$W = \frac{1}{2} \tau_c \gamma_c \quad (1.78)$$

where τ_c is given by Eq. (1.75).

The damping ratio, D , is obtained by combining Eqs. (1.77) and (1.78) with Eq. (1.69):

$$D = \frac{2}{\pi} \left\{ 2 \frac{1 + \frac{\gamma_c}{\gamma_{ref}}}{\left(\frac{\gamma_c}{\gamma_{ref}} \right)^2} \left[\frac{\gamma_c}{\gamma_{ref}} - \ln \left(1 + \frac{\gamma_c}{\gamma_{ref}} \right) \right] - 1 \right\} \quad (1.79)$$

Also, we note that application of l'Hospital rule gives

$$\lim_{\frac{\gamma_c}{\gamma_{ref}} \rightarrow \infty} D = 0 \quad (1.80)$$

It is interesting to note that for the Hardin/Drnevich hysteretic damping and the elastic model, the damping ratio does not depend on G_{max} . Also, D is larger for smaller values of γ_{ref} . For an elastic, cyclic shear test of constant amplitude at constant volume, the use of hysteretic damping produces a response that is independent of the number of cycles performed.

Hysteretic damping with the Mohr-Coulomb model

When hysteretic damping is used with an elastic/plastic model in *FLAC*, the modulus-reduction technique is applied in the elastic range, and natural damping applies in the plastic range. In this case, we combine the Hardin/Drnevich hysteretic damping with a Mohr-Coulomb model. The Mohr-Coulomb model has a constant, tangent, elastic shear modulus, G_{max} , and a constant yield stress, τ_m . The Hardin/Drnevich model is used to provide energy dissipation in the elastic range (but *not* to simulate yielding by means of a hyperbolic plasticity model). Accordingly, the yield level from the hyperbolic law must be higher than the Mohr-Coulomb yield stress. This will be the case provided that the following requirement is met:

$$\gamma_{ref} > \gamma_m \quad (1.81)$$

where

$$\gamma_m = \frac{\tau_m}{G_{max}} \quad (1.82)$$

An initial loading curve involving Mohr-Coulomb yielding, and a loop traced in one cycle of unloading/reloading, are sketched in [Figure 1.55](#):

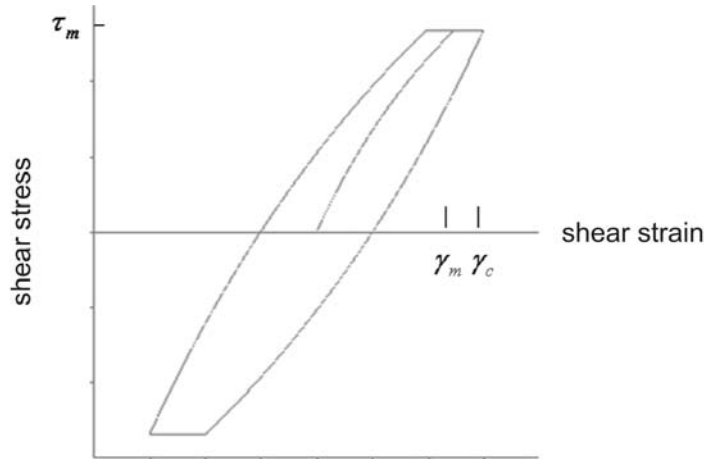


Figure 1.55 *Shear stress/strain cycle – Mohr-Coulomb model with Hardin/Drnevich hysteretic damping*

The elastic range is defined by $\gamma_c < \gamma_m$, where the shear strain, γ_m , is found from the following relation (see [Eq. \(1.71\)](#)):

$$\frac{\tau_m}{G_{max}} = \frac{\gamma_m}{1 + \frac{\gamma_m}{\gamma_{ref}}} \quad (1.83)$$

In the elastic range, $\gamma_c < \gamma_m$, the modulus reduction factor is given by [Eq. \(1.71\)](#), or

$$\frac{G}{G_{max}} = \frac{1}{1 + \frac{|\gamma|}{\gamma_{ref}}} \quad (1.84)$$

In the plastic range, $\gamma_c \geq \gamma_m$, the relation is

$$\frac{G}{G_{max}} = \frac{1}{\left(1 + \frac{\gamma_m}{\gamma_{ref}}\right) \frac{|\gamma|}{\gamma_m}} \quad (1.85)$$

Also, in the plastic range, the energy dissipated in one cycle is the area enclosed by the loop in [Figure 1.55](#). This energy may be expressed as the sum of two contributions:

$$\Delta W = \Delta W_H + \Delta W_{MC} \quad (1.86)$$

where ΔW_H is given by (see [Eq. \(1.77\)](#))

$$\Delta W_H = 4G_{max}\gamma_{ref}^2 \left\{ 2 \left[\frac{\gamma_m}{\gamma_{ref}} - \ln \left(1 + \frac{\gamma_m}{\gamma_{ref}} \right) \right] - \frac{\left(\frac{\gamma_m}{\gamma_{ref}} \right)^2}{1 + \frac{\gamma_m}{\gamma_{ref}}} \right\} \quad (1.87)$$

and ΔW_{MC} is expressed as (see [Eq. \(1.67\)](#))

$$\Delta W_{MC} = 4 \left(\frac{G_{max}}{1 + \frac{\gamma_m}{\gamma_{ref}}} \right) \gamma_m^2 \left(\frac{\gamma_c}{\gamma_m} - 1 \right) \quad (1.88)$$

The maximum stored energy in one cycle is

$$W = \frac{1}{2} \tau_m \gamma_c \quad (1.89)$$

and the damping ratio is

$$D = \frac{1}{4\pi} \frac{\Delta W_H + \Delta W_{MC}}{W} \quad (1.90)$$

After substituting [Eqs. \(1.87\)](#) and [\(1.88\)](#) in [Eq. \(1.90\)](#), we obtain with some manipulation,

$$D = \frac{2}{\pi} \left\{ 2 \frac{1 + \frac{\gamma_m}{\gamma_{ref}}}{\left(\frac{\gamma_m}{\gamma_{ref}} \right)^2} \left[\frac{\gamma_m}{\gamma_{ref}} - \ln \left(1 + \frac{\gamma_m}{\gamma_{ref}} \right) \right] - 1 \right\} \frac{1}{\frac{\gamma_c}{\gamma_{ref}}} + \frac{2}{\pi} \frac{(\gamma_c - \gamma_m)}{\gamma_c} \quad (1.91)$$

The energy dissipation for the three damping cases is compared by exercising the equations for G/G_{max} and D over a cyclic strain range. A *FISH* function, listed in [Example 1.14](#), performs this exercise over a cyclic shear strain, γ_c , from 0.0001 to 4.0. The value for γ_m is set to 0.01 and the value for γ_{ref} is set to 0.02. The results for G/G_{max} versus $\log \gamma_c/\gamma_m$, based upon [Eqs. \(1.65\)](#), [\(1.71\)](#), [\(1.84\)](#) and [\(1.85\)](#), are plotted in [Figure 1.56](#). The results for D versus $\log \gamma_c/\gamma_m$, based upon [Eqs. \(1.70\)](#), [\(1.79\)](#) and [\(1.91\)](#), are plotted in [Figure 1.57](#).

The inclusion of hysteretic damping is shown to reduce the shear modulus from the initial value of G_{max} , and increase the damping ratio (compared to the elastic-only response). The damping ratio increases monotonically with shear strain amplitude, and approaches the asymptotic value of $2/\pi$ for all three cases.

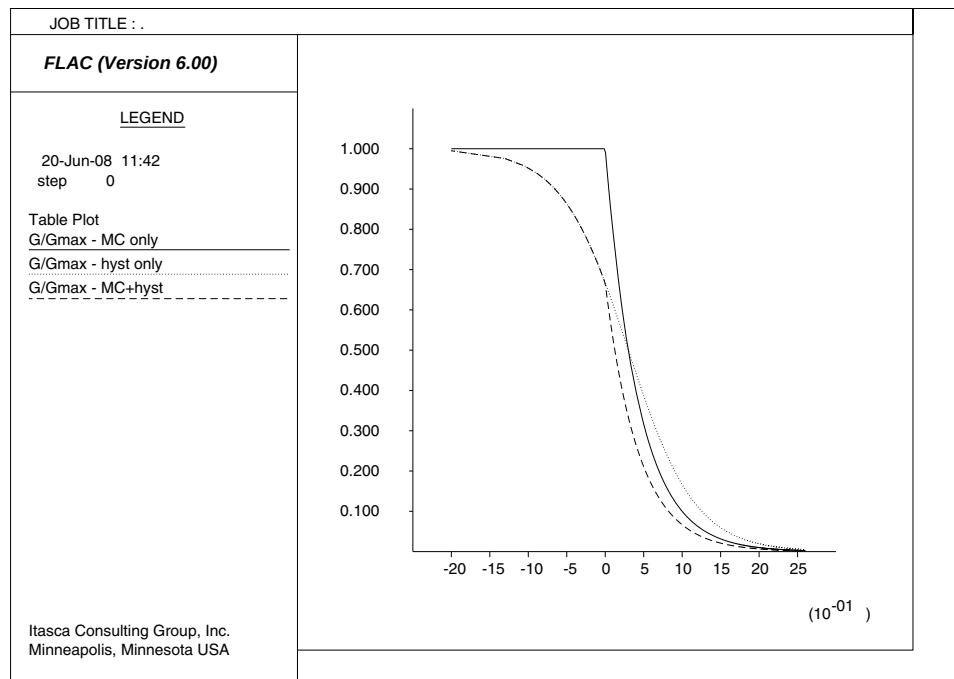


Figure 1.56 Normalized shear modulus vs log normalized shear stain for three damping cases

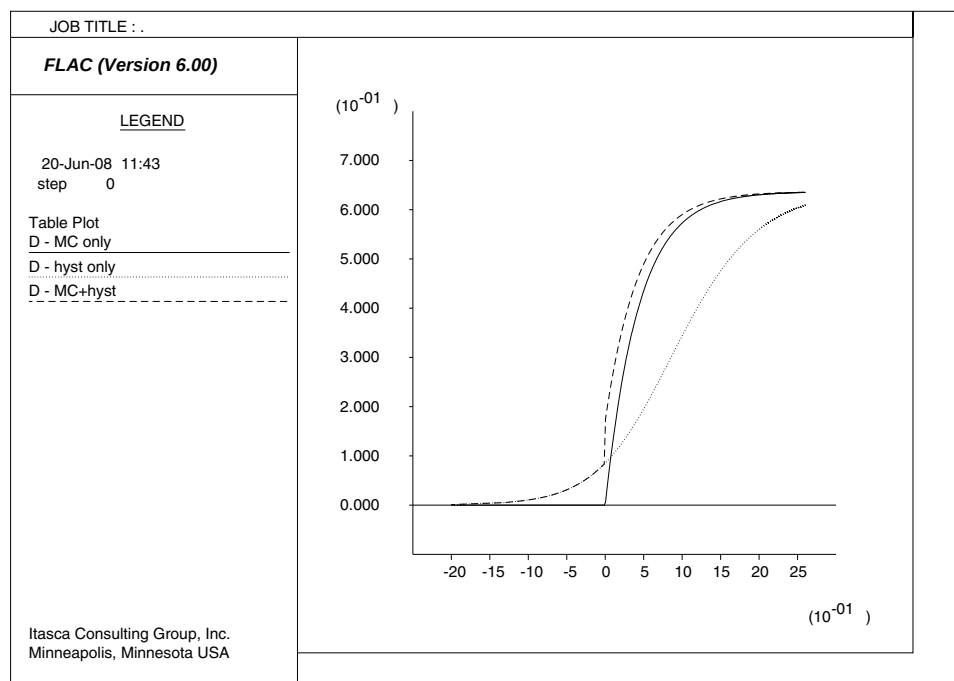


Figure 1.57 Damping ratio vs log normalized shear stain for three damping cases

Example 1.14 Compare damping

```

new
; assume Gmax = 1.
def _damp
  ; strain range
  _r1 = 0.0001 ; min
  _r2 = 4.      ; max
  _rm = 0.01
  _rref = 0.02
  _np = 10000
  _rinc = (_r2-_r1)/float(_np)
  _r = _r1
  loop n (1,_np)
    _rrat = _r / _rm
    if _r <= 0. then
      _ghyst = 1.
      _dhyst = 0.
    else
      _tau = _r/(1.+_r/_rref)
      _ghyst = _tau/_r
      _rf = _r/_rref
      _rf2 = _rf*_rf
      _rln = _rf - ln(1.+_rf)
      _dhyst = 2./pi*(2.*(1.+_rf)/_rf^2*_rln-1.)
    endif
    if _r < _rm then
      _gmohr = 1.
      _dmohr = 0.
    else
      _gmohr = _rm/_r
      _dmohr = 2.*(_r-_rm)/(pi*_r)
    endif
    if _r <= _rm then
      _gcomb = 1./(1.+_r/_rref)
    else
      _gcomb = 1./((1.+_rm/_rref)*_r/_rm)
    endif
    if _r <= _rm then
      _dcomb = _dhyst
    else
      _rf = _rm/_rref
      _rf2 = _rf*_rf
      _rln = _rf - ln(1.+_rf)
      _dcomb = 2./pi*(2.*(1.+_rf)/_rf^2*_rln-1.)*_rref/_r

```

```

        _dcomb = _dcomb + 2./pi*(_r-_rm)/_r
    endif
    _rrat = log(_rrat)
    table(11,_rrat) = _gmohr
    table(21,_rrat) = _dmohr
    table(12,_rrat) = _ghyst
    table(22,_rrat) = _dhyst
    table(13,_rrat) = _gcomb
    table(23,_rrat) = _dcomb
    _r = _r + _rinc
endloop
end
_damp
label table 11
G/Gmax - MC only
label table 12
G/Gmax - hyst only
label table 13
G/Gmax - MC+hyst
label table 21
D - MC only
label table 22
D - hyst only
label table 23
D - MC+hyst
plot hold table 11 line 12 line 13 line
plot hold table 21 line 22 line 23 line
return

```

1.4.4 Dynamic Pore-Pressure Generation

1.4.4.1 Liquefaction Modeling

Liquefaction is defined as the loss of shear strength of soil under monotonic or cyclic loading, arising from a tendency for loose soil to compact under shear loading. The term “liquefaction” was originally coined by Mogami and Kudo (1953). Note that this definition covers both static and dynamic liquefaction; the effective stress does not necessarily have to be zero for a soil to liquefy. In particular, when a saturated cohesionless soil is submitted to rapid static or cyclic loading, the tendency for the soil to densify causes the effective stress to decrease, and the process leads to soil liquefaction.

It is known that pore pressures may build up considerably in some sands during cyclic shear loading. Eventually, this process may lead to liquefaction when the effective stress decreases. Although excess pore pressure is generally associated with liquefaction, it is not the direct cause of liquefaction. In constant volume tests with no applied load, it is the decrease in contact forces between particles that is responsible for the decrease in effective stress. The process is documented by Dineash et al. (2004), who modeled similar tests (in which no change in pore pressure occurs) using the distinct element method. Alternatively, in undrained, simple shear tests under normal pressure, it is the irrecoverable reduction of porosity during cyclic compaction that generates pore pressure and, consequently, a decrease in effective stress.

Dilation plays an important role in the liquefaction process. As soil densifies under repeated shear cycles, grain rearrangement may be inhibited. Soil grains may then be forced to move up against adjacent soil particles, causing dilation to occur, the effective stress to increase and the pore pressure to decrease. Thus, densification is a self-limiting process.

The standard built-in constitutive models in *FLAC* do not model the liquefaction process directly. Coupled dynamic-groundwater flow calculations can be performed with *FLAC*. However, by default, the pore fluid simply responds to changes in pore volume caused by the mechanical dynamic loading; the *average* pore pressure remains essentially constant in the analysis.

There are many different models that attempt to account for pore pressure build-up, but they often do it in an ill-defined manner because they refer to specific laboratory tests. In a computer simulation, there will be arbitrary stress and strain paths. Consequently, an adequate model must be robust and general, with a formulation that is not couched in terms that apply only to specific tests. The following section describes a model that is simple, but that accounts for the basic physical process. In [Section 1.4.4.3](#), a review is presented of more comprehensive models that have been developed, and in [Section 1.4.4.4](#) we compare the simple and comprehensive models.

1.4.4.2 Simple Formulations

As mentioned in the previous section, in reality, pore pressure build-up is a *secondary* effect, although many people seem to think it is the primary response to cyclic loading. The primary effect is the irrecoverable volume contraction of the matrix of grains when a material is taken through a complete strain cycle with the confining stress held constant. Since it is grain rearrangement rather than grain volume change that takes place, the volume of the void space decreases under constant confining stress. If the voids are filled with fluid, then the pressure of the fluid increases and the effective stress acting on the grain matrix decreases. Note that pore pressures would not increase if the test were done at constant volume; it is the transfer of externally applied pressure from grains to fluid that accounts for the fluid-pressure increase.

Finn Model – Martin et al. Formulation

This mechanism is well-described by Martin et al. (1975), who also note that the relation between irrecoverable volume-strain and cyclic shear-strain amplitude is independent of confining stress. They supply the following empirical equation that relates the increment of volume decrease, $\Delta\epsilon_{vd}$, to the cyclic shear-strain amplitude, γ , where γ is presumed to be the “engineering” shear strain:

$$\Delta\epsilon_{vd} = C_1 (\gamma - C_2 \epsilon_{vd}) + \frac{C_3 \epsilon_{vd}^2}{\gamma + C_4 \epsilon_{vd}} \quad (1.92)$$

where C_1 , C_2 , C_3 and C_4 are constants.

Note that the equation involves the accumulated irrecoverable volume strain, ϵ_{vd} , in such a way that the increment in volume strain decreases as volume strain is accumulated. Presumably, $\Delta\epsilon_{vd}$ should be zero if γ is zero; this implies that the constants are related as follows: $C_1 C_2 C_4 = C_3$. Martin et al. (1975) then go on to compute the change in pore pressure by assuming certain moduli and boundary conditions (which are not clearly defined). We do not need to do this. Provided we correctly account for the irreversible volume change in the constitutive law, *FLAC* will take care of the other effects.

Finn Model – Byrne Formulation

An alternative, and simpler, formula is proposed by Byrne (1991):

$$\frac{\Delta\epsilon_{vd}}{\gamma} = C_1 \exp(-C_2 (\frac{\epsilon_{vd}}{\gamma})) \quad (1.93)$$

where C_1 and C_2 are constants with different interpretations from those of Eq. (1.92). In many cases, $C_2 = \frac{0.4}{C_1}$, so Eq. (1.93) involves only one independent constant; however, both C_1 and C_2 have been retained for generality. In addition, a third parameter, C_3 , sets the threshold shear strain (i.e., the limiting shear-strain amplitude below which volumetric strain is not produced).

The shear induced volumetric strain for constant amplitude of cyclic shear strain predicted by this formula is plotted versus number of cycles in [Figure 1.58](#). The formula predicts an increase in shear-induced (compactive) volumetric strain with the level of cyclic shear-strain. Also, for a given strain amplitude, γ , the rate of accumulation decreases with the number of cycles.

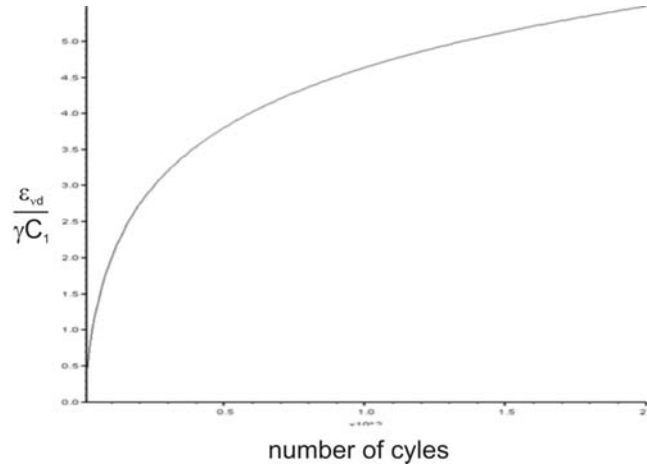


Figure 1.58 Finn/Byrne formula – constant, cyclic shear-strain amplitude

The incremental volumetric behavior of the Finn/Byrne model (at the end of a cycle) may be expressed as

$$\Delta\sigma_m + \alpha\Delta p = K(\Delta\epsilon + \Delta\epsilon_{vd}) \quad (1.94)$$

where $\sigma_m = \sigma_{ii}/3$ is the mean stress, p is pore pressure, α is Biot coefficient ($= 1$ for soil), K is the drained bulk modulus of the soil and ϵ is the volumetric strain. Note that ϵ is positive in extension, while ϵ_{vd} is positive in compression. For undrained conditions, the change in pore pressure is proportional to the change in volumetric strain:

$$\Delta p = -\alpha M \Delta\epsilon \quad (1.95)$$

where M is Biot modulus. After substitution of [Eq. \(1.95\)](#) into [\(1.94\)](#), and solving for $\Delta\epsilon$, we obtain

$$\Delta\epsilon = \frac{\Delta\sigma_m - K\Delta\epsilon_{vd}}{K + \alpha^2 M} \quad (1.96)$$

If the fluid is very stiff compared to the solid matrix ($M \gg K$), Eq. (1.96) predicts no change in volume. Further, using $\Delta\epsilon = 0$ in Eq. (1.94) gives

$$\Delta\sigma_m + \alpha\Delta p = K\Delta\epsilon_{vd} \quad (1.97)$$

Eq. (1.97) predicts a *decrease in magnitude of effective stress with cyclic shear strain* (produced by an increase of shear induced compaction). Under conditions of constant stress, $\Delta\sigma_m = 0$, there will be an increase in pore pressure:

$$\Delta p = K\Delta\epsilon_{vd} \quad (1.98)$$

that is proportional to the drained bulk modulus of the soil. While under free stress conditions, the pore pressure will remain unchanged ($\Delta p = 0$), and the magnitude of the total stress will decrease according to

$$\Delta\sigma_m = K\Delta\epsilon_{vd} \quad (1.99)$$

Note that in both situations, the drained (tangent) bulk modulus, K , plays an important role in determining the magnitude of the cyclic loading impact on effective stress. The Finn/Byrne model, therefore, captures the main physics of liquefaction.

Finn Model Implementation in FLAC

FLAC contains a built-in constitutive model (named the Finn model)* that incorporates both Eq. (1.92) and Eq. (1.93) into the standard Mohr-Coulomb plasticity model – it can be modified by the user as required. The use of Eq. (1.92) or Eq. (1.93) can be selected by setting parameter **ff_switch** = 0 or 1, respectively. As it stands, the model captures the basic mechanisms that can lead to liquefaction in sand. In addition to the usual parameters (friction, moduli, etc.), the model needs the four constants for Eq. (1.92), or three constants for Eq. (1.93). For Eq. (1.92), Martin et al. (1975) describe how these may be determined from a *drained* cyclic test. Alternatively, one may imagine using some trial values to model an undrained test with *FLAC*, and compare the results with a corresponding laboratory test. The constants could then be adjusted to obtain a better match. (See Example 1.15 for an example.) For Eq. (1.93), Byrne (1991) notes that the constant, C_1 , can be derived from relative densities, D_r , as follows:

$$C_1 = 7600(D_r)^{-2.5} \quad (1.100)$$

* A *FISH* constitutive model is also provided for the Finn model (see “FINN.FIS” in Section 3 in the *FISH* volume).

Further, using an empirical relation between D_r and normalized standard penetration test values, $(N_1)_{60}$,

$$D_r = 15(N_1)_{60}^{\frac{1}{2}} \quad (1.101)$$

then,

$$C_1 = 8.7(N_1)_{60}^{-1.25} \quad (1.102)$$

C_2 is then calculated from $C_2 = \frac{0.4}{C_1}$ in this case. Note that, as expected, the volumetric strain is larger for smaller values of the blow count (compare Eq. (1.102) to Eq. (1.93)). Refer to Byrne (1991) for more details.

In the Finn model there is logic to detect a strain reversal in the general case. In Martin et al. (1975) (and most other papers on this topic), the notion of a strain reversal is clear, because they consider one-dimensional measures of strain. In a two-dimensional analysis, however, there are at least three components of the strain-rate tensor. By eliminating the volumetric strain, we have a 2D “strain space.” In the general case of earthquake loading (where there is vertical as well as horizontal motion), the trajectory of each element in this strain space is very complicated.

For example, Figure 1.59 shows the locus of strain states for a few seconds of typical earthquake shaking, where $e_{11} - e_{22}$ is plotted on the x -axis and $2e_{12}$ is plotted on the y -axis. What is a strain cycle in this case? We adopt a formulation that degenerates to the conventional notion of strain cycle when the amplitude on one axis is zero, or if there is a constant offset in strain. Note that a simple magnitude measure (e.g., distance from the center point) is not good enough. Denoting the two orthogonal strain measures as ϵ_1 and ϵ_2 , we accumulate strain, as follows, from *FLAC*’s “input” strain increments:

$$\epsilon_1 := \epsilon_1 + \Delta e_{11} - \Delta e_{22} \quad (1.103)$$

$$\epsilon_2 := \epsilon_2 + 2\Delta e_{12} \quad (1.104)$$

We use the following scheme to locate extreme points in strain space. Denoting the previous point by superscript ($^\circ$), and the one before that with ($^{\circ\circ}$), the previous unit vector, n_i° , in strain space is computed:

$$v_i = \epsilon_i^\circ - \epsilon_i^{\circ\circ} \quad (1.105)$$

$$z = \sqrt{v_i v_i} \quad (1.106)$$

$$n_i^\circ = \frac{v_i}{z} \quad (1.107)$$

where subscript i takes the values 1,2, and repeated indices imply summation.

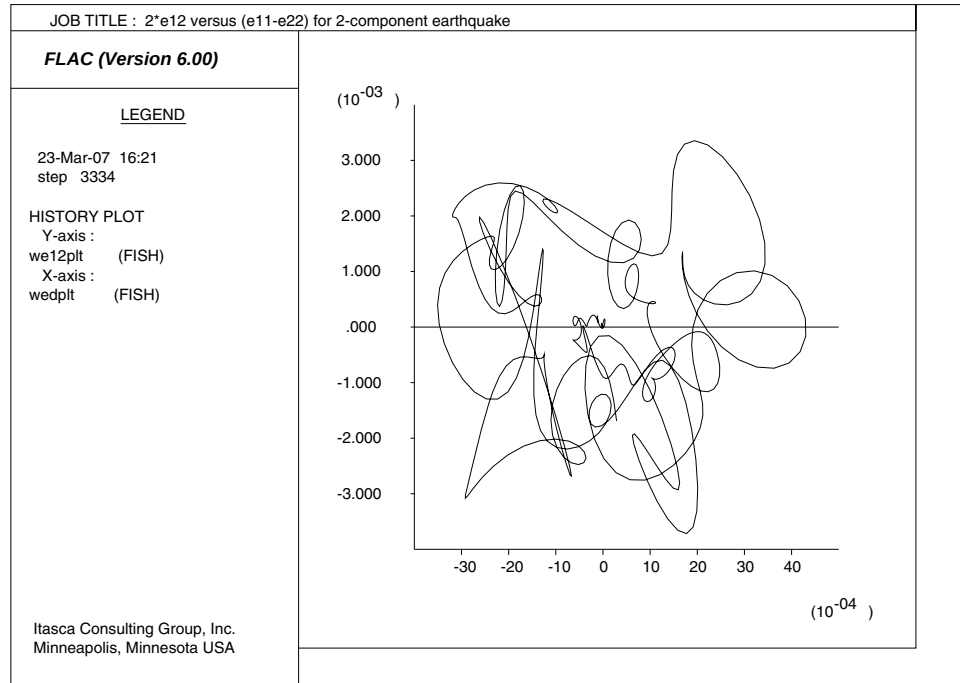


Figure 1.59 Locus in strain space using typical earthquake histories

The perpendicular “distance,” d , from the old point to a new point is given by the dot product of the new vector with the previous unit vector:

$$d = (\epsilon_i - \epsilon_i^\circ) n_i \quad (1.108)$$

We use the rule that d must be negative (so that the new strain segment corresponds to a *reversal* compared to the previous segment). We then monitor the absolute value of d and do the following calculation when it passes through a maximum, $d_{\max}$, provided that a minimum number of timesteps has elapsed (to prevent the reversal logic being triggered again on transients that immediately follow a reversal). This threshold number of timesteps is controlled by the property named **ff_latency**, which is set to 50.0 in the runs reported here.

$$\gamma = d_{\max} \quad (1.109)$$

$$\epsilon_i^{\circ\circ} = \epsilon_i^{\circ} \quad (1.110)$$

$$\epsilon_i^{\circ} = \epsilon_i \quad (1.111)$$

Note that there are two factors of 2 implied in Eq. (1.109) that cancel out: the shear strain is half the excursion $d_{\max}$, but γ is the engineering strain, which is twice *FLAC*'s strain. Having obtained γ , we insert it into Eq. (1.92) and obtain $\Delta\epsilon_{vd}$. We then update ϵ_{vd} , as follows, and save it for use in Eq. (1.92):

$$\epsilon_{vd} := \epsilon_{vd} + \Delta\epsilon_{vd} \quad (1.112)$$

We also save one-third of $\Delta\epsilon_{vd}$, and revise the direct strain increments input to the model at the next cycle:

$$\Delta e_{11} := \Delta e_{11} + \frac{\Delta\epsilon_{vd}}{3} \quad (1.113)$$

$$\Delta e_{22} := \Delta e_{22} + \frac{\Delta\epsilon_{vd}}{3} \quad (1.114)$$

$$\Delta e_{33} := \Delta e_{33} + \frac{\Delta\epsilon_{vd}}{3} \quad (1.115)$$

Note that *FLAC*'s compressive strain increments are negative, and $\Delta\epsilon_{vd}$ is positive. Hence, the mean effective stress decreases.

The logic described above is certainly not perfect, but it seems to work in simple cases. However, the user must verify that the algorithm is appropriate before applying it to real cases. In particular, the number of “cycles” detected depends strongly on the relative magnitude of horizontal and vertical motion. Hence, the rate of build-up of pore pressure will also be sensitive to this ratio. It may be more practical to consider just the e_{12} component of strain for something like a dam, which is wide compared to its height. Ultimately, we need better experimental data for volume changes during complicated loading paths; the model should then be revised accordingly. One effect that has been shown to be very important (see, for example, Arthur et al. 1980) is the effect of rotation of principal axes: volume compaction may occur even though the magnitude of deviatoric strain (or stress) is kept constant. Such rotations of axes occur frequently in earthquake situations. Another effect that is not incorporated into the Finn model is that of modulus increase induced by compaction – it is known that sand becomes stiffer elastically when compaction occurs by cyclic loading. It would be easy for the user to add this modification to the “FINN.FIS” model.

The Finn model is implemented in *FLAC* with the **MODEL** command (i.e., **MODEL finn**). The code must be configured for dynamic analysis (**CONFIG dynamic**) to apply the model. As with the other built-in models, the property names are assigned with the **PROPERTY** command. The following keywords are used to assign properties for the Finn model:

bulk	bulk modulus
cohesion	cohesion
dilation	dilation angle in degrees
ff_c1	Eqs. (1.92) and (1.93) constant C_1
ff_c2	Eqs. (1.92) and (1.93) constant C_2
ff_c3	Eq. (1.92) constant C_3 , and threshold shear strain for Eq. (1.93)
ff_c4	Eq. (1.92) constant C_4
ff_latency	minimum number of timesteps between reversals
ff_switch	= 0 for Eq. (1.92) , and 1 for Eq. (1.93)
friction	friction angle in degrees
shear	shear modulus
tension	tension cutoff

In addition, the following Finn model variables may be printed or plotted:

ff_count	number of shear strain reversals detected
ff_evd	internal volume strain, ϵ_{vd} , of Eqs. (1.92) and (1.93)

Simulation of the Liquefaction of a Layer

The material constants in the Finn model that control pore pressure build-up are related to the volumetric response in a drained test. However, if results are available for an undrained test, then the test itself may be modeled with *FLAC*, and the material constants deduced by comparing the *FLAC* results with the experimental observations. Some adjustment will be necessary before a match is found.

In the following example, a “shaking table” is modeled with *FLAC* – this consists of a box of sand that is given a periodic motion at its base. The motion of the sides follows that of the base, except that the amplitude diminishes to zero at the top (i.e., the motion is that of simple shear). Vertical loading is by gravity only. Equilibrium stresses and pore pressures are installed in the soil, and pore pressure and effective stress (mean total stress minus the pore pressure) are monitored in a zone within the soil. A column of only one zone-width is modeled, since the horizontal variation is of no particular interest here.

[Example 1.15](#) lists the data file for this test, and can be run for both the Martin et al. (1975) formula ([Eq. \(1.92\)](#)) and the Byrne (1991) formula ([Eq. \(1.93\)](#)). (The Byrne parameters are commented out in [Example 1.7](#).) The Byrne parameters correspond to $(N_1)_{60} = 7$, which was selected to produce results that match those based on the given Martin parameters.

The results based on [Eq. \(1.92\)](#) are shown in [Figure 1.60](#), and those based on [Eq. \(1.93\)](#) are shown in [Figure 1.61](#). The figures indicate similar behavior using either formula. Both show how the pore pressure in zone (1,2) builds up with time. The history of effective stress in the same zone is also shown. It can be seen that the effective stress reaches zero after about 20 cycles of shaking (4 seconds at 5 Hz). At this point, liquefaction can be said to occur. This test is strain-controlled in the shear direction. For a stress-controlled test, collapse would occur earlier, since strain cycles would start to increase in amplitude, thus generating more pore pressure.

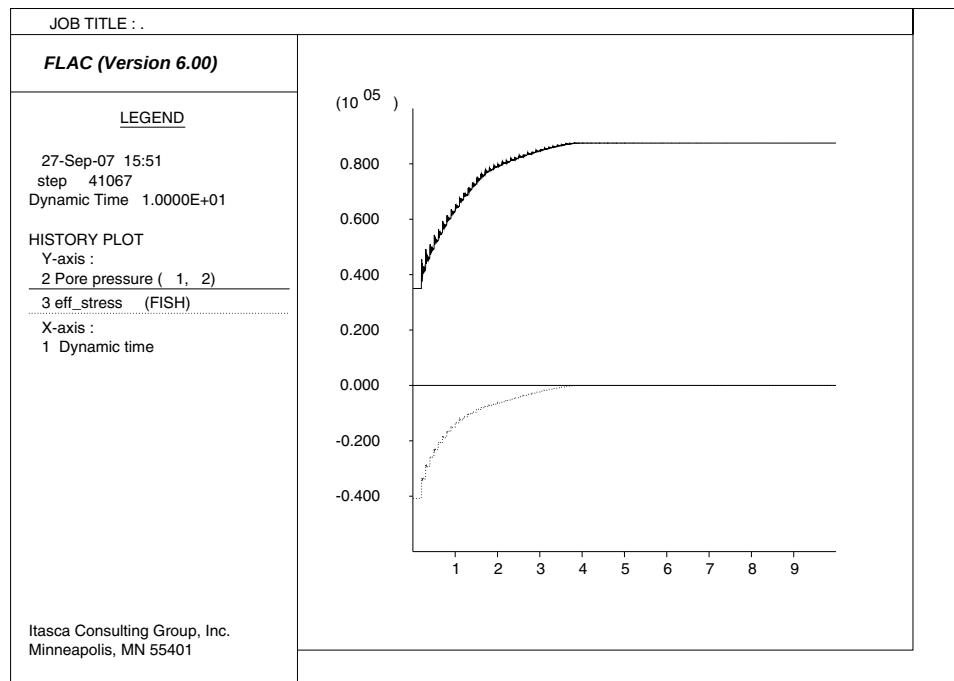


Figure 1.60 Pore pressure (top) and effective stress (bottom) for shaking table, using Eq. (1.92)

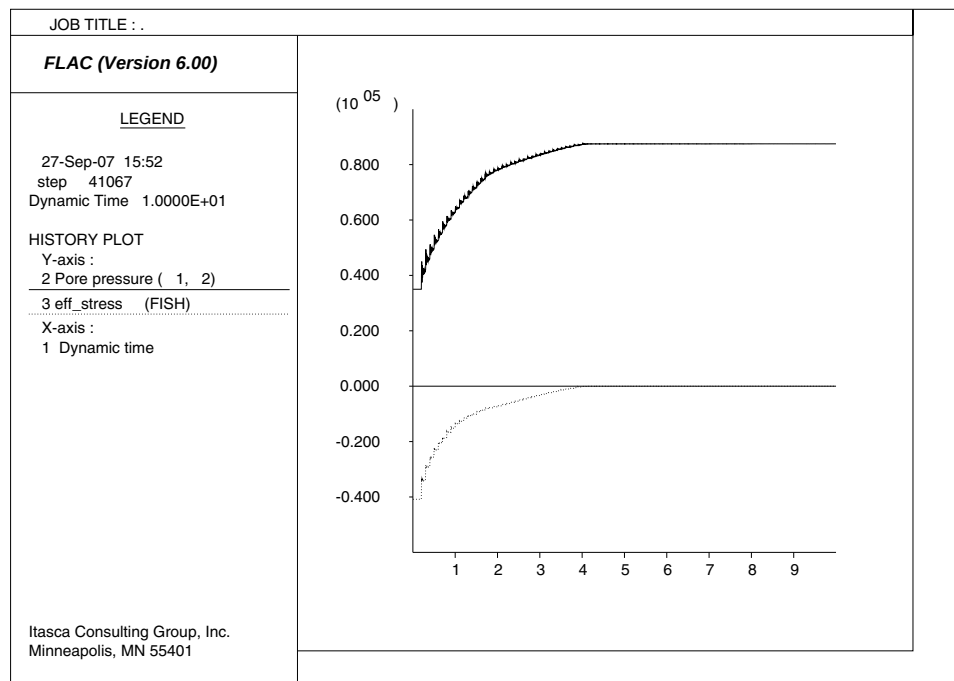


Figure 1.61 Pore pressure (top) and effective stress (bottom) for shaking table, using Eq. (1.93)

Example 1.15 Shaking table test

```

conf dyn gw
; shaking table test for liquefaction
g 1 5
m finn
gen 0 0 0 5 50 5 50 0
fix x y j=1
fix x
set grav 10, flow=off
prop dens 2000 shear 2e8 bulk 3e8
prop fric 35 poros 0.5
water dens 1000 bulk 2e9 tens 1e10
ini pp 5e4 var 0 -5e4
ini syy -1.25e5 var 0 1.25e5
ini sxx -1e5 var 0 1e5 szz -1e5 var 0 1e5
prop ff_latency=50
; parameters for Martin formula
;prop ff_switch = 0
;prop ff_c1=0.8 ff_c2=0.79
;prop ff_c3=0.45 ff_c4=0.73
; parameters for Byrne formula
prop ff_switch = 1
def _setCoeff_Byrne
    ff_c1_ = 8.7*exp(-1.25*ln(n1_60_))
    ff_c2_ = 0.4/ff_c1_
    ff_c3_ = 0.0000
end
set n1_60_ = 7
_setCoeff_Byrne
prop ff_c1=ff_c1_ ff_c2=ff_c2_
prop ff_c3=ff_c3_
set ncwrite=50
def sine_wave
    while_stepping
        vv = ampl * sin(2.0 * pi * freq * dytime)
        loop j (1,jzones)
            vvv = vv * float(jgp - j) / float(jzones)
            loop i (1,igp)
                xvel(i,j) = vvv
            end_loop
        end_loop
    end
end
def eff_stress
    eff_stress = (sxx(1,2)+syy(1,2)+szz(1,2))/3.0 + pp(1,2)

```

```

    settlement = (ydisp(1,jgp)+ydisp(2,jgp))/2.0
end
set dy_damp=rayl 0.05 20.0
his dytime
his pp i 1 j 2
his eff_stress
his settlement
his nstep 20
set ampl=0.005 freq=5.0
solve dyt=10.0
save Byrne.sav
;
new
conf dyn gw
; shaking table test for liquefaction
g 1 5
m finn
gen 0 0 0 5 50 5 50 0
fix x y j=1
fix x
set grav 10, flow=off
prop dens 2000 shear 2e8 bulk 3e8
prop fric 35 poros 0.5
water dens 1000 bulk 2e9 tens 1e10
ini pp 5e4 var 0 -5e4
ini syy -1.25e5 var 0 1.25e5
ini sxx -1e5 var 0 1e5 szz -1e5 var 0 1e5
prop ff_latency=50
; parameters for Martin formula
prop ff_switch = 0
prop ff_c1=0.8 ff_c2=0.79
prop ff_c3=0.45 ff_c4=0.73
; parameters for Byrne formula
;prop ff_switch = 1
def _setCoeff_Byrne
    ff_c1_ = 8.7*exp(-1.25*ln(n1_60_))
    ff_c2_ = 0.4/ff_c1_
    ff_c3_ = 0.0000
end
set n1_60_ = 7
;_setCoeff_Byrne
;prop ff_c1=ff_c1_ ff_c2=ff_c2_
;prop ff_c3=ff_c3_
set ncwrite=50
def sine_wave
    while_stepping

```

```
vv = ampl * sin(2.0 * pi * freq * dytime)
loop j (1,jzones)
  vvv = vv * float(jgp - j) / float(jzones)
  loop i (1,igp)
    xvel(i,j) = vvv
  end_loop
end_loop
end
def eff_stress
  eff_stress = (sxx(1,2)+syy(1,2)+szz(1,2))/3.0 + pp(1,2)
  settlement = (ydisp(1,jgp)+ydisp(2,jgp))/2.0
end
set dy_damp=rayl 0.05 20.0
his dytime
his pp i 1 j 2
his eff_stress
his settlement
his nstep 20
set ampl=0.005 freq=5.0
solve dyt=10.0
save Martin.sav
```

1.4.4.3 Comprehensive Liquefaction Constitutive Models

The state of practice for seismic analysis involving liquefiable materials is currently experiencing a shift from using empirical schemes (first developed in the 1970s) to simulate liquefaction, to time-marching numerical methods incorporating liquefaction constitutive models currently at various stages of development. There are several ways in which liquefaction behavior is included in numerical methods, ranging from total-stress empirical schemes to estimate liquefaction conditions (e.g., the UBCTOT model – see Beaty and Byrne 2000), to simple effective-stress shear-volume coupling schemes (e.g., the Finn model described in [Section 1.4.4.2](#) and the URS model – see Dawson et al. 2001), to more comprehensive constitutive models (e.g., the UBCSAND model – see Byrne et al. 1995; and bounding surface models such as the model described in Wang 1990 and the model described in Papadimitriou et al. 2001) that address cyclic shearing via kinematic hardening.

To help practicing engineers choose a procedure best-suited to their needs, a selection of approaches is outlined below, ranging from simple to elaborate in terms of complexity and model parameter determination. This is not a comprehensive list, but is a selection that illustrates the different types of liquefaction models that have been developed, and have been used in *FLAC*.^{*} One important factor to keep in mind is that, in engineering practice, the use of a very complicated model for liquefaction analyses is often hardly justified, considering the many uncertainties with respect to soil properties and earthquake motions, and the numerous approximations which must be made (see Dawson et al. 2001). Before describing the individual models, it will be helpful to review the current state of practice for liquefaction analysis. (See Byrne et al. 2006 for further discussion on state-of-practice analysis.)

State of Practice – The standard practice approach for liquefaction analysis of earthquake loading is based on a total-stress analysis in which it is assumed that the liquefiable soil remains undrained at the in-situ void ratio (Byrne and Wijewickreme 2006). Typically, this analysis approach is divided into three steps:

1. *Triggering Evaluation*: Typically, an equivalent-linear elastic, dynamic analysis (such as *SHAKE*) using strain-compatible moduli and damping is conducted for the design earthquake. The cyclic stress ratio (CSR)** is evaluated from the numerical simulation and compared to the value of cyclic resistance that the soil has because of its density

* The liquefaction models discussed in this section have been implemented in *FLAC* either as *FISH* or C++ user-defined constitutive models. Contact the authors of these liquefaction models in order to receive additional information about the model, or to inquire about receiving a copy of the model. Itasca does *not* provide technical support for these models.

** Cyclic Stress Ratio: ratio of maximum dynamic shear stress to the initial vertical effective stress prior to the earthquake at a specific location.

(CRR),*** derived from empirical curves. A factor of safety against triggering liquefaction is evaluated using the ratio of CRR and CSR (e.g., see Byrne and Anderson 1991, and Youd et al. 2001).

2. *Flow Slide Assessment*: Post-liquefaction (undrained) strengths are assigned in zones predicted to liquefy from the triggering evaluation analysis, and a standard limit-equilibrium analysis is carried out to evaluate the factor of safety against a flow slide. Post liquefaction strengths may be derived from penetration resistance (corrected blow count $(N_1)_{60}$) using empirical charts (e.g., see Seed and Harder 1990, and Olson and Stark 2002).
3. *Seismic Displacements*: Displacements are evaluated using the Newmark approach (see Newmark 1965). In this step, the potential sliding block of soil is modeled as a rigid mass resting on an inclined plane. The design time history of acceleration is applied at the base, and the equation of motion is solved to obtain the displacement of the mass caused by the shaking.

The main shortcomings of the standard practice approach are that the three aspects of liquefaction (triggering, flow slide and deformation) are treated sequentially, when in reality they may interact locally in various zones of the soil structure and affect the overall behavior of the soil mass. Also, no direct account is made of excess pore-pressure redistribution and dissipation.

Total-Stress Synthesized Procedure

The synthesized procedure of Beaty and Byrne (2000) uses *FLAC* and the UBCTOT constitutive model to combine the three steps of the standard practice approach (triggering, flow slide and estimate of liquefaction-induced displacements) into one single analysis. The procedure, which assumes undrained behavior, uses a (two-dimensional) total-stress approach to liquefaction analysis, and relies on adjustment of liquefied element properties (stiffness and strength) at the instant of triggering of liquefaction. The main features of the UBCTOT model are summarized below.

A seismic analysis using the synthesized procedure starts from a static state of equilibrium for the *FLAC* model. The seismic analysis is conducted in total-stress space. UBCTOT uses Mohr-Coulomb elasto-plastic logic with zero friction and a value of cohesion equal to the undrained shear strength, in combination with Rayleigh damping. The elastic shear modulus is assigned a value of G_{max} multiplied by a Modulus Reduction Factor (MRF). As opposed to equivalent-linear methods, this approach is not iterative, and appropriate values of MRF and damping are selected at the start of the seismic analysis. Triggering of liquefaction is based on changes of shear stress on the horizontal plane, τ_{xy} . The irregular shear-stress history caused by the earthquake is interpreted in each *FLAC* zone as a succession of half-cycles with the contribution to triggering determined by the maximum value of τ_{cyc} , defined as the difference between τ_{xy} and the initial horizontal shear-stress prior to earthquake loading (i.e., the static bias). A cumulative damage approach is used to combine the effects of each half-cycle. The approach converts the nonuniform history into an equivalent series of uniform stress cycles with amplitude equal to τ_{15} (i.e., the value of τ_{cyc} required to cause liquefaction in 15 cycles, which is approximately the number of cycles in a magnitude 7.5

*** Cyclic Resistance Ratio: commonly taken as the value of CSR that causes liquefaction in 15 cycles of dynamic loading.

earthquake). This is done using an empirical chart giving the cyclic stress ratio (CSR) versus cycles to liquefaction. Several property changes are imposed when liquefaction is detected in a *FLAC* zone: a residual shear strength is assigned; a reduced loading stiffness is used; and unloading uses a stiffer modulus than loading, according to a bilinear stiffness model. Also, a hydrostatic stress state is imposed when a zone experiences a shear-stress reversal. Finally, reduced viscous damping is assigned in a liquefied zone. The constitutive model also has logic to account for anisotropy in stiffness and strength. See Beaty (2001) for additional information.

The UBCTOT model removes some of the limitations associated with the sequential approach to problem solving used in the state-of-practice procedure, while relying on similar empirical charts for triggering of liquefaction and residual strength.

Some of the drawbacks of the model are:

- the use of equivalent modulus ratio that may not capture the pre-liquefaction phase well;
- the cyclic shear stresses are accounted for on the horizontal plane only;
- the simplified manner in which the undrained shear strength is specified;
- pore pressure is not taken into account explicitly; and
- liquefaction due to monotonic loading is not considered.

Loosely Coupled Effective-Stress Procedure

The URS model is a loosely coupled effective-stress constitutive model to generate pore pressure from shear stress cycles using the Seed cyclic stress approach (Seed and Idriss 1971). This model is built around the standard *FLAC* Mohr Coulomb model. The (two-dimensional) model counts shear stress cycles by tracking the shear stress acting on horizontal planes (τ_{xy}) and looking for stress reversals. The cyclic stress ratio (CSR) of each cycle is measured, and this is used to compute the incremental “damage” that is then translated into an increment of excess pore pressure. The procedure is “loosely coupled” because pore pressures are only computed after each 1/2 cycle of strain or stress as the analysis proceeds. The model incorporates residual strength (a critical parameter for seismic stability analyses) by using a two-segment failure envelope consisting of a residual cohesion value and zero friction angle that is extended to meet with the traditional Mohr-Coulomb failure envelope.

The model is simple, robust and practice-oriented; it is based on the widely accepted cyclic-stress approach with input parameters readily obtainable from routine field investigations. (Note that liquefaction due to monotonic loading is not considered.) A disadvantage of the model is that liquefaction-induced consolidation settlements are not captured, because the actual physical mechanism of liquefaction, whereby pore pressure is generated through contraction of the soil skeleton, is bypassed. The model is applicable to problems where slope movements due to reduced shear strength are the main concern (such as seismic stability of dams, and waterfront retaining structures), while shaking-induced consolidation settlements are of secondary importance. See Roth et al. (1991), Inel et al. (1993), Roth et al. (1993), and Perlea et al. (2008) for some field applications.

The URS model is similar to the built-in Finn model (described in [Section 1.4.4.2](#)), which is also considered a loosely coupled effective-stress model. The primary difference is that in the Finn model, the volumetric strains induced by cyclic loading are evaluated based on an experimental curve of irrecoverable volumetric strain versus number of constant amplitude cycles. Pore pressures are then generated from these volumetric strains, as well as from contraction of the soil skeleton. Also, the Finn model in *FLAC*, at present, does not include a post-liquefaction residual strength. Further discussion and numerical testing of the Finn model is presented in [Section 1.4.4.4](#).

Fully Coupled Effective-Stress Procedure

The UBCSAND model is a fully coupled (kinematic hardening) effective-stress constitutive model to predict seismic response and liquefaction of cohesionless soils in plane strain problems. The model uses an elasto-plastic formulation, based on an assumed hyperbolic relation between stress ratio and plastic shear strain, similar to the Duncan and Chang (1970) formulation. It is applicable for monotonic as well as cyclic loading (e.g., see Byrne et al. 2003, 2006).

The model implementation is a modified form of the built-in Mohr-Coulomb model in *FLAC* that accounts for a strain-hardening frictional behavior, neglects cohesion, and applies to plane strain conditions. The hardening law is a hyperbolic function of plastic shear strain. Unloading is assumed to be nonlinear elastic, with bulk and shear modulus as functions of mean (in-plane) effective stress. Stress reversal is detected by a change of sign in horizontal shear stress, τ_{xy} . Reloading is elasto-plastic, with the yield locus reset to the value at the reversal point. Plastic flow is non-associated; the logic is based on a variation of Rowe stress-dilatancy theory. According to this theory, there is a constant-volume stress ratio, ϕ_{cv} , below which the material contracts (i.e., for mobilized friction, ϕ_m , smaller than ϕ_{cv}), while for higher stress ratios (i.e., for $\phi_m > \phi_{cv}$), the material dilates. The effect of relative density is addressed through the choice of material properties. Most properties are calibrated to field experience as well as centrifuge tests, and are conveniently related to blow count, $(N_1)_{60}$.

The model is able to capture the stiff pre-liquefaction stage, the onset of liquefaction at the appropriate number of cycles, and the very much softer post-liquefaction response observed in cyclic, simple shear-constant volume tests.

The coupled effective-stress approach corrects many drawbacks of the previous approaches. Although most parameters are related to blow count, and rely on a growing body of data and experience, it is always good practice to check on model parameters for each layer (using numerical simulation of a simple shear test) to verify that, if it has to liquefy in N cycles according to the field data during dynamic loading, it will. Also, because comparison with standard procedures may not be straightforward, it is recommended that the model be used with supervision from an experienced practitioner. With time, and with the increase of its usage, the model should become more prominent and be used for problems ranging from simple to complex with little effort.

The primary disadvantages are that (1) the logic for detection of stress reversal is based on horizontal shear stress only, and (2) the formulation applies only to two-dimensional analysis. See [Section 1.4.4.4](#) for additional discussion and numerical testing of the UBCSAND model.

Fully Coupled Effective-Stress Bounding-Surface Procedure

Bounding surface plasticity provides a framework to account for cyclic stress reversal in two and three dimensions (e. g., see Dafalias 1986, and Wang 1990). The models developed by Wang (1990) (herein named the WANG model) and Papadimitriou et al. (2001) (herein named the PAPADIMITRIOU model) are two (kinematic-hardening) constitutive models that have been implemented in *FLAC* based on that logic.

WANG Model – The WANG model is an effective stress, bounding-surface hypoplasticity model for (cohesionless) soil that is capable of reproducing, in detail, typical monotonic and cyclic, drained and undrained, hardening and softening behavior observed in classical laboratory tests on initially dense and loose soils (Wang, 1990). (The term hypoplasticity characterizes the dependence of loading and plastic strain-rate directions on stress-rate direction.)

The model formulation includes a noncircular pyramidal failure (bounding) surface, a loading surface, a surface of phase transformation (at which contractive behavior changes to dilative during shearing), and a critical state surface (defining an ultimate state in which the sand deforms at constant volume under constant stress). The three-dimensional effective-stress model requires the specification of 15 constants for a given sand (eight parameters are required for two-dimensional analysis). One disadvantage is that model calibration is an arduous task because most model constants are not related to properties with which the practitioner is familiar, and the body of available parameter data is not yet sufficiently well-developed. Also, comparison to state-of-practice analysis is not straightforward. The relation between cyclic stress ratio, number of cycles to liquefaction and normalized blow count can be used to calibrate the model (Wang et al. 2001), but there is no *direct* relation between empirical rules used in standard practice and theoretical laws used in the model theory. The WANG model is a sophisticated research tool for laboratory-scale experiments. Application of the model to study boundary-value problems at the field scale should probably not be attempted without the advice of the model developer, whose assistance may be required for model calibration, interpretation of results, and support on possible issues with numerical implementation.

PAPADIMITRIOU Model – The PAPADIMITRIOU model is an effective stress, bounding-surface model for loose and dense sand that is based on critical state elasto-plasticity (Papadimitriou et al. 2001, Papadimitriou and Bouckovalas 2002). The model applies to monotonic as well as cyclic loading (in two and three dimensions) of noncohesive soils under small and large strains. The model uses a kinematic hardening noncircular cone as the (loading) yield surface. In addition to bounding and dilatancy (marking the transition between contractive and dilatants behavior) surfaces, the model also contains a critical-state surface (defining an ultimate state in which the sand deforms at constant volume under a constant shear and confining stress). The model contains a total of 14 parameters: 11 of the parameters can be derived from in situ and laboratory tests, while the remaining three must be derived indirectly via trial-and-error simulations of drained and undrained laboratory tests. (Note that each parameter set is independent of initial and drainage conditions, as well as cyclic shear-strain amplitude.)

The model has the capability to reproduce, qualitatively, the characteristic behavior observed in cyclic experiments, including the degradation of shear modulus and increase of hysteretic damping with cyclic shear strain amplitude, the shear and volumetric strain accumulation at a decreasing rate with increasing number of cycles, and the increase in liquefaction resistance with density.

Comparisons with centrifuge experiments have been made (Andrianopoulos et al. 2006a), and the ability of the model to study a practical problem of geotechnical earthquake engineering has been demonstrated (Andrianopoulos et al. 2006b). The disadvantages are the model calibration, which is a rather tedious procedure and requires a test database not readily available in most cases, and the long computational time required for the solution of practical problems. Also, comparison to standard practice is not straightforward. Application of the model to study boundary-value problems at the field scale is not recommended at this time without the assistance of the model developer, for model calibration, interpretation of results, and eventual support for issues related to numerical implementation.

1.4.4.4 Comparison of Simple and Comprehensive Liquefaction Models

FLAC is able to follow the full nonlinear stress/strain behavior of soil or rock, provided that a suitable constitutive model is provided. In particular, a comprehensive constitutive soil model for dynamic liquefaction analysis should exhibit hysteresis loops (and, hence, damping) and progressive volume change with continuing cyclic shearing (to produce liquefaction). One such model is the UBCSAND model (described previously in [Section 1.4.4.3](#)). Alternatively, simple formulations for hysteretic damping (as discussed in [Section 1.4.3.4](#)) and liquefaction modeling (as discussed in [Section 1.4.4.2](#)) can be combined in a simple elastic/plastic model (such as Mohr-Coulomb) to produce comparable liquefaction behavior.

In this section, the UBCSAND model is compared to a simple liquefaction scheme (Finn/Byrne liquefaction formulation in the Mohr-Coulomb model) incorporating hysteretic (Hardin/Drnevich model) damping. To compare the predictions of the Finn/Byrne model to the UBCSAND model, we consider a constant amplitude, constant volume, cyclic-shear test. There is no applied load; therefore, the pore pressure remains constant. First, we compare the volumetric behavior of the two models, and then we compare the energy dissipation.

The *FLAC* grid for this test consists of one zone with unit dimensions. The initial conditions of the test are:

$$\sigma_{yy} = -150.0$$

$$\sigma_{xx} = -100.0$$

$$\sigma_{zz} = -125.0$$

$$p = 50.0$$

The material properties for the UBCSAND model are:

$$\phi_{cv} = 33$$

$$\phi_f = \phi_{cv} + \frac{(N_1)_{60}}{10}$$

$$P_a = 100.0$$

where ϕ_f is the ultimate friction angle, P_a is the reference pressure, and $\frac{(N_1)_{60}}{10}$ is the normalized standard penetration test value.

The Finn/Byrne model properties used for the comparison are:

$$\phi = \phi_f$$

$$\psi = 0$$

$$K = K_{ini}^{UBCSAND}$$

$$G = G_{ini}^{UBCSAND}$$

The Finn/Byrne elastic properties, K and G , are selected to be equal to the values of tangent bulk and shear moduli recorded after the first step of calculation in the UBCSAND model simulation.

The simulations are conducted for four values of $(N_1)_{60}$ (5, 10, 20 and 30), and for two values of shear strain amplitude (0.01% and 0.02%).

Volumetric behavior – The volumetric behavior of the Finn/Byrne model in a constant-volume cyclic, shear test of constant amplitude under no applied load is calculated to be (see Eq. (1.99))

$$\Delta\sigma_m = K \Delta\epsilon_{vd} \quad (1.116)$$

Integration of the Finn/Byrne formula (Eq. (1.93)) with respect to cycle number, for constant shear strain amplitude gives

$$\epsilon_{vd} = \frac{\gamma C_1}{0.4} \ln(1 + 0.4n) \quad (1.117)$$

where n is the number of cycles. After substitution of Eq. (1.117) into Eq. (1.116), and some manipulation, we obtain

$$\frac{\Delta\sigma_m}{K \gamma C_1} = \frac{1}{0.4} \ln(1 + 0.4n) \quad (1.118)$$

Thus, in the framework of the Finn/Byrne model, and for the proposed cyclic test, the dimensionless stress measure appearing on the left side of Eq. (1.118) is predicted to be solely a function of the number of cycles.

For comparison of model behavior, we use the following dimensionless pressure measure:

$$-\frac{\Delta P'}{K \gamma C_1} \quad (1.119)$$

where P' is the mean in-plane effective pressure,

$$P' = -\frac{\sigma'_{xx} + \sigma'_{yy}}{2} \quad (1.120)$$

and $\Delta P'$ is the difference between the current and initial value of P' for the test. K is a constant equal to the UBCSAND constant $K_{ini}^{UBCSAND}$, and C_1 is related to the blow count, as defined in Eq. (1.102).

The Finn/Byrne model and UBCSAND model pressure predictions are compared for four different values of blow count in Figures 1.62 through 1.65. Also, in each figure, two different values of cyclic shear strain are considered.

The plots in Figures 1.62 through 1.64 show that for the Finn/Byrne model, the dimensionless pressure measure is independent of shear-strain modulus. This is expected, provided the response remains elastic as in Figures 1.62 through 1.64. This is in contrast to the UBCSAND behavior, which shows higher pressure measure for the higher amplitude of shear strain. This difference is attributed in part to the value of bulk modulus, which is constant for the Finn/Byrne model, and evolving with stress level, P' , and other quantities in the UBCSAND formulation (the K value used for scaling pressure is the constant bulk-modulus value assigned in the Finn/Byrne model, and equal to $K_{ini}^{UBCSAND}$).

Note that both the Finn/Byrne and UBCSAND pressure plots for $\gamma = 0.2\%$ exhibit a plateau at a value of dimensionless pressure of approximately 7 in Figure 1.65. This plateau corresponds to yielding of the sample at the ultimate value of friction angle ϕ_f .

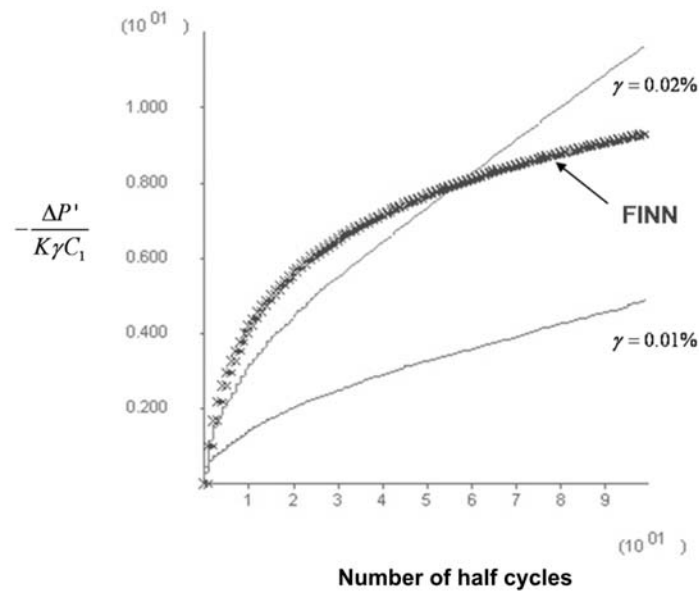


Figure 1.62 Pressure response in cyclic shear test for Finn/Byrne and UBCSAND models – $(N_1)_{60} = 30$

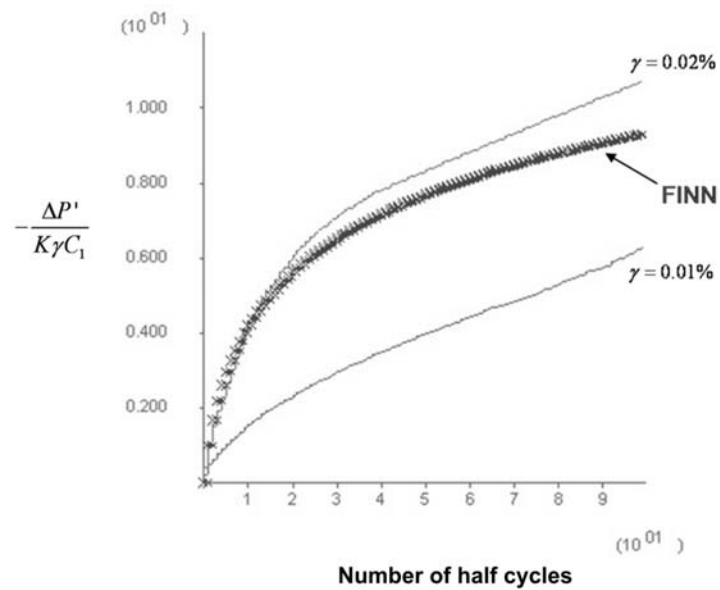


Figure 1.63 Pressure response in cyclic shear test for Finn/Byrne and UBC-SAND models – $(N_1)_{60} = 20$

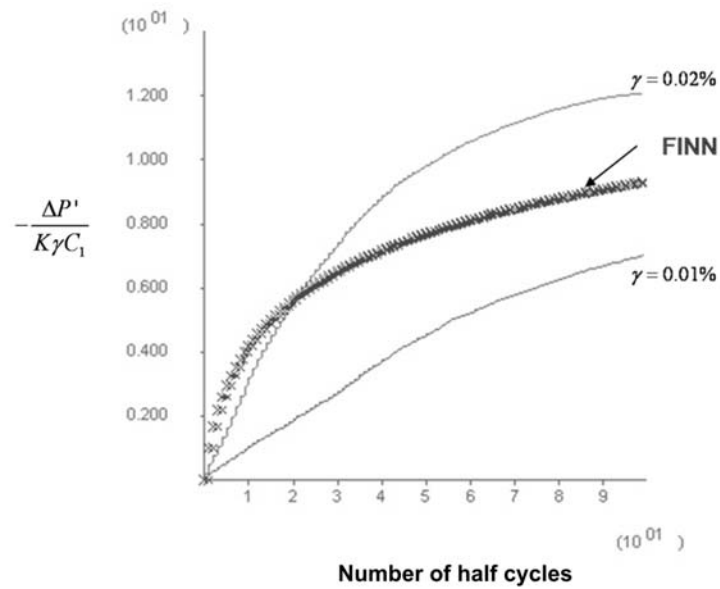


Figure 1.64 Pressure response in cyclic shear test for Finn/Byrne and UBC-SAND models – $(N_1)_{60} = 10$

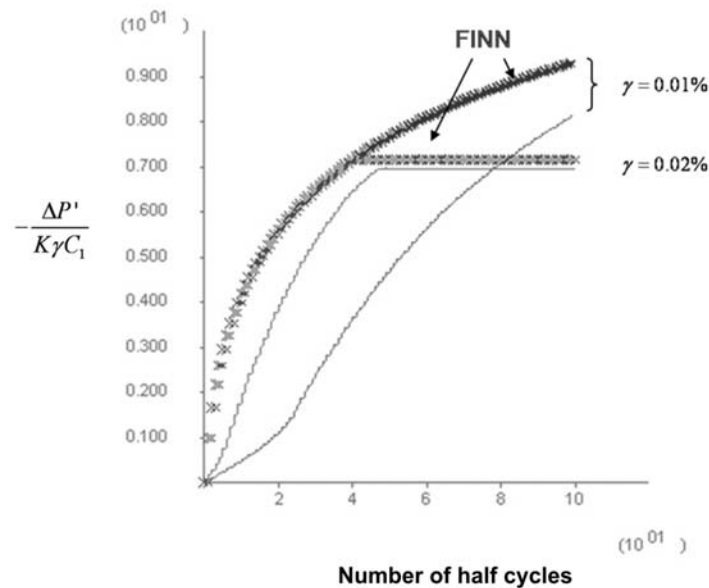


Figure 1.65 Pressure response in cyclic shear test for Finn/Byrne and UBC-SAND models – $(N_1)_{60} = 5$

Damping behavior – The Finn/Byrne model, which builds on the logic of a Mohr-Coulomb model, naturally accounts for damping arising from plastic flow (see [Section 1.4.3.11](#)). However, the model does not account automatically for cyclic energy dissipation during the elastic response. Some additional form of damping must be provided, in the form of Rayleigh or hysteretic damping. In this section, we apply hysteretic damping (with the Hardin/Drnevich formulation) to the Finn/Byrne model, and compare the damping ratio and shear modulus reduction factor from a cyclic shear test, at constant amplitude and constant volume, to those derived from the same test using the UBCSAND model.

We note that although the Finn/Byrne scheme in *FLAC* is intended for liquefaction modeling, it may still affect the response of a constant-volume shear test because a reduction in effective stress results in a reduction in shear strength.

The *FLAC* model, initial conditions and test properties are the same as those used previously for the comparison of volumetric responses. The values of shear and bulk moduli for the Finn/Byrne model are chosen to be equal to the values of tangent shear and bulk moduli recorded after the first calculation step in the UBCSAND model simulation. Also, friction is set equal to the ultimate value specified for the UBCSAND model, and dilation is zero.

The simulations are conducted for four values of blow count: $(N_1)_{60} = 5, 10, 20$ and 30 . Six values of shear strain amplitude are considered, ranging from 0.01% to 0.05% at an interval of 0.01% . The damping ratio and modulus-reduction ratio are calculated from the second loop of the constant-amplitude shear test. A convention is adopted for definition of the ratios because the strain loop is not “closed” for all cases investigated. [Figure 1.66](#) shows a typical loop with the parameters used for the definition of the ratios.

The energy dissipated in the cycle, ΔW , is calculated as the (algebraic) area under the loading curve (from τ^i to τ^m) minus the (algebraic) area under the unloading curve (from τ^m to τ^f). The “peak energy” in the cycle of amplitude, γ , is evaluated using $W = \tau^{av} \gamma / 2$. The damping ratio is then evaluated using the small ratio formula (i.e., $D = \Delta W / (4\pi W)$). The shear modulus ratio is estimated using the ratio of $(\tau^{av} + \tau^m) / (2\gamma)$ to the initial value of shear modulus, G . Also, for the model comparison, the Hardin/Drnevich parameter, γ_{ref} , is calibrated to match the damping ratio of the UBCSAND model at 0.03% strain.

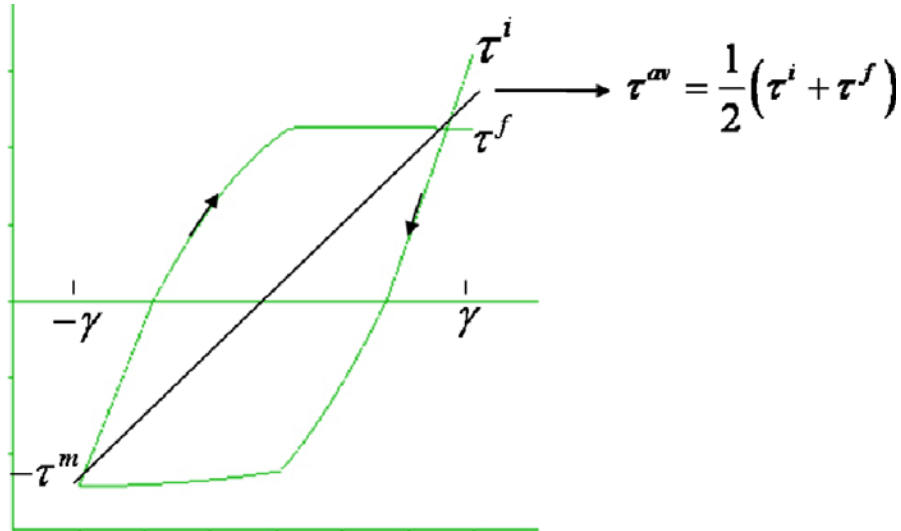


Figure 1.66 Definition of parameters used for damping and shear modulus measures

The results for damping ratio and modulus ratio for the four different values of blow count are shown in [Figures 1.67](#) through [1.70](#).

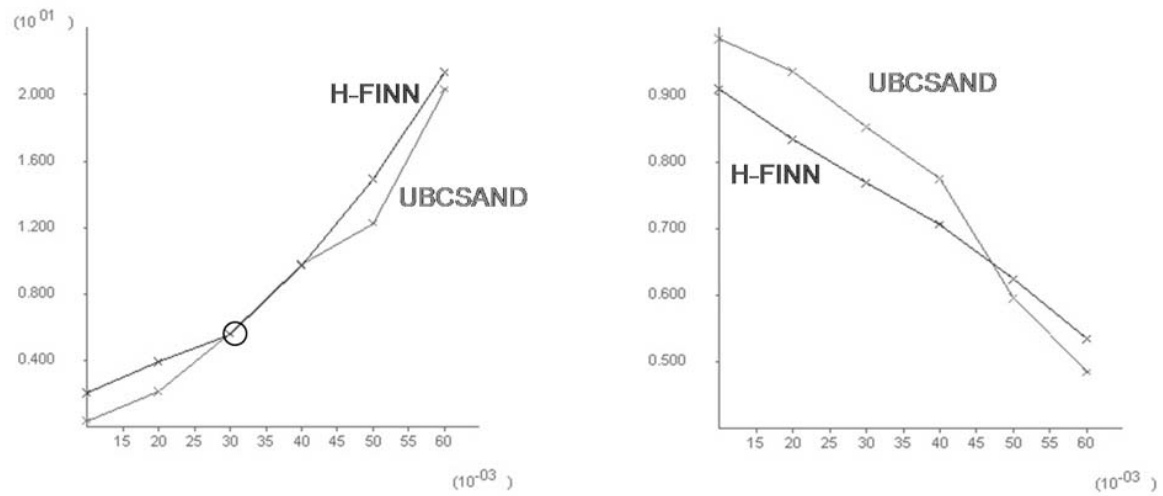


Figure 1.67 Damping ratio (%) and G/G_{max} versus shear strain (%) in cyclic shear test – $(N_1)_{60} = 30$

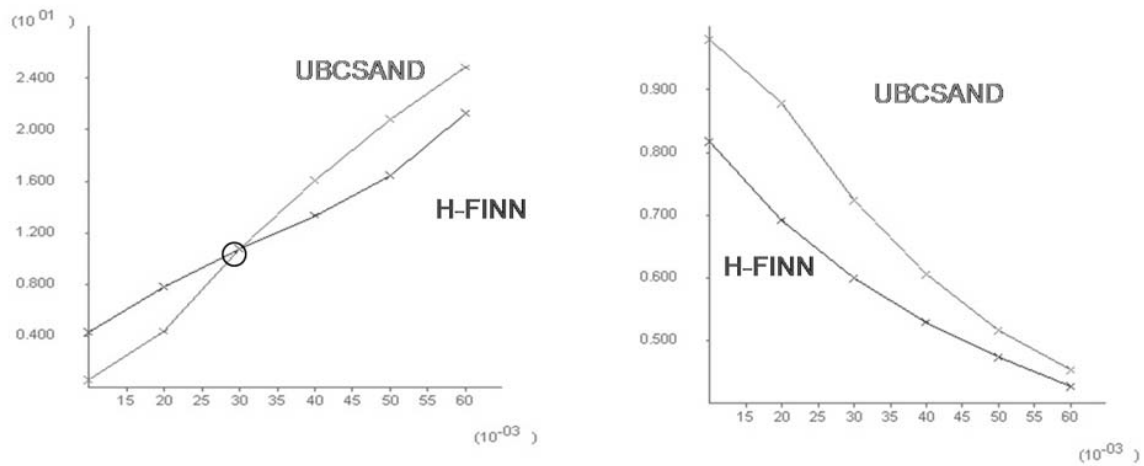


Figure 1.68 Damping ratio (%) and G/G_{max} versus shear strain (%) in cyclic shear test – $(N_1)_{60} = 20$

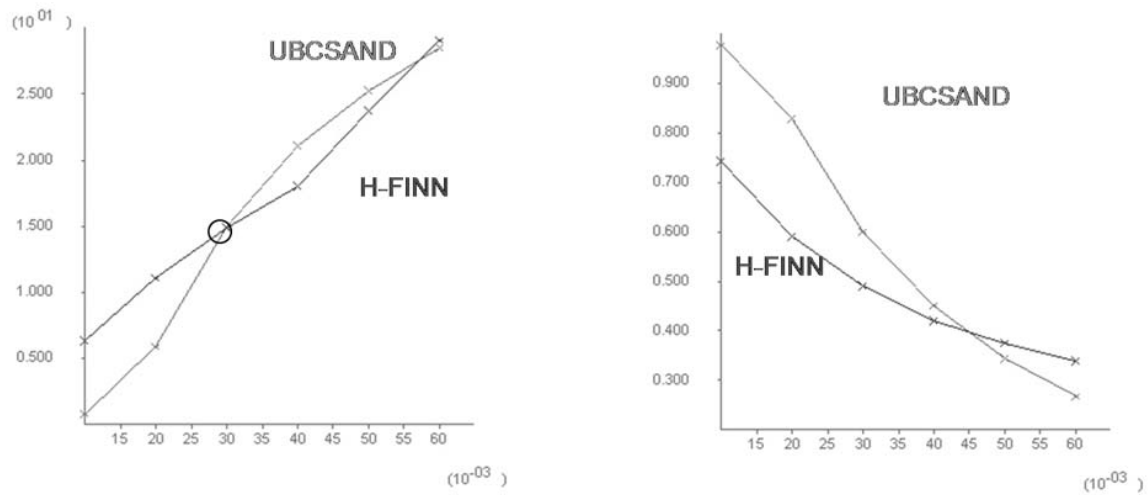


Figure 1.69 Damping ratio (%) and G/G_{max} versus shear strain (%) in cyclic shear test – $(N_1)_{60} = 10$

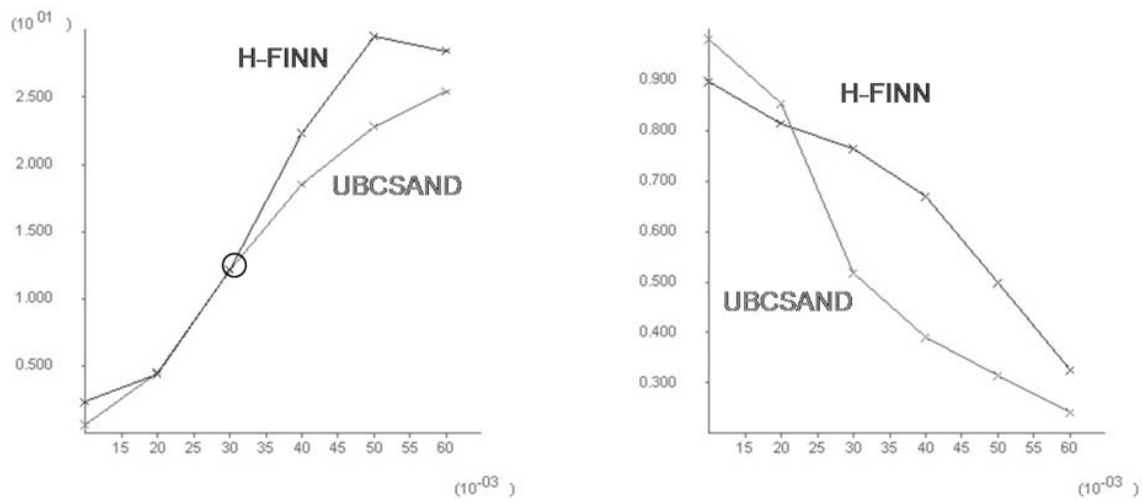


Figure 1.70 Damping ratio (%) and G/G_{max} versus shear strain (%) in cyclic shear test – $(N_1)_{60} = 5$

The results of the simulations show similar damping trends for the Finn/Byrne model with hysteretic damping and the UBCSAND model for a shear cycle of constant amplitude and constant volume. Also, the values of damping ratio and shear-modulus reduction ratio compare rather well given the complex elastic/plastic behavior involved. (The complexity is unavoidable because of the difficulty in exercising both models in only the elastic range.)

The main advantages of using the Finn/Byrne model combined with hysteretic damping are: (1) the model reproduces the main mechanisms of liquefaction; (2) it is a robust approach that works both in two-dimensional and three-dimensional analysis (because the logic for stress reversal is valid in both 2D and 3D); and (3) it gives a fair comparison to UBCSAND model behavior.

The inconveniences of the Finn/Byrne model with hysteretic damping are: (1) the pore-pressure generation is not smooth (because the pore-pressure update occurs when a half-cycle is completed); (2) the approach does not apply to monotonic loading; (3) the elastic stiffness, K , does not evolve with effective stress level; (4) the damping ratio is rather high during plastic flow; (5) the approach applies to over-consolidated soil (because the Mohr-Coulomb envelope does not evolve); and (6) the response may be too conservative (because dilation is assumed constant).

The UBCSAND model is a more comprehensive approach. The advantages of this approach are: (1) the correct physics are produced, based on laboratory test behavior; (2) the model works for monotonic and cyclic loading; (3) it is calibrated extensively with field case histories; (4) it is flexible and easy to use (e.g., most of the material properties are related to blow count); (5) the evolution of physical variables, including volumetric strain, is continuous; and (6) dilation is accommodated for stress states above a constant volume friction angle.

The disadvantages of the UBCSAND model are: (1) the logic for stress reversal is based on horizontal shear stress only; and (2) the formulation of the model applies only to two-dimensional analysis.

1.5 Solving Dynamic Problems

Approaches for modeling dynamic problems are described in the following three subsections. The first subsection ([Section 1.5.1](#)) discusses procedures for dynamic, mechanical-only calculations. The second subsection ([Section 1.5.2](#)) discusses dynamic coupled analyses, including the effect of groundwater on the dynamic response. The first two subsections include simple examples to illustrate the modeling approaches. In the third subsection ([Section 1.5.3](#)), recommended steps are given for a full-scale seismic analysis of an earth structure with the soil behavior represented by a nonlinear effective-stress material model including liquefaction behavior. [Section 1.6](#) contains an example that illustrates the application of these steps.

1.5.1 Procedure for Dynamic Mechanical Simulations

Dynamic analysis is viewed as a loading condition on the model, and as a distinct stage in a modeling sequence, as described in [Section 3.5](#) in the **User's Guide**. A static equilibrium calculation always precedes a dynamic analysis. There are generally four components to the dynamic analysis stage:

1. Ensure that model conditions satisfy the requirements for accurate wave transmission (by adjusting zone sizes with the **GENERATE** command – see [Section 1.4.2](#)). This check must be performed even before the static solution is performed, because gridpoints must not be relocated by the user after the calculation starts.
2. Specify appropriate mechanical damping, representative of the problem materials and input frequency range. Use the **SET dy.damp** or **INITIAL dy.damp** command, as described in [Section 1.4.3](#).
3. Apply dynamic loading and boundary conditions (by using the **APPLY** and **INTERNAL** commands – see [Section 1.4.1](#)). A given time history may need to be filtered in order to comply with the requirements noted in [Section 1.4.2](#).
4. Set up facilities to monitor the dynamic response of the model (by using the **HISTORY** command).

The procedure for dynamic analysis is illustrated by [Example 1.16](#), and then in [Example 1.17](#). The model is greatly simplified for rapid execution, but it still illustrates the steps in a dynamic analysis. Consider the problem of a structure built at the top of a soil slope. The slope is initially stable under the applied structural loading. The data file for the initial static loading stage is given below. The stress state of the model at equilibrium is shown in [Figure 1.71](#).

Example 1.16 Initial conditions for the slope problem

```
config dyn  ex 5
gr 20,10
m ss
gen line 5,3 9,10
```

```
mark i=1,6 j=4
mod null reg=1,10
prop s=400.0e6 b=666.67e6 d=1700 fri=40 coh=1.0e5 ten=1e10 ctab 1
table 1 0 1e5 2.0e-3 1e5 2.0e-3 0.0 3.0e-3 0.0 5.0e-3 0.0 1e-2 0.0
his nste=1
his ydis i=10 j=10
his unbal
his yvel i=10 j=10
fix x i=1
fix x i=21
fix x y j=1
set grav=9.81
set dyn off
solve
save stage1.sav
struct prop=1001 e=18e9 i=0.0104 a=.5 dens 2000.0
struct prop=1002 e=200e9 i=2.3e-5 a=4.8e-3 dens 2000.0
struct beam beg gr 11,11 end gr 12,11 seg=1 pr=1001
struct beam beg gr 12,11 end gr 13,11 seg=1 pr=1001
struct beam beg gr 13,11 end gr 14,11 seg=1 pr=1001
struct beam beg node 1 end 10,13 seg=2 pr 1002
struct beam beg 10,13 end 13,13 seg=2 pr=1002
struct beam beg 13,13 end node 4 seg=2 pr=1002
struct node=6 load 0 -1e6 0
struct node=8 load 0 -1e6 0
save stage2.sav
solve
save stage3.sav ; equilibrium with structure
```

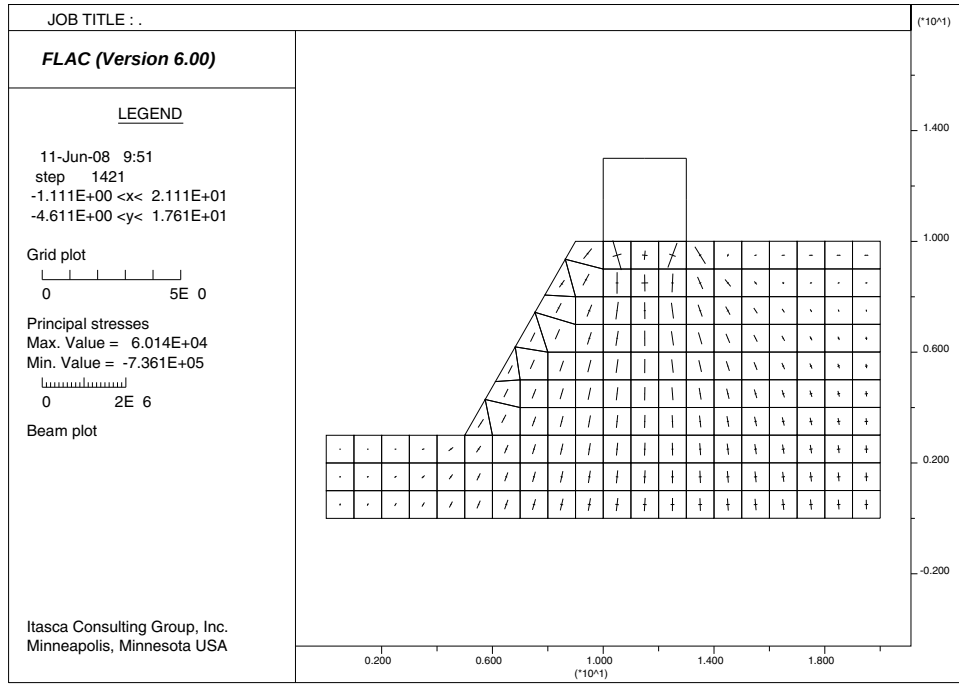


Figure 1.71 Initial equilibrium of structure on soil slope

The slope material is simulated as a strain-softening soil; the cohesion weakens as a function of plastic strain. This example demonstrates the development of slope failure as a consequence of loss of material strength following dynamic loading.

The four steps identified previously are now followed to prepare for dynamic analysis:

1. **Check Wave Transmission** – The dynamic loading for this problem is a sinusoidal velocity wave applied at the base of the model in the x -direction. The wave has an amplitude of 1 m/sec and a frequency of 10 Hz.

Based upon the elastic properties for this problem, the compressional and shear wave speeds are (from Eqs. (1.16) and (1.17))

$$C_p = 840 \text{ m/sec}$$

$$C_s = 485 \text{ m/sec}$$

The largest zone dimension for this model is 1 m. Based upon Eqs. (1.37) and (1.29), the maximum frequency which can be modeled accurately is

$$f = \frac{C_s}{\lambda} = \frac{C_s}{10 \Delta l} \approx 48 \text{ Hz}$$

Therefore, the zone size is small enough to allow velocity waves at the input frequency to propagate accurately.

2. **Specify Damping** – The plastic flow associated with the strain-softening model can dissipate most of the energy and, hence, tends to make the selection of damping parameters less critical to the outcome of the analysis. This model was run with no damping, and with a small amount of Rayleigh damping (5%, at the natural frequency), to evaluate the influence of damping.

To estimate the lowest natural frequency for this model (used as a Rayleigh damping parameter), [Example 1.16](#) is run with **SET dyn on** and with no damping. A plot of velocity history ([Figure 1.72](#)) indicates that the dominant natural frequency of the system is approximately 25 Hz. This is unrealistically high, but the value reflects the simplifications made for this example.

3. **Apply Dynamic Loading and Boundary Conditions** – The **APPLY** command is used with the **hist** keyword to specify the dynamic input. The *FISH* function **wave** supplies the history (a sinusoidal wave of 1 m/sec amplitude, 10 Hz frequency and 0.25 sec duration). Free-field boundaries are invoked along the left and right boundaries to absorb energy.
4. **Monitor Dynamic Response** – Three velocity histories are located in the model: the first at the position of the applied input wave; the second along the slope face; and the third within the grid.

The data file for the dynamic stage is reproduced in [Example 1.17](#):

Example 1.17 Dynamic excitation of the slope problem

```

restore stage3.sav
def wave
; sinusoidal wave : ampl = 1 m/sec, freq = 10 Hz, duration = .25 sec
  freq = 10
  wave = 1.0 * sin(2.0*pi*freq*dytime)
  if dytime > 0.25 then
    wave = 0.0
  end_if
end
; set dy_damp struct rayl 0.05 25
; set dy_damp rayl 0.05 25
apply ff
apply xvel=1.0 hist=wave j=1
apply yvel=0.0 j=1
set large
set dyn on
set dytime=0.0
ini xvel=0 yvel=0 xdis=0 ydis=0
hist reset

```

```

hist dytime
hist xvel  i=8 j=7
hist xvel  i=8 j=1
hist xvel  i=18 j=10
solve dytime = 0.5
save stage3.sav

```

The response of the slope at 0.5 sec (0.25 sec after the dynamic wave is stopped) is shown in [Figure 1.73](#). A rotational failure mechanism develops beneath the structure, resulting from the loss of cohesive strength. The velocity histories in [Figure 1.74](#) illustrate the input history (at $i = 8, j = 1$), the continuous movement at the slope face (at $i = 8, j = 7$), and the gradual return to equilibrium at a position remote from the slope (at $i = 18, j = 10$).

The response is similar for both no damping and 5% damping, although velocities are lower for the damped case. To see this, [Example 1.17](#) may be rerun with the **SET dy_damp rayl** and **SET dy_damp struct rayl** commands enabled (i.e., with the comment characters removed). The results are shown in [Figures 1.75](#) and [1.76](#). Note, also, that the structural damping has a minor influence on these results; if only **SET dy_damp rayl** is applied, the results are nearly the same as those shown in [Figures 1.75](#) and [1.76](#).

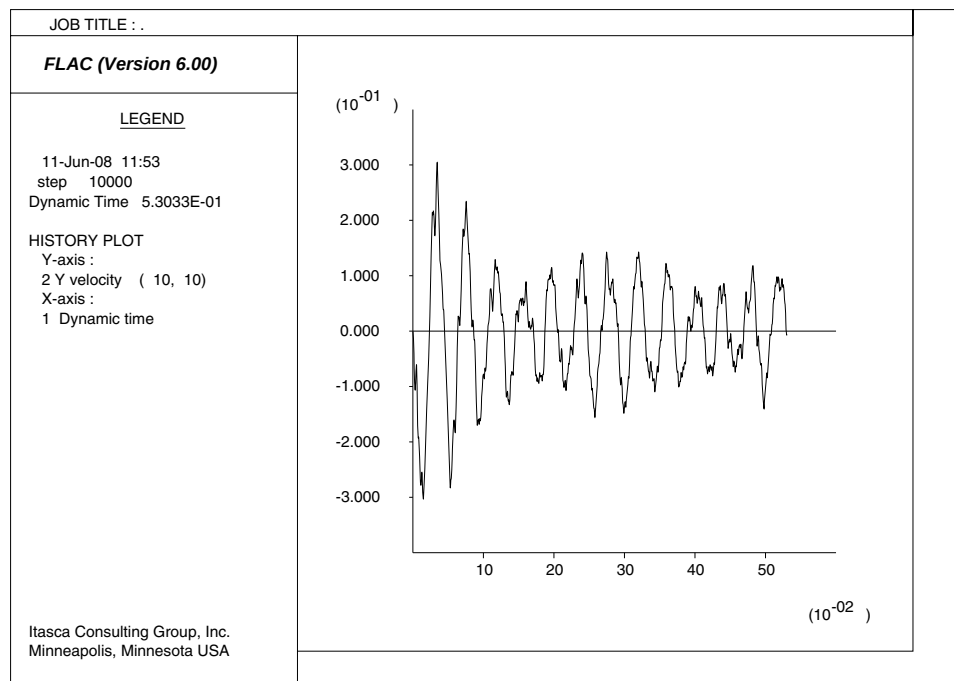


Figure 1.72 Velocity history, used to estimate lowest natural frequency

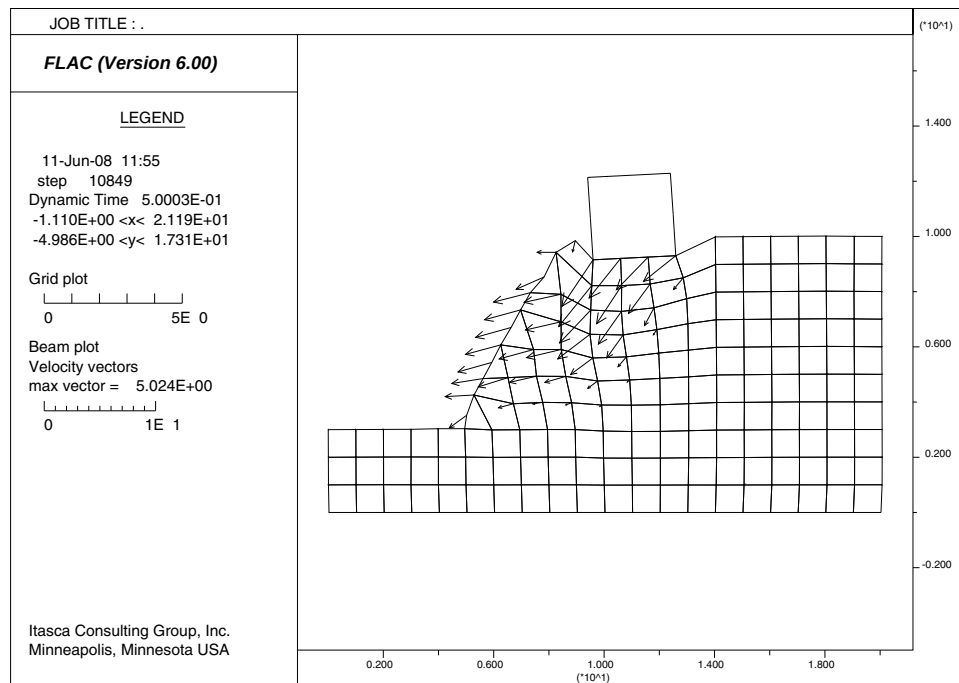


Figure 1.73 Slope failure resulting from dynamic loading
 – undamped simulation

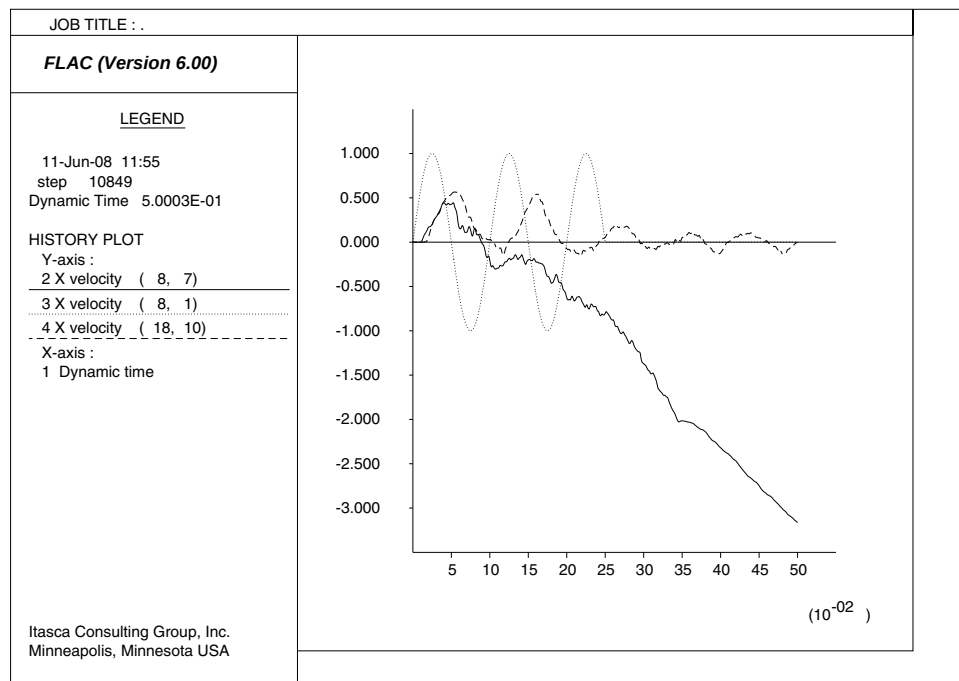


Figure 1.74 Velocity histories at base, slope face and remote from slope
 – undamped simulation

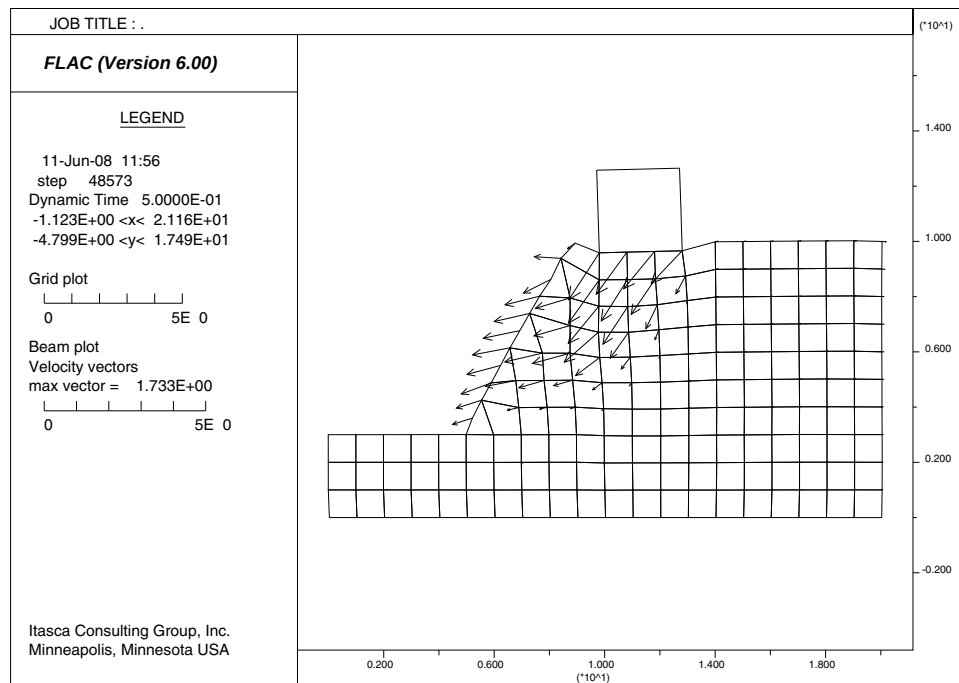


Figure 1.75 Slope failure resulting from dynamic loading
– with Rayleigh damping for soil and structure

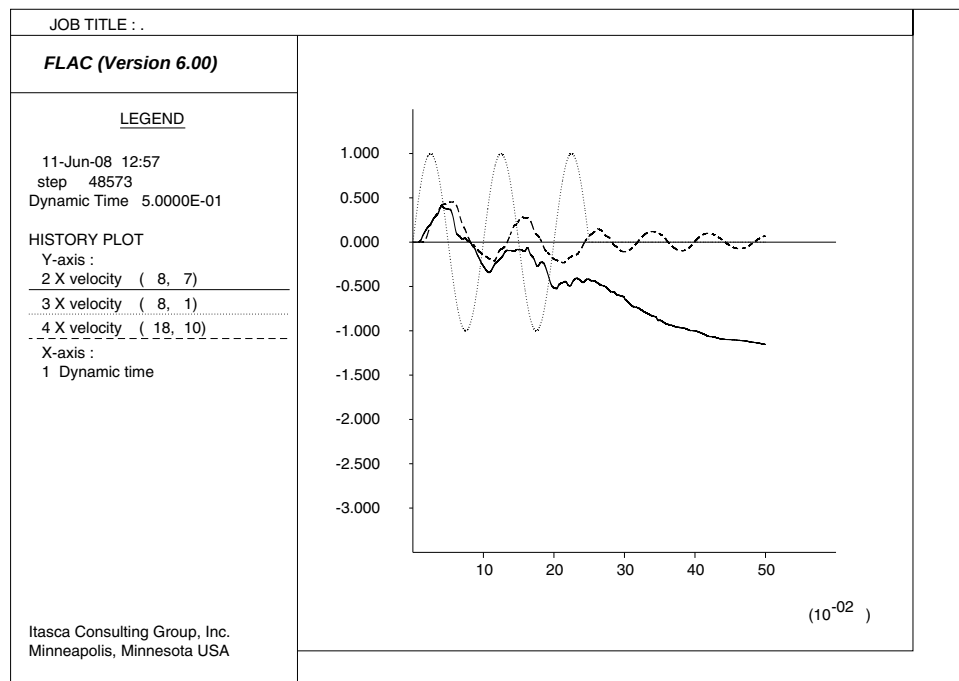


Figure 1.76 Velocity histories at base, slope face and remote from slope
– with Rayleigh damping for soil and structure

1.5.2 Procedure for Dynamic Coupled Mechanical/Groundwater Simulations

1.5.2.1 Undrained Analysis

Prior to performing a dynamic simulation with groundwater present, an equilibrium state must be obtained. This consists of several stages, which are illustrated by an analysis of the earthquake response of an idealized dam resting on a foundation (Figure 1.77 shows the original shape of the dam). Note that this dynamic example is not very realistic, as it subjects the dam to a few cycles of very high amplitude; however, it runs quickly and illustrates some important points.

First, the foundation is set in place and brought to equilibrium. The data for this initial stage is given in Example 1.18.

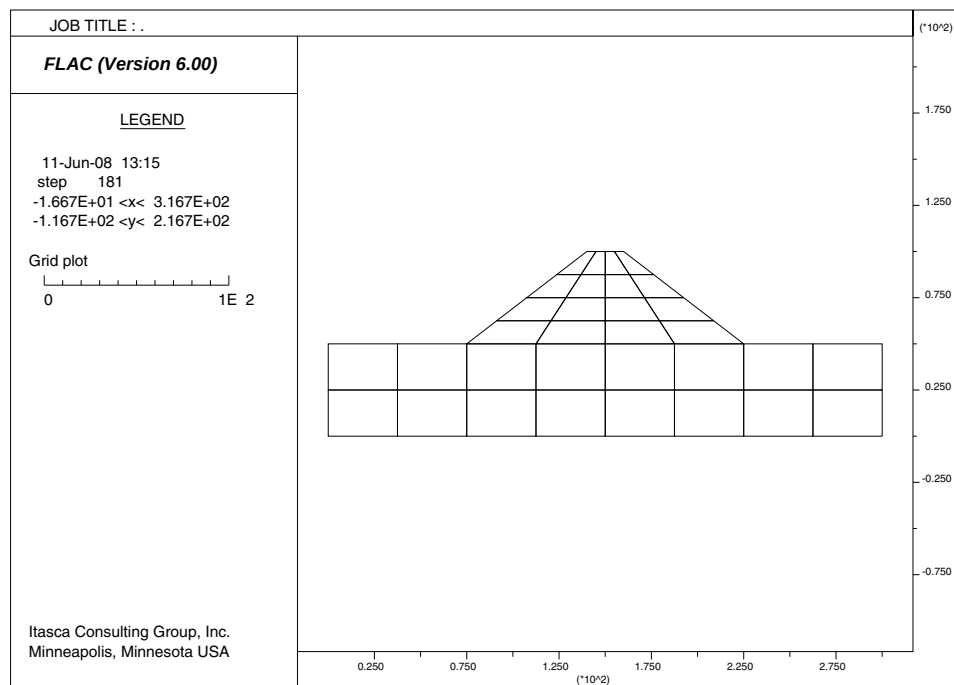


Figure 1.77 Dam resting on foundation

Example 1.18 Placement of foundation material

```
config dyn gw ex 5
grid 8 6
gen 0 0 0 50 300 50 300 0 i=1,9 j=1,3
gen 0 50 0 100 300 100 300 50 i=1,9 j=3,7
gen same 140 100 160 100 same i=3,7 j=3,7
model mohr j=1,2
prop dens 0.0017 poros 0.3 j=1,2
```

```

prop bulk 666.67 shear 400.0    j=1,2
prop cohes 0.2   fric 35        j=1,2
water bulk 0 dens 0.001 tens 1e10
prop perm 1e-8
ini syy=-1.0    var=0 1.0    j=1,2
ini sxx=-0.75   var=0 0.75   j=1,2
ini szz=-0.75   var=0 0.75   j=1,2
ini pp = 0.5    var=0 -0.5   j=1,3
fix x i=1
fix x i=9
fix y j=1
his unbal
set grav=10
save step0.sav ; equilibrium ... no steps necessary

```

- 1. Gravity Compaction of the Dam** – We create the dam in a single placement of material, which is saturated (see [Example 1.19](#) for the data file). If we are not interested in the *time* of settlement, we can set the bulk modulus of water to zero for this stage, so that numerical convergence is rapid.

Example 1.19 Gravity compaction of dam

```

rest step0.sav
set flow=off dyn=off
model mohr i=3,6 j=3,6
prop dens=0.0017 poros=0.3    i=3,6 j=3,6
prop bulk=333.33 shear=200.0  i=3,6 j=3,6
prop cohes=0.1 fric=35        i=3,6 j=3,6
water bulk 0 dens 0.001 tens 1e10
prop perm 1e-8
hist xdisp ydisp i=5 j=6
solve
save step1.sav

```

- 2. Fill Reservoir: Mechanical Response** – By applying a mechanical pressure to the upstream face of the dam, the dam responds mechanically. Note that this stage is imagined to take place rapidly, so that fluid flow is still not allowed.

Example 1.20 Fill reservoir

```

rest step1.sav
app press 0.5 var 0 -0.5 from 1,3 to 3,7
solve
save step2.sav

```

- 3. Allow Phreatic Surface to Develop** – Again, assume that we are only interested in the final flow pattern, not in the *time* it takes to occur. (If consolidation time *is* important, then consult [Section 1.8.6 in Fluid-Mechanical Interaction](#).) To allow rapid adjustment of the phreatic surface, we set the fluid modulus to a low value (1 MPa, compared with the “real” value of 2×10^3 MPa). We also do the fluid calculation and the mechanical adjustment separately (since the fully coupled solution takes much longer) – i.e., for this stage, **flow=on** and **mech=off**. The tensile limit for water is set to zero so that a phreatic surface develops. Pore pressure is applied to the upstream face, with fixed saturation of 1.0; on the other surfaces, pore pressure is fixed at its default value of zero.

Example 1.21 Develop phreatic surface in dam

```
rest step2.sav
water tens=0 bulk=1.0
app pp 0.5 var 0 -0.5 from 1,3 to 3,7
fix sat i=1,3 j=3
fix sat i=3 j=3,7
fix pp i=4,7 j=7
fix pp i=7 j=3,7
fix pp i=7,9 j=3
fix pp i=9
set flow=on mech=off ncwrite=50
his pp i 4 j 3
his pp i 4 j 2
his pp i 4 j 1
solve
save step3.sav
```

- 4. Mechanical Adjustment to New Flow Field** – Once the equilibrium flow field is established, we need to do a final mechanical adjustment, because: (a) some of the material is now partially saturated so the gravity loading is less; and (b) the effective stress has changed, which may cause plastic flow to occur. During this stage, we prevent fluid flow and pore pressure changes (setting fluid modulus temporarily to zero), since we are not concerned with the consolidation process here.

Example 1.22 Mechanical adjustment to new flow field

```
rest step3.sav
set flow=off mech=on ncwrite=10
water bulk=0
his reset
hist unbal
hist xdisp ydisp i=5 j=6
solve
```

```
water bulk=2e3
save step4.sav
```

We now have a system that is in mechanical and fluid equilibrium, ready for dynamic excitation; the fluid modulus is at the value for pure water (no entrained air). Note that the separation into several stages (just fluid or just mechanical) was done to reduce calculation time. The fully coupled simulation could be done if required.

- 5. Apply Dynamic Excitation to Dam** – The dynamic simulation may now be done. What is being modeled is the response of the dam and its trapped groundwater. It is assumed here that no fluid flow occurs, and that no pore-pressure generation occurs due to particle rearrangement. However, pore pressure changes do occur because of the dynamic volume changes induced by the seismic excitation. The excitation is by rigid sinusoidal shaking at the base of the foundation.

Example 1.23 Apply dynamic excitation to dam

```
rest step4.sav
set large, dyn=on ncwrite=20
def sine_wave
  sine_wave = 10.0*sin(2.0*pi*freq*dytime)
end
set dy_damp=rayleigh 0.05 1.5
set dytime 0.0 freq=0.5
ini xvel=0.0 yvel=0.0 xdisp=0.0 ydisp=0.0
prop tens=1e10
apply ff
apply yvel=0 xvel 1.0 hist sine_wave j=1
his reset
his dytime
his pp i 4 j 3
win 75 250 0 175
solve dyt 10
save step5.sav
```

Figure 1.78 shows the deformed grid, and Figure 1.79 shows the pore-pressure history in zone (4,3). The dashed lines represent the original shape.

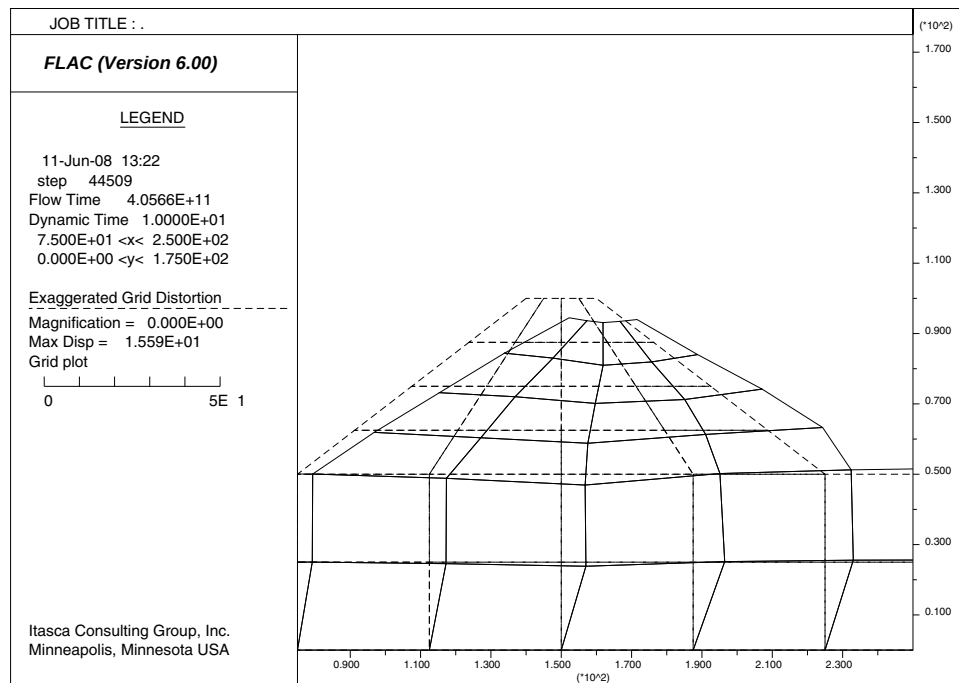


Figure 1.78 Deformation after 10 sec of shaking – Mohr-Coulomb model

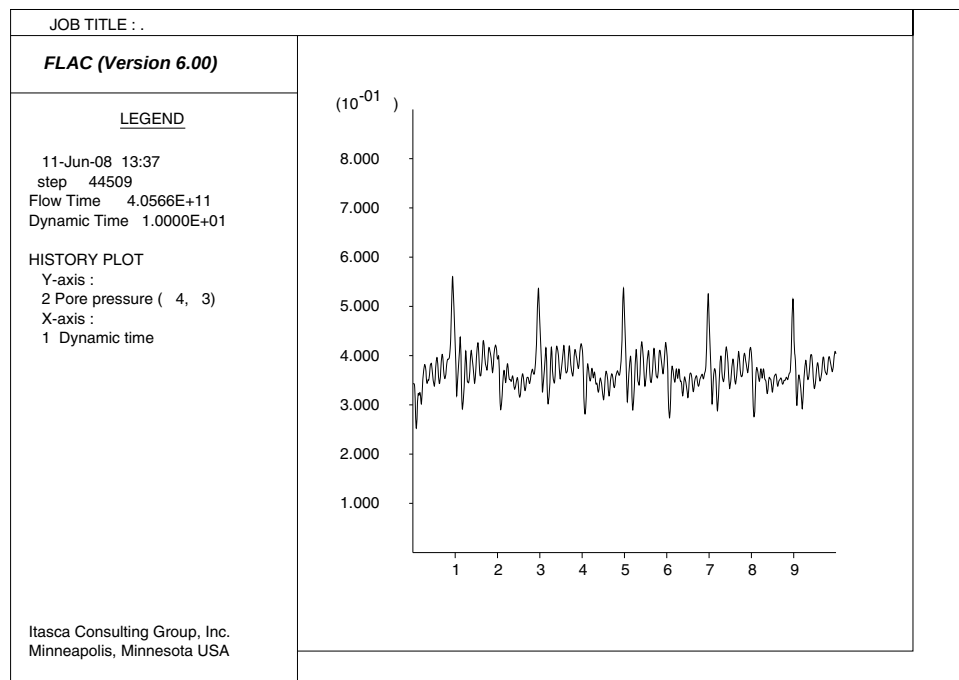


Figure 1.79 Pore pressure history at zone (4,3) – Mohr-Coulomb model

1.5.2.2 Dynamic Pore-Pressure Generation

The dynamic simulation given previously may be repeated with the Finn constitutive model, which replaces the Mohr-Coulomb model in the dam; the stresses remain, even though the model is replaced. The constants C_1 , C_2 , C_3 and C_4 are given the values that Martin et al. (1975) provide in their paper (although the condition $C_1 C_2 C_4 = C_3$, mentioned earlier, is violated slightly).

Example 1.24 Apply dynamic excitation to dam with `finn` model

```

rest step4.sav
mod finn i=3 6 j 3 6
prop bulk=333.33 shear=200.0 coh=0.1 i=3,6 j=3,6
prop fric=35 ff_latency=50 i=3,6 j=3,6
prop ff_c1=0.8 ff_c2=0.79 i=3,6 j=3,6
prop ff_c3=0.45 ff_c4=0.73 i=3,6 j=3,6
prop tens=1e10
set large dyn=on ncwrite=20
def sine_wave
  sine_wave = 10.0*sin(2.0*pi*freq*dytime)
end
set dy_damp=rayleigh 0.05 1.5
set dytime 0.0 freq=0.5
ini xvel=0.0 yvel=0.0 xdisp=0.0 ydisp=0.0
his reset
his dytime
his pp i 4 j 3
set step 100000 clock 1000000
win 75 250 0 175
; prop perm 0.1 i 3 6 j 3 6
; set flow on
apply ff
apply yvel=0 xvel 1.0 hist sine_wave j=1
solve dyt 10
save step5_f.sav

```

The same quantities as before are plotted – see [Figures 1.80](#) and [1.81](#). We now have considerable pore pressure build-up, and there is much larger horizontal movement in the dam; undoubtedly, liquefaction is occurring.

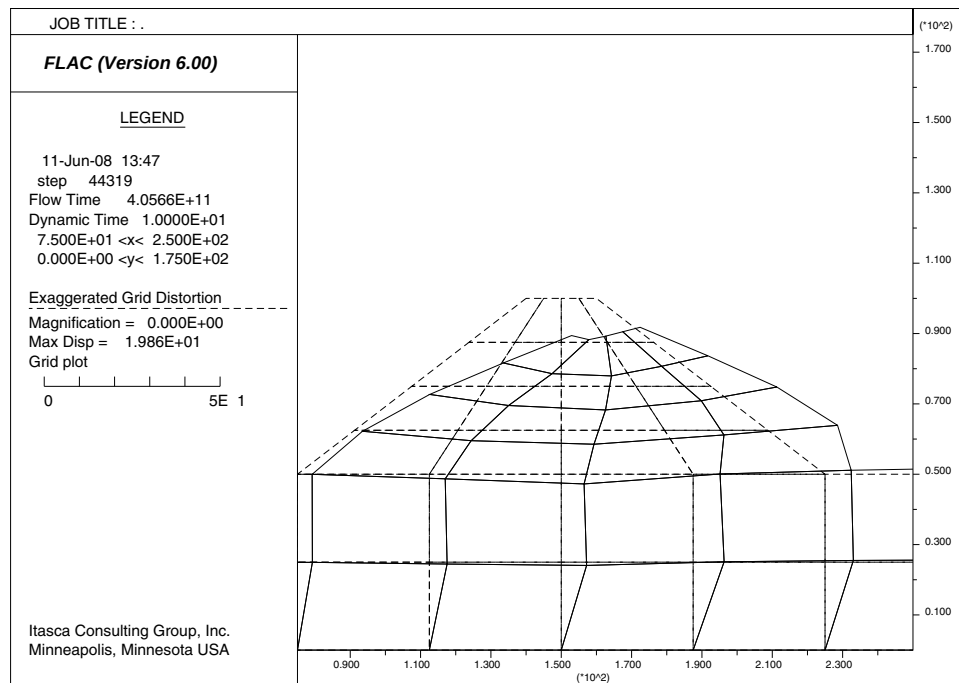


Figure 1.80 Deformation after 10 seconds of shaking – finn model

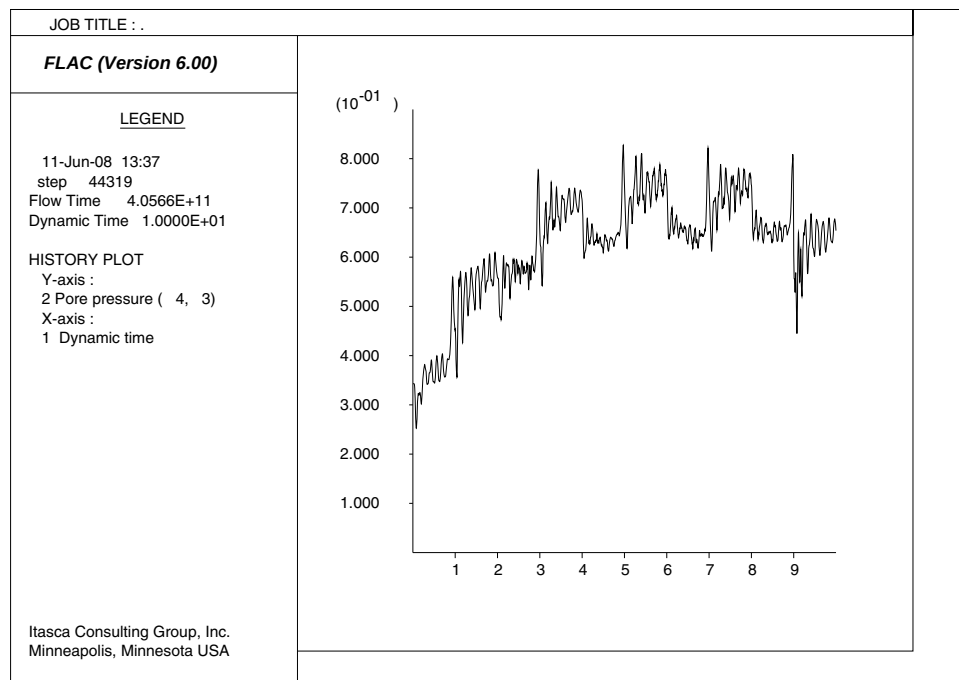


Figure 1.81 Pore-pressure history at zone (4,3) – finn model

1.5.2.3 Coupled Flow and Dynamic Calculation

Although very little dissipation of pore pressure is likely during seismic excitation in most structures, *FLAC* is able to carry out the groundwater flow calculation in parallel with the dynamic calculation. When both dynamic and groundwater options are selected together (**SET flow=on dyn=on**), the two timesteps (groundwater and dynamic) are forced to be equal; the overall timestep is set to whichever is the smallest. The previous example may be repeated with the addition of the following lines (the Finn model generates pore pressures and the flow logic dissipates them):

```
prop perm 0.5 i 3 6 j 3 6
set flow=on
```

The permeability is unrealistically high, for demonstration purposes. The resulting plot of pore pressure in zone (4,3) is shown in [Figure 1.82](#). There is clear evidence of pore pressures dropping off in the later stages of the simulation, but the situation is complicated because pressures generated in other zones appear to flow into zone (4,3) initially. Note that it is possible – in principle – for the pore pressure in a particular zone to *increase* when dissipation is allowed, if the surrounding zones contribute excess fluid.

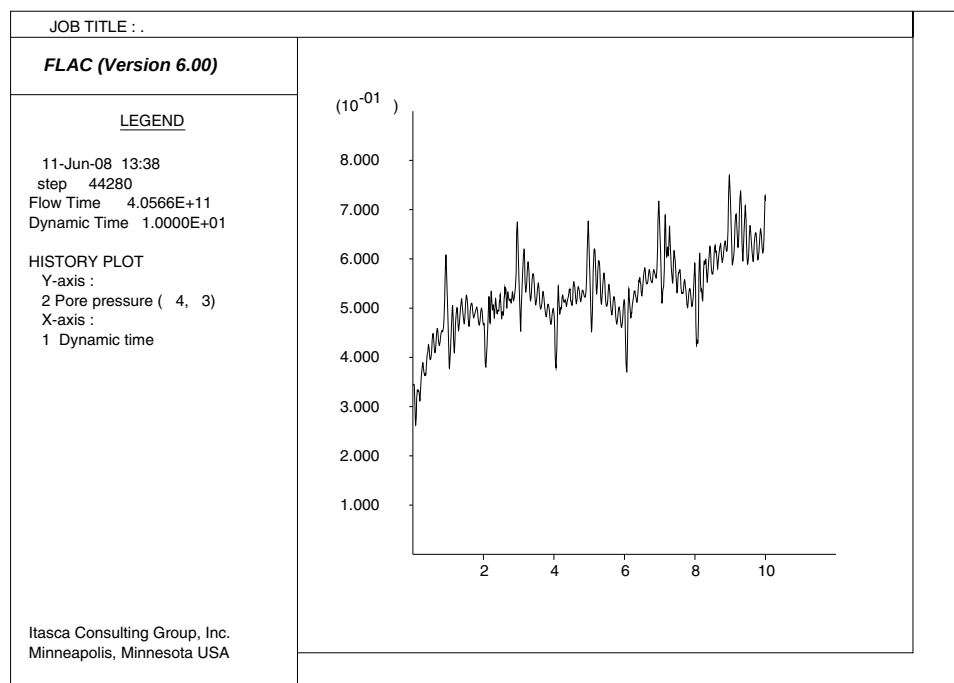


Figure 1.82 Pore pressure history at zone (4,3) – finn model, with dissipation

1.5.3 Recommended Steps for Seismic Analyses

A seismic analysis of an earth structure can be quite complicated, especially if there is a potential for dynamic liquefaction of the soil materials. The following steps are recommended to perform a seismic analysis using a nonlinear effective-stress material model. In this procedure, the nonlinear stress/strain behavior is represented by a “simple” elastic/plastic (Mohr-Coulomb) model, with formulations added to simulate liquefaction and hysteretic damping of the soils during cyclic loading. In general, these steps for performing a seismic analysis can also be applied for a more comprehensive liquefaction model.

The steps are listed and described below. [Section 1.6](#) provides an example application of this procedure for a seismic analysis of an embankment dam.

1. **Determine representative static and dynamic material characteristics** – The effective-stress analysis approach in *FLAC* is performed as a fully coupled mechanical-fluid flow analysis, which requires the selection of **CONFIG gw**, and assignment of drained stiffness and strength material properties, and the unsaturated (“dry”) density (or unit weight). In some situations, only undrained properties may be available. However, it is possible, with certain assumptions, to estimate drained properties from undrained properties. See [Section 1.9.4](#) in **Fluid-Mechanical Interaction** for guidelines on relating undrained properties to drained properties.

In addition, the fully coupled effective-stress calculation requires that the water bulk modulus be specified explicitly. The water bulk modulus must be selected carefully for this type of analysis. The behavior of the model depends on the stiffness ratio, R_k , as defined by [Eq. \(1.67\)](#) in [Section 1.8.1](#) in **Fluid-Mechanical Interaction**, and repeated here:

$$R_k = \frac{K_w/n}{K + 4G/3} \quad (1.121)$$

where K_w is the water bulk modulus, n is the porosity, and K and G are the bulk and shear moduli of the unsaturated soil. There is a temptation to decrease the water bulk modulus arbitrarily in order to increase the timestep (and reduce the simulation time). However, there are two cases to consider when selecting K_w :

1. If the calculated value of R_k based upon the given soil properties and an assumed value of $K_w = 4.18 \times 10^7$ psf, or 2 GPa (pure water), is greater than 20, then the water bulk modulus can be reduced such that $R_k = 20$ without affecting the results significantly (and reducing the simulation time).
2. If the calculated value for R_k , using the actual water bulk modulus, is less than 20, then that value of water bulk modulus should be used.

The water bulk modulus can be made to depend on the local elastic moduli, and can be specified differently for different soil units, provided that the two conditions listed above are satisfied.

The material response to dynamic cyclic loading is typically quantified by a shear modulus degradation curve and a damping ratio curve. Representative curves should be selected for each material in order to provide an accurate representation of wave attenuation and energy dissipation during dynamic loading. In addition, it may be necessary to make the curves depth-dependent to make the energy dissipation more realistic (e.g., see Darendeli 2001).

As discussed in [Section 1.4.4.3](#), a comprehensive material model to represent liquefaction behavior can be quite complex. The simplest forms of liquefaction models, as discussed in [Section 1.4.4.2](#), are based on a volume-change modification to the Mohr-Coulomb model. In this case, the only additional property is the blow count associated with the material. For example, see the Finn/Byrne formula defined by [Eq. \(1.93\)](#) and [Eq. \(1.102\)](#).

2. **Evaluate seismic motion characteristics** – A design earthquake ground motion is often provided as an acceleration record. It is typically an outcrop motion that is often recorded at a rock outcrop. It is important to know the location of the recorded motion because the motion may need to be modified for input to the *FLAC* model. The deconvolution analysis that is performed to obtain the appropriate input motion is discussed below in Step 6.

The characteristics of the seismic motion should always be checked because these characteristics can influence the model conditions. As discussed in [Section 1.4.2](#), the frequency content of the input motion affects the selection of mesh size for accurate wave propagation. [Eq. \(1.29\)](#) should be used to choose the appropriate maximum zone size for an accurate representation. If the highest frequencies associated with the input motion necessitate an extremely fine mesh (and a correspondingly small timestep), it may be possible to remove the high frequency components to permit a coarser mesh. If most of the power for the input is contained in the lower frequency components (say 80% to 90%) then the history can be filtered to remove the higher frequencies without significantly affecting the results. The *FISH* function ‘FFT.FIS’ (see [Section 3](#) in the ***FISH* volume**) can be used to evaluate the frequency content of the wave. The *FISH* function ‘FILTER.FIS’ which is also described in [Section 3](#) in the ***FISH* volume** can be used to perform the filtering.

For assessing frequency content, it is suggested that the input motion be evaluated in the form of a particle velocity (not acceleration) history. (An acceleration record can be converted into a velocity record using ‘INT.FIS,’ described in [Section 3](#) in the ***FISH* volume**.) In a plane wave propagating through a continuous medium, it is easy to show (e.g., see textbooks on wave propagation) that stress $\sigma = C\rho v$ (where C is the wave speed, ρ is the density and v is the particle velocity). Therefore, when considering the potential for yield or failure (which is determined by the level of induced stress), it is the particle velocity that is most relevant when evaluating the dominant frequencies, because the induced stress is directly proportional to velocity, not acceleration.

The input record should also be checked for baseline drift (i.e., continuing residual displacement after the motion has finished, see [Section 1.4.1.2](#)). The *FISH* function ‘INT.FIS,’ is used to integrate the velocity record to produce the displacement wave

form related to the input acceleration. If needed, a baseline correction can be performed by adding a low frequency sine wave to the velocity record; the sine wave parameters are adjusted so that the final displacement is zero. An example is given in [Section 1.6.1](#); see [Example 1.25](#).

3. **Estimate material damping parameters to represent inelastic cyclic behavior** – When using a simple elastic-perfectly plastic material model (such as the Mohr-Coulomb model) in a seismic analysis, it is necessary to incorporate additional material damping to account for cyclic energy dissipation during the elastic part of the response, as discussed in [Section 1.4.3.11](#). A difficult aspect is how to determine the appropriate material damping input. Generally, two different schemes are used: either Rayleigh damping (as described in [Section 1.4.3.1](#)), or hysteretic damping (as described in [Section 1.4.3.4](#)).

The equivalent-linear method, as applied, for example, in program *SHAKE*, is one way to estimate material damping input for the Rayleigh damping scheme and the hysteretic damping scheme. An equivalent-linear analysis is performed on a soil column, representative of the site conditions, using the shear wave speeds and densities for the different soil layers in the column, the modulus reduction and damping ratio curves selected as representative of the materials, and the target earthquake design motion for the site. Elastic strain-compatible values are then determined for the shear-modulus reduction factors and damping ratios. Average modulus-reduction factors and damping ratios can be estimated for each soil layer; these are the input parameters for the model with Rayleigh damping.

The anticipated range of cyclic shear-strain magnitudes for the given site conditions is needed to specify a best-fit range for the modulus reduction and damping ratio curves used with hysteretic damping (as discussed in [Section 1.4.3.5](#)). This range can be estimated from the range of equivalent uniform cyclic strains provided from a *SHAKE* simulation.

Note that these estimates are derived from the equivalent-linear analysis that assumes a low level of nonlinearity. Some adjustment in parameters may be required, especially if the actual model exhibits strong nonlinearity. This is discussed further in Step 7.

4. **Create appropriate model grid for accurate wave propagation** – The characteristics of the input motion are used to help select the appropriate mesh size and adjust the input wave record in order to provide an accurate solution in the seismic analysis. (See [Section 1.4.2](#) for further information on the relation between wave propagation characteristics and mesh size.)

In most seismic simulations, especially if excessive motion and strains are anticipated, a large-strain simulation should be performed in order to provide a more accurate deformation solution. When significant deformation and distortion of the grid is anticipated, it is important to minimize the number of triangular-shaped zones in the mesh and, in particular, those along slope faces. Triangular zones along slope faces are prone to become badly distorted during large-strain calculations, because triangular zones do not contain overlaid sets of subzones. Quadrilateral-shaped zones contain two overlaid sets

of subzones, which provide a more accurate calculation for materials undergoing plastic yield. (See [Sections 1.3.2](#) and [1.3.3.2](#) in **Theory and Background**.)*

- 5. Calculate static equilibrium state for site** – It is important to model the construction sequence of the earth structure as closely as possible in order to provide a reasonable representation of the initial, static shear stresses in the structure. This is important, particularly in a liquefaction analysis, because the initial static shear stresses can affect the triggering of liquefaction.

Simple analyses typically assume that the initial shear modulus (G_{max}) is uniform throughout each material unit. It may be more appropriate to vary G_{max} for soils as a function of the in-situ effective stress (e.g., see Kramer 1996). For example, it may be considered that the maximum shear modulus varies as a function of effective stress as defined by the Seed et al. (1986) expression,

$$G_{max} = 21.7 \times P_a \times K_{2,max} \left[\frac{\sigma'_m}{P_a} \right]^{0.5} \quad (1.122)$$

where P_a is the atmospheric pressure, $K_{2,max}$ is a constant determined from the relative density and σ'_m is the effective mean stress. This initial modulus variation can be implemented via *FISH* during the static loading stage.

It is important, if hysteretic damping is applied in the seismic simulation, to check the initial shear stress at the static equilibrium state. The hysteretic damping formulation is assumed to initiate hysteresis from an initial value of zero shear strain. If the initial shear stresses are high, then the shear stress and shear strain state for hysteretic damping may not be compatible. If shear stresses are high at the static equilibrium state of the model, then in order to ensure that shear stresses and strains are consistent during the dynamic phase, hysteretic damping should be invoked before the model is brought to the initial equilibrium state. See [Section 1.4.3.6](#) for a recommended procedure to incorporate hysteretic damping during the static solution stage.

- 6. Apply deconvoluted dynamic loading derived from target seismic record for site** – The input seismic record to the *FLAC* model should produce a calculated motion that can be matched to the target design earthquake motion. If the target motion is at a different location than the model input motion, then a deconvolution analysis should be performed to apply an input motion that will produce a motion that is comparable to the target motion at the target's location.

* If a badly distorted zone causes a calculation to stop prematurely in a mesh containing a few triangular zones, it may be possible to prevent this by increasing the strength of the individual zone. This should not significantly affect the model results, provided that the strengths of only a few zones are changed. Alternatively, the automatic rezoning logic in *FLAC* can be used to correct the mesh automatically during cycling when zones become badly distorted. See [Section 6](#) in **Theory and Background**.

It is important to note that a different form of the deconvoluted motion is applied for a rigid-base model than for a compliant-base model, in order to reproduce the target motion. (See [Section 1.4.1.6](#) for further explanation.) For the rigid-base boundary, the input motion to *FLAC* is the within motion calculated by *SHAKE-91* at the specified depth. For the compliant-base boundary, the input motion is the upward-propagating motion, which is half of the outcrop motion output by *SHAKE* at the specified depth. [Figure 1.83](#) illustrates the different base input motions:

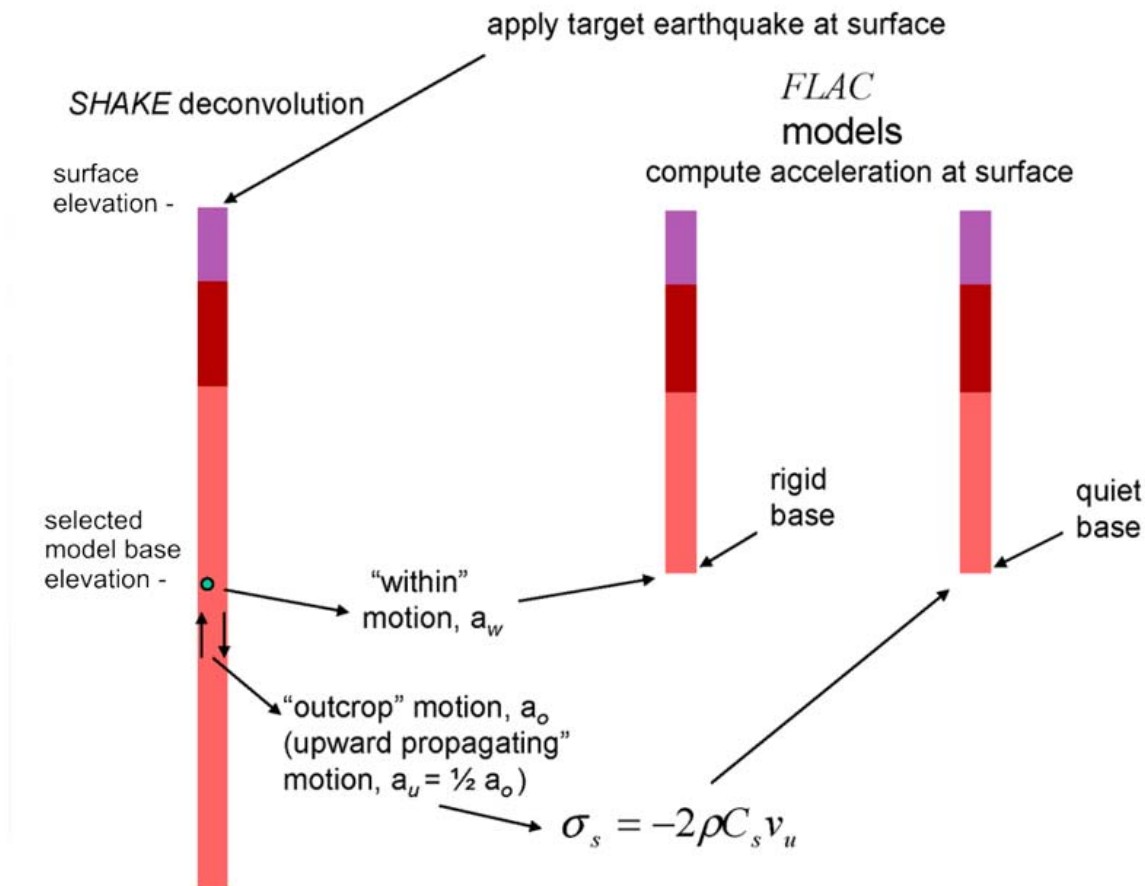


Figure 1.83 Deconvolution analysis to obtain base input motion

The dynamic boundary and loading conditions should be specified in a specific order for a seismic analysis. This is because the free-field boundaries that are used to represent the lateral extent of the far-field are assigned properties and initial conditions that are transferred from the main-grid zones adjacent to the free-field boundaries. Any changes to these zones or gridpoints after the free field is assigned are not seen by the free field.

The following sequence is recommended to assign conditions for a dynamic calculation, to ensure that these conditions are applied properly:

1. In the **SETTINGS** tool, turn on the dynamic calculation mode (**DYNA** tool) and the large-strain calculation mode (**MECH** tool).
2. Input the filtered and baseline-corrected input velocity that corresponds to the acceleration at the base of the model derived from the deconvolution analysis of the target acceleration. The input velocity can be called into *FLAC* via the **RUN/CALL** tool.
3. In the **UTILITY/HISTORY** tool, select various parameters to monitor during the dynamic simulation, such as gridpoint accelerations and zone pore pressures and stresses.
4. In the **IN SITU/INITIAL** tool, initialize the displacements and velocities in the grid, and specify the type of mechanical damping and parameters.
5. In the **IN SITU/APPLY** tool, press the **FREE-FIELD** button to assign the free-field boundaries. Then, assign the dynamic boundaries for the seismic loading.

In order to apply quiet boundary conditions along the same boundary as the dynamic input, the dynamic input must be applied as a stress boundary, because the effect of the quiet boundary will be nullified if the input is applied as an acceleration (or velocity) wave. A velocity record is converted into a shear-stress boundary condition using a two-step procedure:

1. Convert the velocity wave into a shear stress wave using the formula

$$\sigma_s = -2(\rho C_s) v_s \quad (1.123)$$

where: σ_s = applied shear stress;
 ρ = mass density;
 C_s = speed of *s*-wave propagation through medium; and
 v_s = input shear particle velocity.

Note that the factor of two in [Eq. \(1.123\)](#) accounts for the input energy dividing into downward- and upward-propagating waves.

2. Monitor the *x*-acceleration at the foundation surface during the dynamic run to compare this acceleration to the target acceleration. If material failure occurs within the model, this will affect the attenuation of the wave. Also, if the model is shallow, the free surface can cause an increase in the velocity and

acceleration that could extend to the base.* Some adjustment to the input stress wave may be required in order to produce an acceleration that is comparable to the target acceleration.

- 7. Perform undamped, elastic-material simulation** – Before running a dynamic model with actual material strength and damping properties, an elastic simulation should be made without damping, to estimate the maximum levels of cyclic strain and natural frequency ranges of the model system. If the cyclic strains are large enough to cause excessive reductions in shear modulus, then the use of additional damping is questionable. In such a case, the damping will be performing outside of its intended range of application. The model properties and input amplitude should be checked if excessive strains are calculated throughout the model.

Velocity histories should be monitored throughout the model to provide an estimate of the dominant natural frequencies of the model system. Also, shear-strain histories should be recorded to estimate maximum cyclic shear strain levels when no material damping is provided.

- 8. Perform simulations with damping and actual strength properties** – The results of the undamped run can be used to help select appropriate damping characteristics for the materials in the model. Additional damping may be prescribed for the model in order to damp the natural oscillation modes identified from the undamped simulation.

Acceleration histories should be checked first in the initial run with damping and actual strength properties, and compared to the target motion for the site. Some adjustment to the model input motion may be required to improve the comparison.

It is important to monitor several different variables during the seismic shaking phase. For example, shear stress/shear strain plots can illustrate the level of hysteretic damping that occurs throughout the model. Excess pore-pressure histories can help quantify the potential for liquefaction, and contours of cyclic pore-pressure ratio can delineate regions of liquefaction in a model. Example variables are shown in the practical exercise in [Section 1.6.1](#).

* This is a result of the velocity-doubling effect of the free surface. Note that the effect of a free surface on a simple sinusoidal velocity wave extends beneath the surface to, approximately, a depth of one-fourth of the wavelength of the wave transmitted through the medium. (The extent of velocity doubling can be shown simply by applying a wave to the base of a column of zones with a free surface, and monitoring the maximum amplitude experienced by each zone as the wave travels through the column.)

1.6 Example Application of a Seismic Analysis

An example application of a nonlinear seismic analysis is presented in this section. The example illustrates several of the topics discussed in this volume. The data files for this example are contained in the “ITASCA\FLAC600\Dynamic” directory.

1.6.1 Seismic Analysis of an Embankment Dam

1.6.1.1 Problem Statement

An analysis of the seismic performance of an embankment dam should consider static-equilibrium and coupled groundwater conditions, as well as fully dynamic processes. This includes calculations for (1) the state of stress prior to seismic loading, (2) the reservoir elevation and groundwater conditions, (3) the mechanical behavior of the foundation and embankment soils including the potential for liquefaction, and (4) the site-specific ground motion response. This example presents a *FLAC* model for an embankment dam that demonstrates a procedure to incorporate these processes and calculations in the seismic analysis.

The example is a simplified representation of a typical embankment dam geometry. The dam is 130 ft high and 1120 ft long, and is constructed above a layered foundation of sandstone and shale materials. The crest of the dam is at elevation 680 ft when the seismic loading is applied. The embankment materials consist of a low-permeability, clayey-sand core zone with upstream and downstream shells of gravelly, clayey sands. These soils are considered to be susceptible to liquefaction during a seismic event. The materials in this analysis are defined as foundation soils 1 and 2 and embankment soils 1 and 2, as depicted in [Figure 1.84](#).

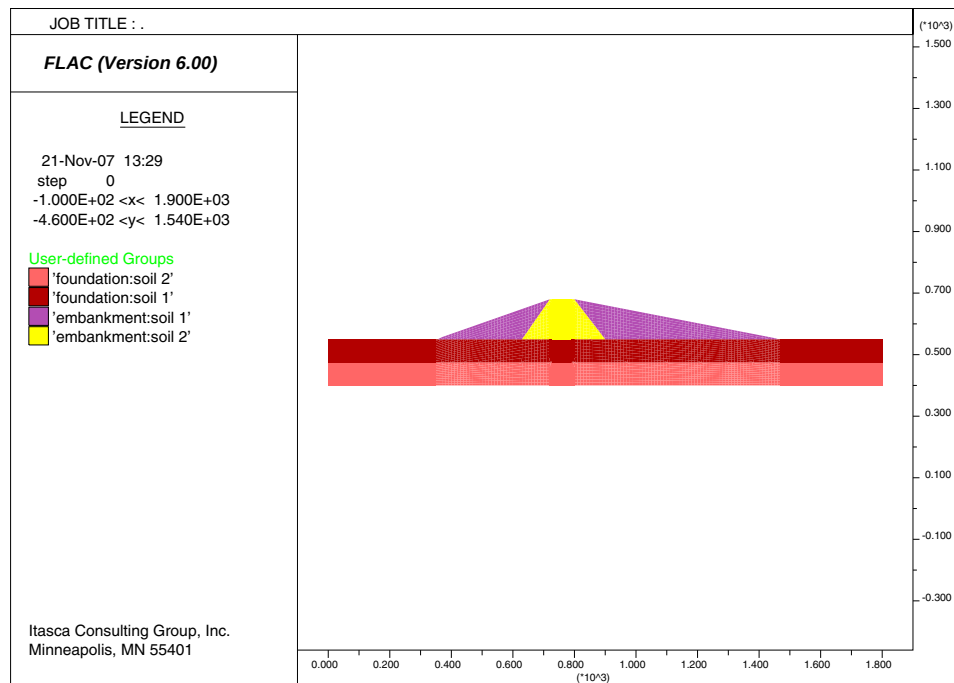


Figure 1.84 Embankment dam

The earth dam is subjected to seismic loading representative of the 1987 Loma Prieta earthquake in California. The earthquake target motion for this model is taken from that recorded at the left abutment of the Lexington Dam during the Loma Prieta earthquake and, for this analysis, the input is magnified somewhat and assumed to correspond to the acceleration at the surface of the foundation soils at elevation 550 ft. The target record is provided in the file named “ACC_TARGET.HIS.” The estimated peak acceleration is approximately 12 ft/sec^2 (or 0.37 g), and the duration is approximately 40 sec. The record is shown in [Figure 1.85](#):

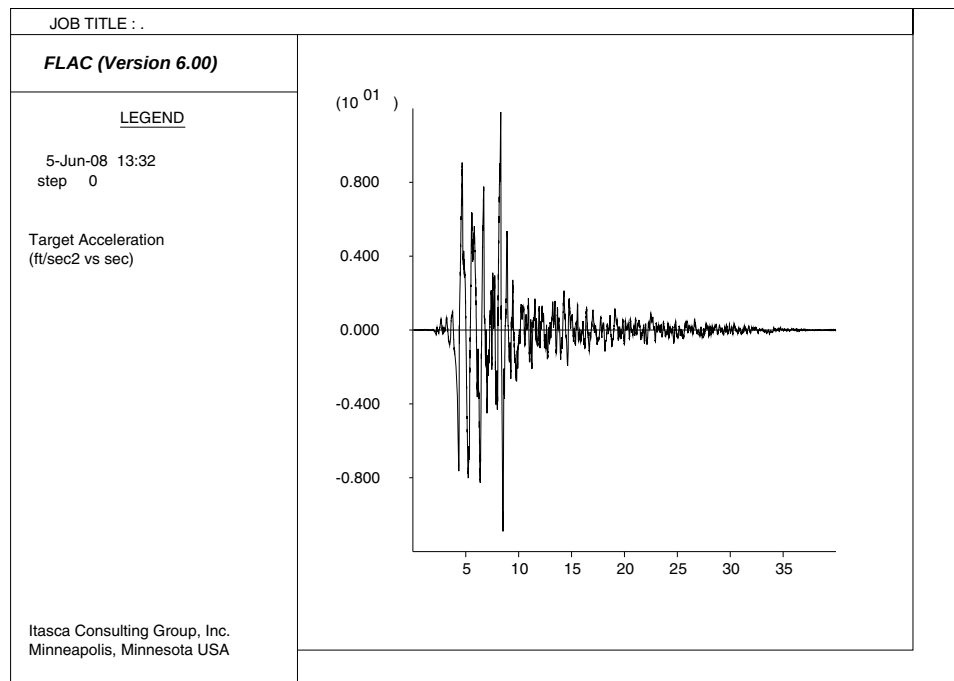


Figure 1.85 *Horizontal acceleration time history at elevation 550 ft – target motion*

1.6.1.2 Modeling Procedure

This example illustrates a recommended procedure to simulate seismic loading of an embankment dam with *FLAC*. A coupled effective-stress analysis is performed using a “simple” material model to simulate the behavior of the soils, including liquefaction. The soil behavior is based upon the Mohr-Coulomb plasticity model with material damping added to account for cyclic dissipation during the elastic part of the response and during wave propagation through the site. Liquefaction is simulated by using the Finn/Byrne model, which incorporates the Byrne (1991) relation between irrecoverable volume change and cyclic shear-strain amplitude (see [Eq. \(1.93\)](#)) into the Mohr-Coulomb model. The recommended steps to develop parameters for the simple model are described below.

The procedure is divided into eight steps:

1. Determine representative static and dynamic material characteristics of the soils, and estimate representative material properties. This includes an estimate for material damping parameters to represent the inelastic cyclic behavior of the materials.
2. Calculate the deconvoluted dynamic loading for the base of the model, derived from the target seismic record for the site, and evaluate the seismic motion characteristics.
3. Adjust input motion for accurate wave propagation, and create an appropriate model grid.
4. Calculate the static equilibrium state for the site including the steady-state groundwater conditions with the reservoir at full pool.
5. Apply the dynamic loading conditions.
6. Perform an undamped elastic material simulation to estimate the dominant frequencies of the site resonance and the maximum cyclic shear strains for the given site conditions.
7. Run a series of simulations with actual strength properties and representative damping, assuming the soils do not liquefy in order to evaluate the model response.
8. Perform the seismic calculation assuming the soils can liquefy.

These steps are described separately in the sections below.

1.6.1.3 Estimate Representative Material Properties

The foundation and embankment soils are modeled as elastic-perfectly plastic Mohr-Coulomb materials. Drained properties are required because this is an effective-stress analysis. The properties for the different soil types are listed in [Table 1.3](#):

Table 1.3 Drained properties for foundation and embankment soils

	Foundation		Embankment	
	Soil 1	Soil 2	Soil 1	Soil 2
Moist unit weight (pcf)	125	125	113	120
Young's modulus (ksf)	12,757	12,757	6,838	6,838
Poisson's ratio	0.3	0.3	0.3	0.3
Cohesion (psf)	83.5	160	120	120
Friction angle (degrees)	40	40	35	35
Dilation angle (degrees)	0	0	0	0
Porosity	0.3	0.3	0.3	0.3
Hydraulic conductivity (ft/sec)	3.3×10^{-6}	3.3×10^{-7}	3.3×10^{-6}	3.3×10^{-7}

The dynamic characteristics of all of the soils in this model are assumed to be governed by the modulus reduction factor (G/G_{max}) and damping ratio (λ) curves, as shown in [Figures 1.86 and 1.87](#), and denoted by the “SHAKE91” legend. These curves are considered to be representative of clayey soils with an average unit weight of 125 pcf and an average shear modulus of 6270 ksf; the data is derived from the input file supplied with *SHAKE-91* (for more information see <http://nisee.berkeley.edu/software/>).

The dynamic characteristics of the soils are simulated in the *FLAC* model in two different ways, for comparison. Simulations are made with either Rayleigh damping ([Section 1.4.3.1](#)) or hysteretic damping ([Section 1.4.3.4](#)) included with the Mohr-Coulomb model and with the Finn/Byrne model, to evaluate and compare their representation of the inelastic cyclic response of the soils during dynamic loading.

The equivalent linear program *SHAKE-91* is run to estimate material damping parameters to represent the inelastic cyclic behavior of the soils in the *FLAC* model, based upon the curves in [Figures 1.86 and 1.87](#). A *SHAKE-91* free-field column model is created for the foundation soils. The *SHAKE-91* analysis is performed using the shear wave speeds, densities, the modulus-reduction and damping-ratio curves for the two foundation soil layers, and the target earthquake motion specified for the site. Strain-compatible values for the shear-modulus reduction factors and damping ratios throughout the soil column are determined from the *SHAKE* analysis.

Average modulus-reduction factors and damping ratios can then be estimated for the foundation soils based upon the values calculated by *SHAKE-91*. The selected damping ratio and modulus-reduction parameters correspond to the equivalent uniform strain (which is taken as 50% of the maximum strain) for each layer. In this exercise, one value is selected as representative for all materials. The maximum equivalent uniform strain for the soils is calculated to be 0.08%, the average damping ratio is 0.063 and the average modulus reduction factor is 0.8. The damping ratio and modulus reduction factor values are the input parameters when Rayleigh damping is applied in the *FLAC* model. For a more comprehensive analysis, different values may be selected for the different soil units.

The maximum equivalent uniform shear strain calculated from the *SHAKE-91* simulation is used to specify the range over which the modulus reduction and damping ratio curves developed for hysteretic damping should best-fit the representative curves for the soils. The default hysteretic damping function ([Eq. \(1.48\)](#)) is used to best-fit the modulus-reduction factor and damping-ratio curves. The parameter values $L_1 = -3.156$ and $L_2 = 1.904$ for the default model provide a reasonable fit to both curves over the range of 0.08% strain, as shown in [Figures 1.86 and 1.87](#). Note that the parameters for the default model may need to be adjusted after the maximum shear strains are calculated from an undamped elastic *FLAC* model of the entire site (see [Section 1.6.1.9](#)).

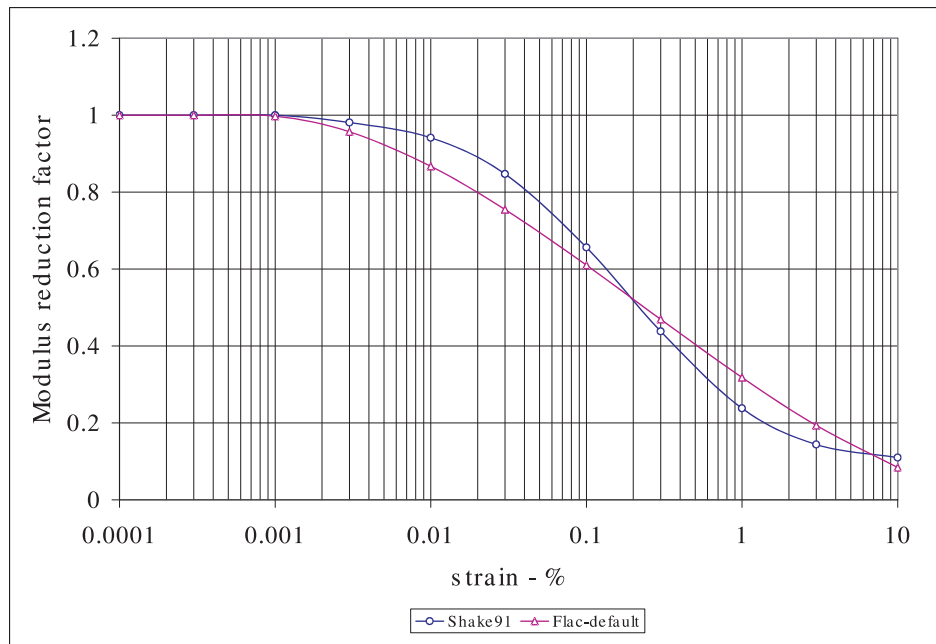


Figure 1.86 *Modulus reduction curve for clayey soils (from SHAKE-91 data)
FLAC default hysteretic damping with $L_1 = -3.156$ and
 $L_2 = 1.904$*

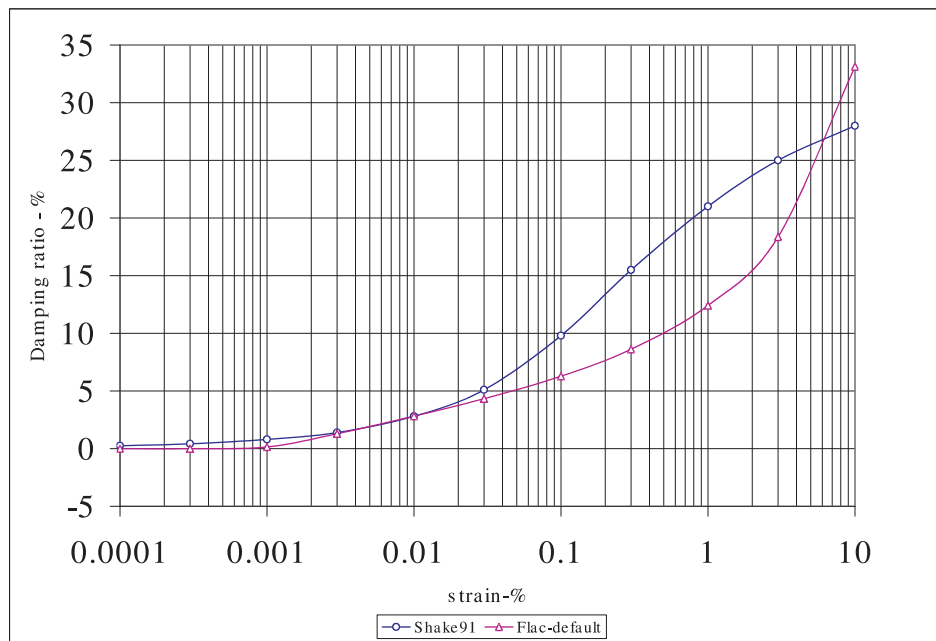


Figure 1.87 *Damping ratio curve for clayey soils (from SHAKE-91 data)
FLAC default hysteretic damping with $L_1 = -3.156$ and
 $L_2 = 1.904$*

The dynamic simulations in this example are fully coupled effective-stress calculations, which require that the water bulk modulus be specified explicitly. For the properties listed in [Table 1.3](#), and assuming the actual modulus $K_w = 4.18 \times 10^6$ psf for the site, the value of R_k (see [Eq. \(1.121\)](#)) for the foundation soils is approximately 0.8, and for the embankment soils it is approximately 1.5. In this example, a uniform value of $K_w = 4.18 \times 10^6$ psf is selected as representative of the actual condition.

The liquefaction condition is estimated for embankment soils in terms of Standard Penetration Test results. A normalized Standard Penetration Test value, $(N_1)_{60}$, of 10 is selected as representative for these soils. This value is used to determine the parameters C_1 and C_2 in the liquefaction model in *FLAC* (selected by setting the property **ff_switch** = 1 for the Finn model – Byrne formulation). For a normalized SPT blow count of 10, the Finn/Byrne model parameters are $C_1 = 0.4904$ and $C_2 = 0.8156$. See [Section 1.4.4.2](#) for a description of the formulation, and see Byrne (1991) for a discussion on the derivation of these parameters.

1.6.1.4 Perform Deconvolution Analysis and Estimate Seismic Motion Characteristics

The target motion provided for this example, [Figure 1.85](#), is assumed to correspond to the motion at the ground surface of the foundation soils near the site.* It is necessary to modify this motion to apply the appropriate seismic input at the base of the model (in this case at elevation 400 ft). The appropriate input motion at depth is computed by performing a deconvolution analysis using the equivalent-linear program *SHAKE*. This approach is reasonable, provided the model exhibits a low level of nonlinearity. A check on the approach is made in [Section 1.6.1.9](#).

SHAKE-91 is used in this example to estimate the appropriate motion at depth corresponding to the target (surface) motion. This deconvoluted motion should then produce the target motion at the surface. In this example, the upward-propagation motion calculated from *SHAKE-91* is used with a compliant-base boundary. (See [Figure 1.83](#).) Note that *SHAKE-91* accelerations are in g's versus seconds, and need to be converted into ft/sec² versus seconds when applied in *FLAC*. The input record (i.e., the upward-propagating motion from the deconvolution analysis and converted to ft/sec²) is in the file named "ACC.DECONV.HIS," and is shown in [Figure 1.88](#).

A Fast Fourier Transform (FFT) analysis of the input acceleration record (using "FFT.FIS" in [Section 3](#) in the *FISH* volume) results in a power spectrum as shown in [Figure 1.89](#). This figure indicates that the dominant frequency is approximately 1 Hz, the highest frequency component is less than 10 Hz, and the majority of the frequencies are less than 5 Hz. The dominant frequencies of the input velocity are also checked by first converting the acceleration record into a velocity record (using the *FISH* function "INT.FIS," described in [Section 3](#) in the *FISH* volume, to integrate the acceleration record), and then performing an FFT analysis to determine the power spectrum. [Figure 1.90](#) shows the resulting power spectrum for the velocity record. The dominant frequency is seen to be less than 1 Hz.

* This target motion is used for illustrating the principles of deconvolution analysis and may not be the typical case encountered in many practical situations. It is more common that the outcrop motion, recorded at the top of bedrock underlying the soil layers, is the target motion. Deconvolution in this case is described in Mejia and Dawson (2006), and is summarized in [Section 1.4.1.6](#).

The input record is also checked for baseline drift. The *FISH* function “INT.FIS” is used to integrate the velocity record again to produce the displacement wave form related to the input acceleration. The resulting residual displacement is found to be approximately 0.3 ft. A baseline correction is performed by adding a low frequency sine wave to the velocity record; the sine wave parameters are adjusted so that the final displacement is zero. (See “BASELINE.FIS” in [Example 1.25](#).) The uncorrected and corrected resultant displacement histories are shown in [Figure 1.91](#).

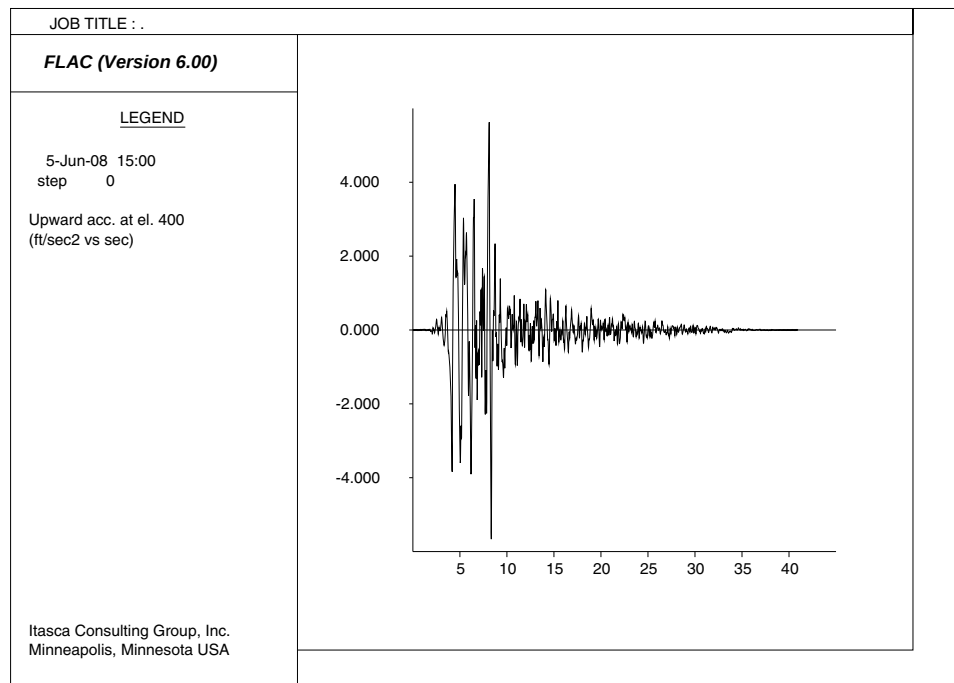


Figure 1.88 *Horizontal acceleration time history at elevation 400 ft
(upward-propagating motion from deconvolution analysis)*

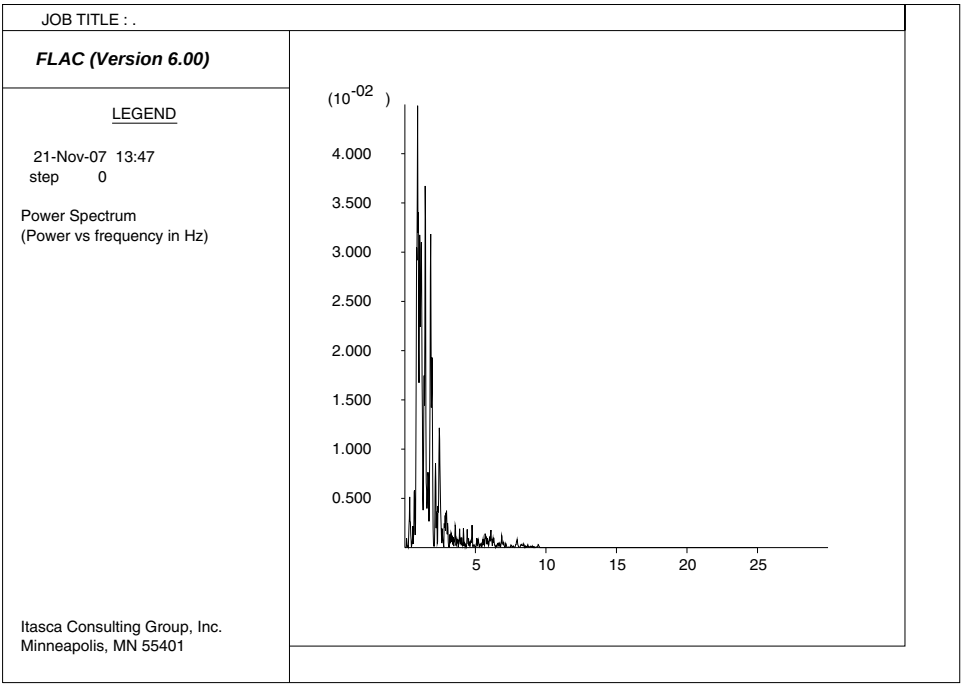


Figure 1.89 Power spectrum of input acceleration

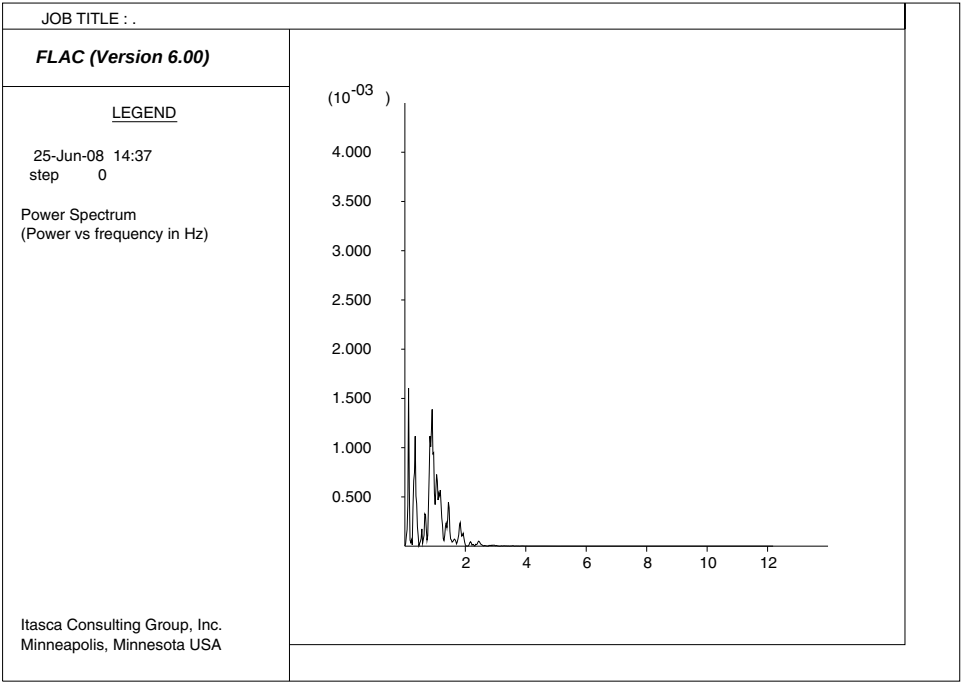


Figure 1.90 Power spectrum of input velocity

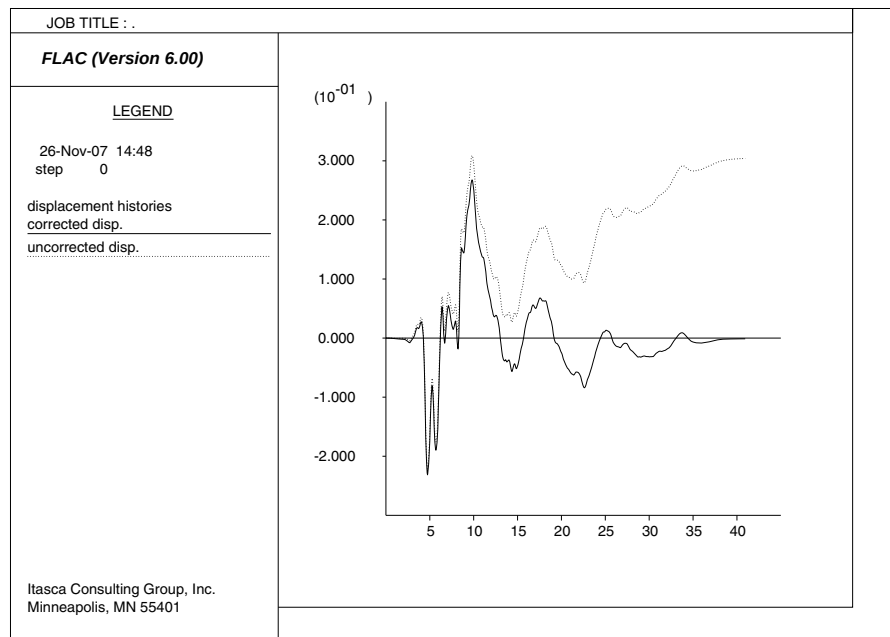


Figure 1.91 *Uncorrected and corrected displacement histories*

Example 1.25 *BASELINE.FIS – Baseline drift correction*

```

;Name:baseline
;Diagram:
;Note:Perform baseline correction with low frequency sine wave
;Input:itab_unc/int/102/uncorrected velocity table
;Input:itab_corr/int/120/low frequency sine wave correction
;Input:drift/float/0.3/residual displ. at end of record
;Input:ttime/float/40.0/total time of record
;Input:itab_cvel/int/104/baseline corrected velocity
def baseline
  npnts = table_size(itab_unc)
  loop ii (1,npnts)
    tt = float(ii-1) * ttime / float(npnts)
    vv = pi * tt / ttime
    cor_d = drift * pi / (2.0 * ttime)
    ytable(itab_corr,ii) = -(cor_d*sin(vv))
    xtable(itab_corr,ii) = tt
    ytable(itab_cvel,ii) = ytable(itab_corr,ii) + ytable(itab_unc,ii)
    xtable(itab_cvel,ii) = xtable(itab_unc,ii)
  endloop
end

```

1.6.1.5 Adjust Input Motion and Mesh Size for Accurate Wave Propagation

The mesh size for the *FLAC* model is selected to ensure accurate wave transmission (see [Section 1.4.2](#)). Based upon the elastic properties listed in [Table 1.3](#), embankment soil 2 has the lowest shear wave speed (840 ft/sec). If the largest zone size in the *FLAC* model is selected to be 10 ft in order to provide reasonable runtimes for this example, then the maximum frequency that can be modeled accurately is

$$f = \frac{C_s}{10 \Delta l} \approx 8.4 \text{ Hz} \quad (1.124)$$

Before applying the acceleration input record, it is filtered to remove frequencies above 5 Hz (by using the *FISH* function “FILTER.FIS” described in [Section 3](#) in the *FISH* volume). This filtering value is selected to account for the reduction in shear wave speed that may occur in some of the materials during the dynamic loading stage, as indicated in [Figure 1.86](#). The acceleration history filtered at 5 Hz is shown in [Figure 1.92](#), the power spectrum for the filtered acceleration wave is shown in [Figure 1.93](#), and the power spectrum for the corresponding velocity wave is shown in [Figure 1.94](#). Note that the difference between the frequency content of the unfiltered and filtered acceleration and velocity waves is minor (compare [Figures 1.89](#) and [1.90](#) to [Figures 1.93](#) and [1.94](#)).

The data file “INPUT.DAT,” listed in [Example 1.26](#), includes the different steps performed in filtering the input acceleration record, integrating this record to produce velocity and displacement histories, and correcting for baseline drift. The resultant, corrected velocity record, stored in table 104 in this data file, is the input motion for the embankment dam analysis.

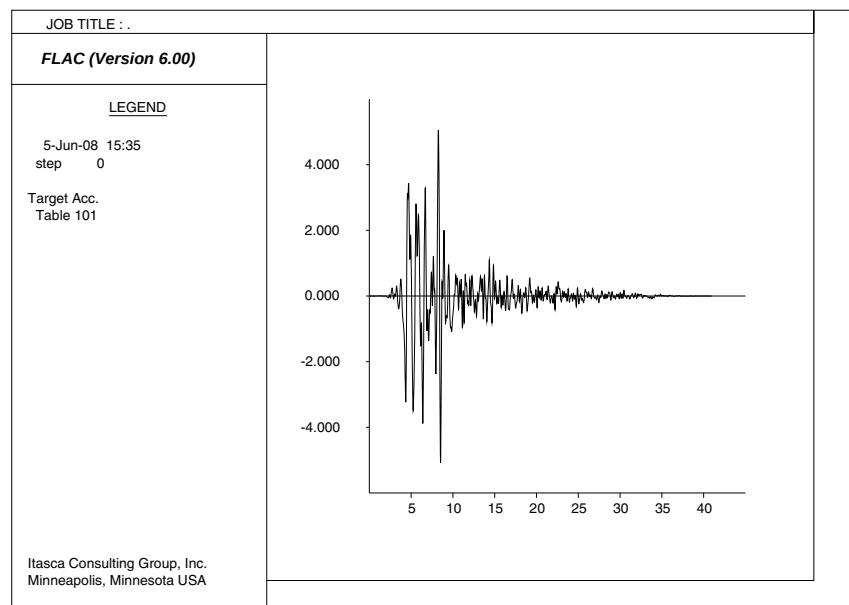


Figure 1.92 Horizontal acceleration time history at elevation 400 ft (upward-propagating motion from deconvolution analysis) with 5 Hz filter and baseline corrected

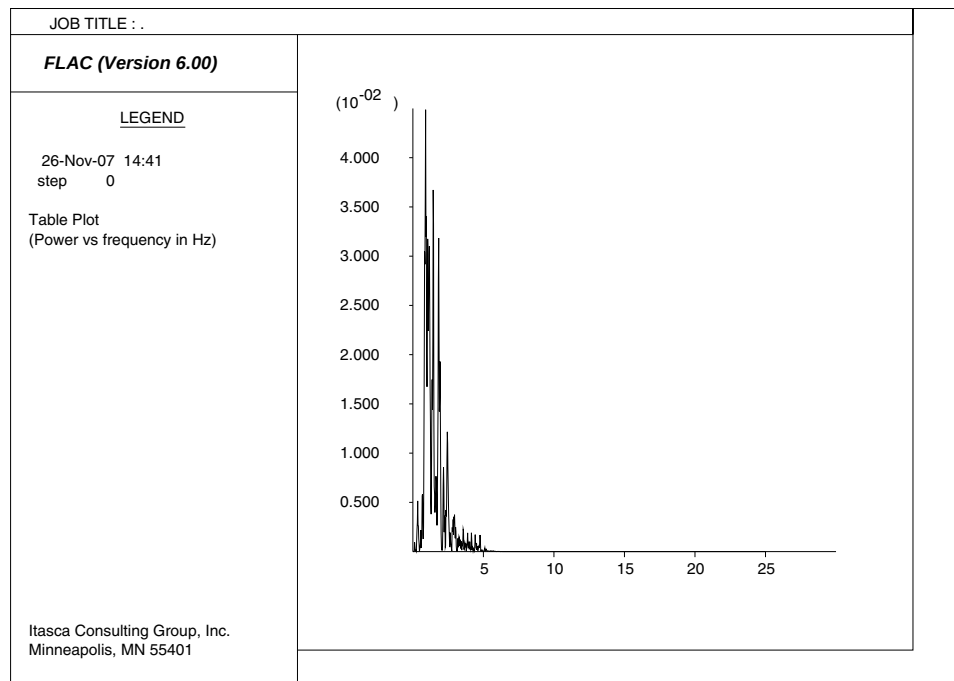


Figure 1.93 *Power spectrum of horizontal acceleration time history with 5 Hz filter*

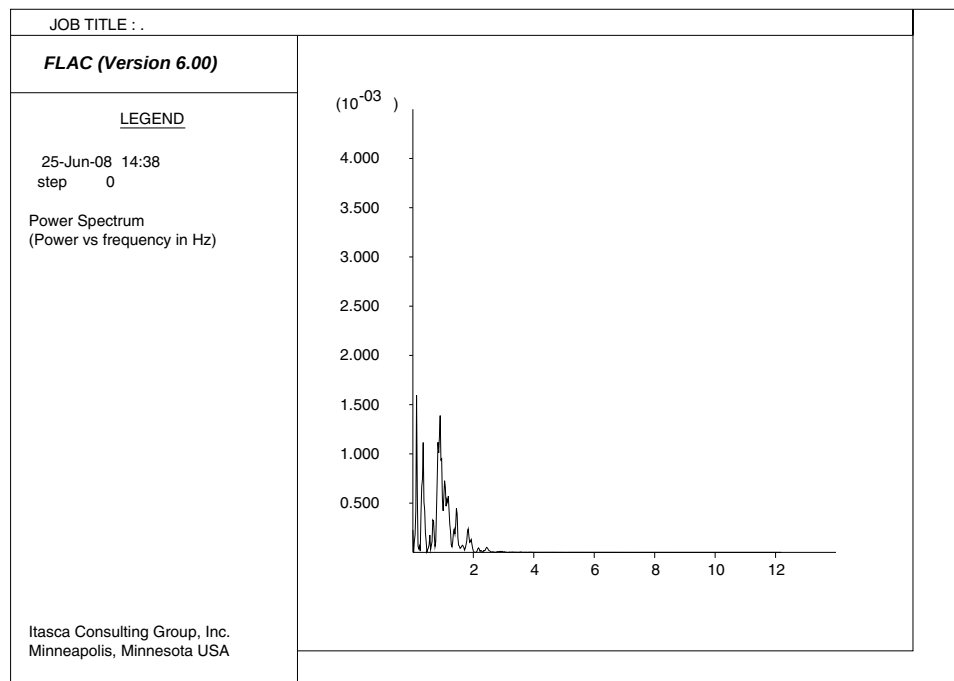


Figure 1.94 *Power spectrum of horizontal velocity time history with 5 Hz filter of acceleration history*

Example 1.26 INPUT.DAT – Input wave characterization

```
config
hist 100 read acc_deconv.his
hist write 100 table 100
save inp1.sav
;
set echo off
call fft_tables.fis
set fft_inp1=100 fft_inp2=110
fft_tables
set echo off
call Fft.fis
fftransform
set echo off
call INT.FIS
set int_in=100 int_out=201
integrate
set echo off
set fft_inp1=201 fft_inp2=210
fft_tables
set echo off
fftransform
save inp2.sav
;
restore inp1.sav
set echo off
call Filter.fis
set filter_in=100 filter_out=101 Fc=5
filter
set echo off
call fft_tables.fis
set fft_inp1=101 fft_inp2=110
fft_tables
set echo off
call Fft.fis
fftransform
save inp3.sav
;
set echo off
call INT.FIS
set int_in=101 int_out=102
integrate
set echo off
call INT.FIS
```

```
set int_in=102 int_out=103
integrate
save inp4.sav
;
restore inp3.sav
set echo off
call INT.FIS
set int_in=101 int_out=102
integrate
set echo off
call baseline.fis
set itab_unc=102 itab_corr=120 drift=0.3 ttime=40.0
set itab_cvel=104
baseline
set echo off
call INT.FIS
set int_in=104 int_out=103
integrate
set echo off
call INT.FIS
set int_in=102 int_out=105
integrate
set echo off
set fft_inp1=104 fft_inp2=210
fft_tables
set echo off
fftransform
save inp5.sav
;
;*** plot commands ****
;plot name: input acc
plot hold table 100 line
;plot name: acc - fft
plot hold table 110 line
;plot name: vel - fft
plot hold table 210 line
;plot name: input disp
plot hold table 103 line
;plot name: input vel
plot hold table 102 line
;plot name: corr. vel
plot hold table 104 line
```

The input motion can be generated for this example by using the *GIIC*. The procedure to create the filtered and baseline-corrected input motion and save it as table 104 is as follows. The acceleration history (“ACC_DECONV.HIS”) is read into *FLAC* via the **READ** button in the **UTILITY/HISTORY** tool. The **EXECUTE** button should be pressed to execute the command. The history is then converted into a table by pressing the **HISTORY -> TABLE** button in the **UTILITY/HISTORY** tool. The dialog shown in [Figure 1.95](#) appears, and the acceleration history (previously assigned ID number 100) is converted into a table (designated by ID number 100).

The power spectrum is calculated in a two-step procedure. First, the *FISH* function named “FFT_TABLES.FIS” (accessed from the **TABLES** menu item in the **UTILITY/FISHLIB** tool) is used to assign tables for inputting the acceleration history and storing the calculated power spectrum. And then “FFT.FIS” is executed from the same location to create the power spectrum.

The acceleration record in table 100 is converted into a velocity record, using *FISH* function “INT.FIS” accessed from the **TABLES** menu item in the **UTILITY/FISHLIB** tool, and stored in table 201. A power spectrum is calculated for the velocity record using the same procedure as for the acceleration record.

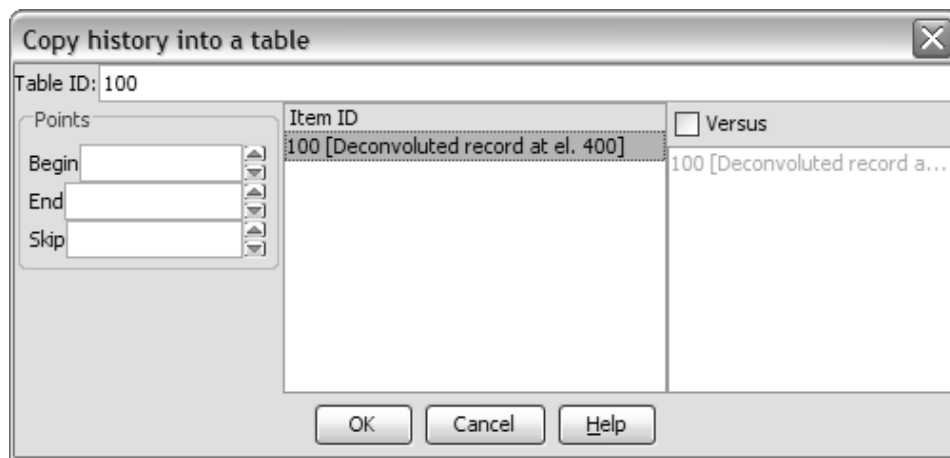


Figure 1.95 Copy history into a table

The *FISH* function “FILTER.FIS” is accessed from the **TABLES** menu item in the **UTILITY/FISHLIB** tool to filter the acceleration at 5 Hz. The filtered table is given the ID number 101, and the cutoff frequency is set to 5 Hz, as shown in the dialog in [Figure 1.96](#). **OK** is pressed to execute this *FISH* function and create the filtered record.

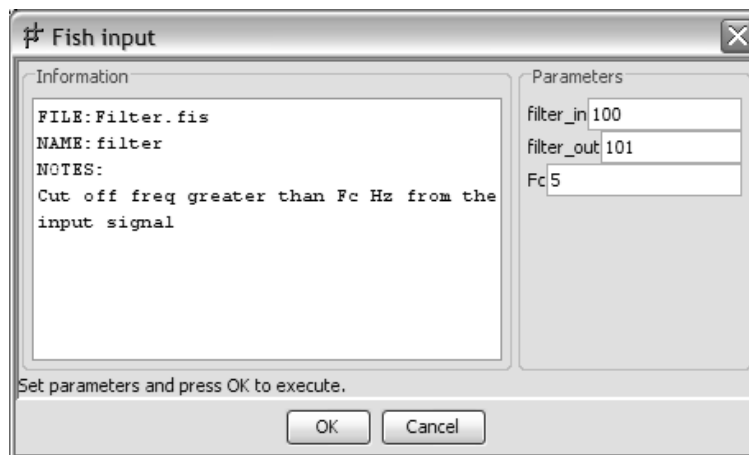


Figure 1.96 Input for **filter FISH** function

In order to perform the baseline correction, the filtered acceleration in table 101 is integrated (with “INT.FIS”) to produce a velocity record and stored in table 102. Then, a low frequency sine wave is added to this velocity record to produce a final displacement of zero. The sine wave is given in “BASELINE.FIS” in [Example 1.25](#). This *FISH* function is accessed from the *FISH editor* resource pane. The input dialog for this *FISH* function is shown in [Figure 1.97](#). The corrected velocity wave is written to table 104. Table 104 is written to a file named “TABLE104.DAT” by selecting SAVE in the UTILITY/TABLE tool.

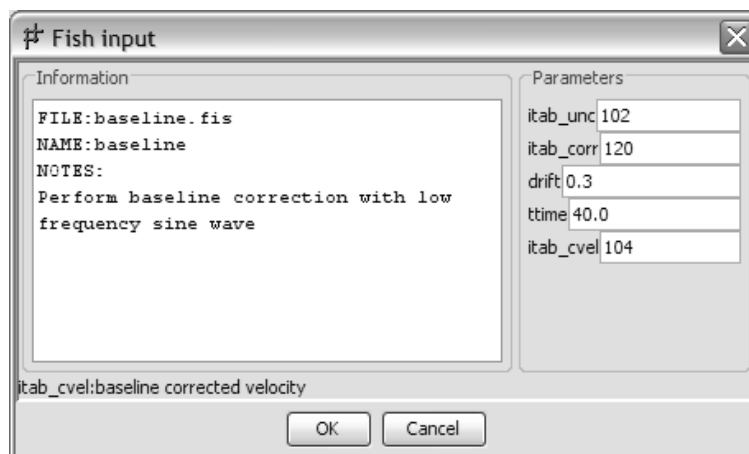


Figure 1.97 Input for **baseline FISH** function

1.6.1.6 FLAC Model Project Setup and Grid Creation

The *FLAC* model options selected for this analysis are shown activated in the *Model options* dialog displayed in [Figure 1.98](#). The dynamic analysis and groundwater flow options are selected. Advanced constitutive models are also included in order to access the Finn model, which will be used for the liquefaction calculation phase. The Imperial system of units is specified for this analysis.

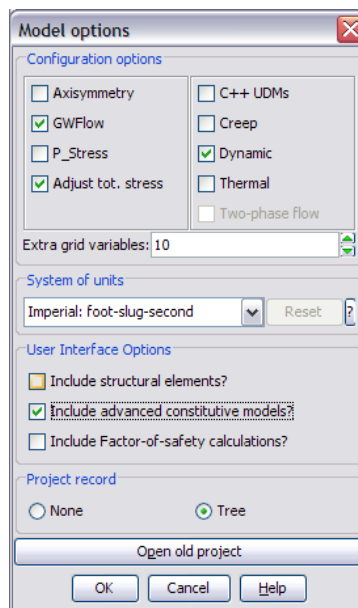


Figure 1.98 Model options selected for the embankment dam example

The embankment dam model can now be created. (The commands that will be generated for this model are listed in “EARTH DAM.DAT” in [Example 1.30](#).)

The dynamic calculation phase is performed using the large-strain mode in *FLAC*. When significant deformation and distortion of the grid is anticipated, as in this example, it is important to minimize the number of triangular-shaped zones in the mesh and, in particular, those along slope faces, as discussed in [Section 1.5.3](#), Step 4. A special *FISH* tool is provided in the `UTILITY/FISHLIB` library to assist with the creation of a mesh for this model. The **gentabletop** tool, shown in the *Fish Library* dialog in [Figure 1.99](#), is used to transform a grid to fit an irregular upper surface defined by a table. The grid beneath the table is adjusted to provide a uniform spacing of quadrilateral zones below the table surface.

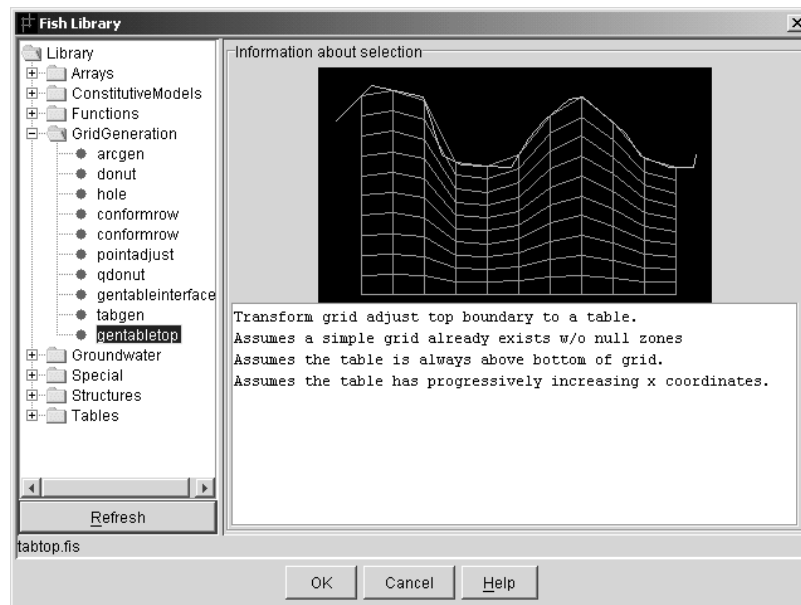


Figure 1.99 *gentabletop FISH function in “TABTOP.FIS”*

The grid zoning is defined first using the **GENERATE/SIMPLE** tool. The grid dimensions selected for this model are shown in the *Range* dialog of the **SIMPLE** tool, in [Figure 1.100](#). The number of zones selected for this model (180×28) corresponds to a zone size of 10 ft.

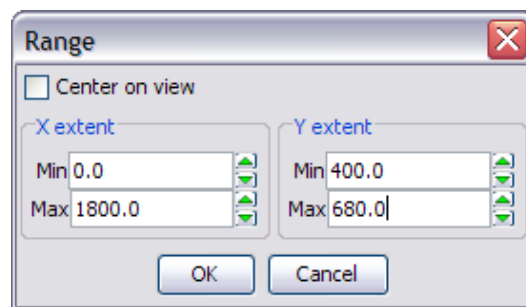


Figure 1.100 *Grid parameters entered in the Range dialog*

After this grid is created, the **UTILITY/TABLE** tool is used to define the slope of the embankment dam. [Figure 1.101](#) shows the pairs of x - and y -values that are entered in the *Edit Table points* dialog to define the surface.

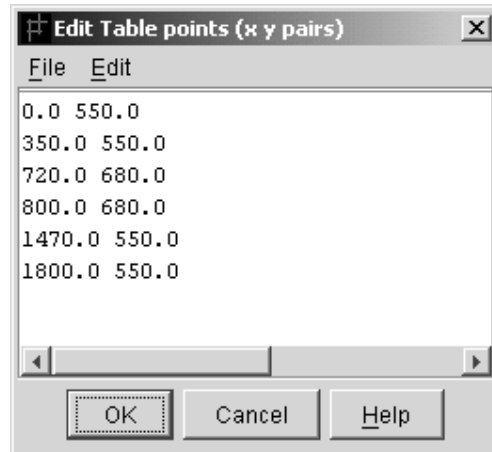


Figure 1.101 *Edit Table points dialog showing x - and y -values defining embankment dam surface*

After this table is defined, the **gentabletop** tool is executed from the **UTILITY/FISHLIB** library to create the grid. The resulting mesh is shown in [Figure 1.102](#). Note that the grid is totally composed of quadrilateral-shaped zones.

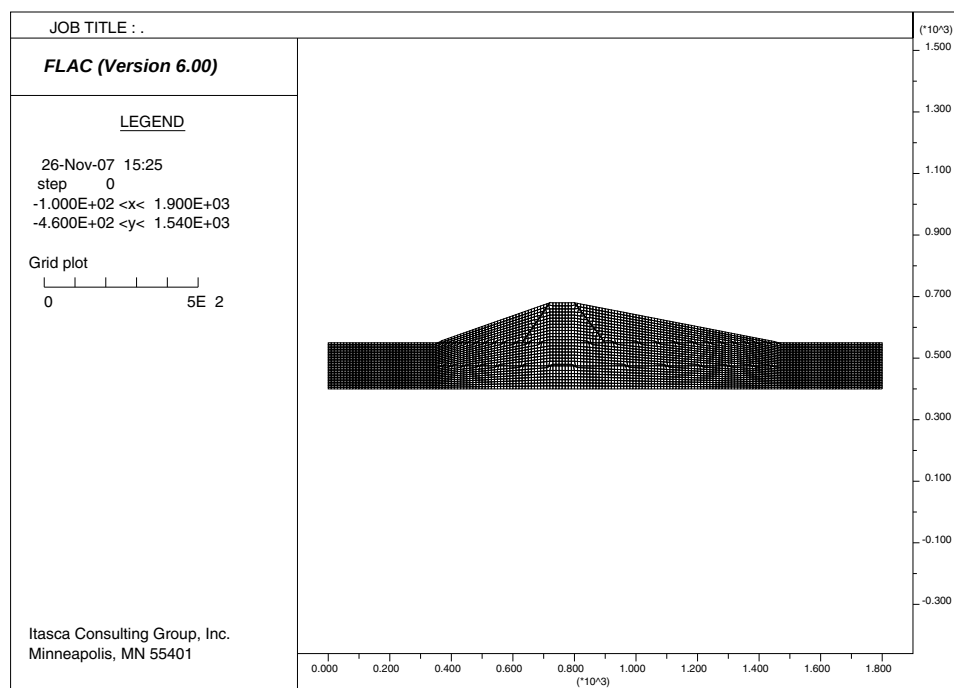


Figure 1.102 *Mesh created with the **gentabletop** tool*

The foundation soil layers and the embankment core and shell regions are delineated by lines generated using the **ALTER/SHAPE** tool. Then the different materials and properties, listed in [Table 1.3](#), are specified, corresponding to Mohr-Coulomb materials, and assigned using the **MATERIAL/ASSIGN** and **MATERIAL/GWPROP** tools. The resulting model is shown in [Figure 1.103](#). Note that some triangular zones are created within the mesh when the different soil regions are defined (see [Figure 1.104](#)). Triangular zones are also created at the slope toe and crest. It is difficult to eliminate triangular zones completely in this model. However, there are only a small number of these zones along the slope face, and the strengths of these zones can be readily adjusted if there is a distortion problem.

The model state, after the geometry shaping is complete, is saved in the *GIIC Project Tree* with the name “EDAM1.SAV”; and after the materials have been added, with the name “EDAM2.SAV.” The model is now ready to begin the analysis stage.

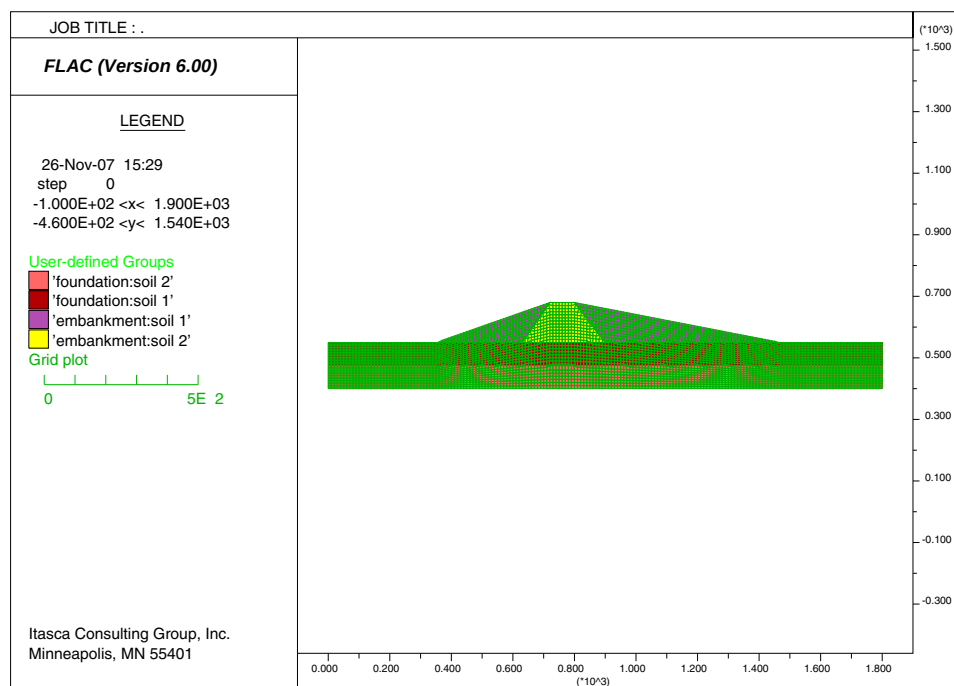


Figure 1.103 *Embankment dam model with foundation and embankment soils assigned*

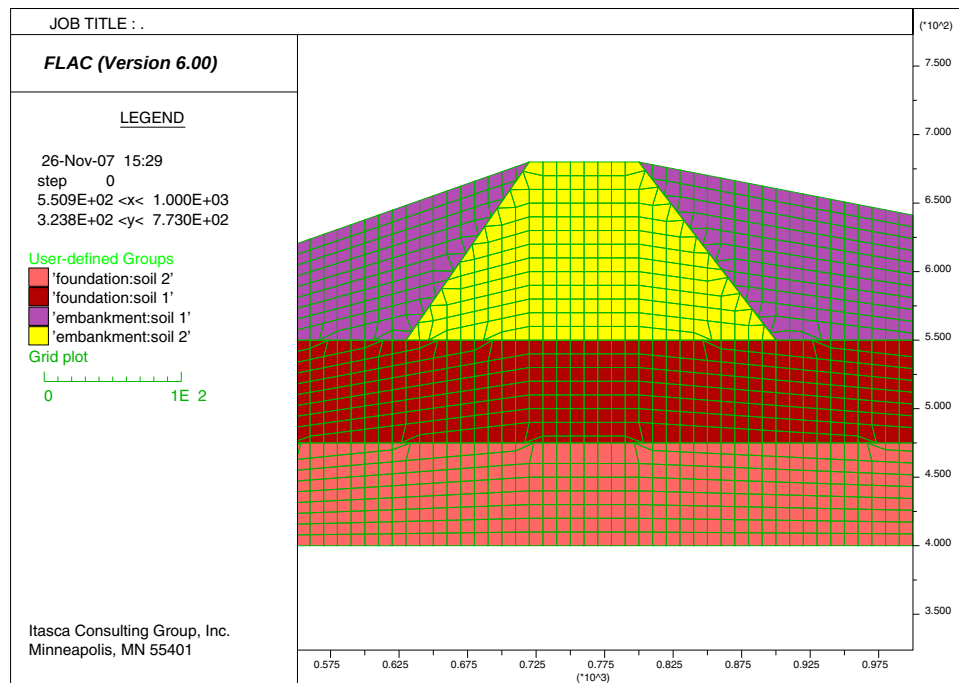


Figure 1.104 Close-up view of embankment dam model

1.6.1.7 Establish Initial State of Stress

State of Stress before Raising Reservoir Level

The analysis is started from the state before the embankment is constructed. The construction process may affect the stress state, particularly if excess pore pressures develop in the soils and do not dissipate completely during the construction stages. The embankment can be constructed in stages, with a consolidation time specified in the *FLAC* model, if pore-pressure dissipation is a concern. In this example, the excess pore pressures are assumed to dissipate before a new lift of embankment material is placed.

It should be noted that staged modeling of the embankment lift construction also provides a better representation of the initial, static shear stresses in the embankment. This is important, particularly in a liquefaction analysis, because the initial, static shear stresses can affect the triggering of liquefaction. In this simplified example, the embankment is placed in one stage. However, it is recommended that the lift construction stages be simulated as closely as is practical, in order to provide a realistic representation of the initial stress state.

The embankment materials are temporarily removed from the model by using the **MATERIAL/CUT&FILL** tool. These materials will be added back after the calculation for the initial equilibrium state of the foundation. The boundary conditions are specified using the **IN SITU/FIX** tool. Note that the bottom of the model is fixed from movement in both the x - and y -directions. If a roller boundary is specified along the bottom boundary, then the foundation is free to slide along the base when the embankment is constructed, which may cause unrealistic failure modes.

The water density of 1.94 slugs/ft^3 and gravitational magnitude of 32.2 ft/sec^2 are assigned, and fluid-flow and dynamic-analysis modes are turned off in the global **SETTINGS** tools.

The most efficient way to achieve an equilibrium stress state in a saturated, horizontally layered soil is to use the special *FISH* tool **inin**, provided in the **UTILITY/FISHLIB** library. This function calculates the pore pressures and stresses automatically for a model containing a phreatic surface. The function requires the phreatic surface height (**wth** = 550 in this example) and the ratios of horizontal to vertical effective stresses (assumed to be **k0x** = **k0z** = 0.5 in this example). The pore pressure, total stress and effective stress distributions are then calculated automatically, accounting for the different soil unit weights, and the position of the water table. The equilibrium state is checked (using the **SOLVE elastic** option in the **RUN/SOLVE** tool). **Figure 1.105** shows the initial pore-pressure distribution in the foundation soils. This state is saved in the *Project Tree* as “EDAM3.SAV.”

The embankment materials can be added to the model in stages, to simulate the construction process, by using the **MATERIAL/CUT&FILL** tool. In this example, both embankment soils 1 and 2 are added simultaneously, and pore pressures are assumed not to change. The displacements resulting from adding the embankment in one step are shown in **Figure 1.106**. This “construction step” is done to simplify the example; a more rigorous analysis should follow the construction sequence as closely as possible, in order to produce a more realistic displacement pattern and initial stress state. The saved state at this stage is named “EDAM4.SAV.”

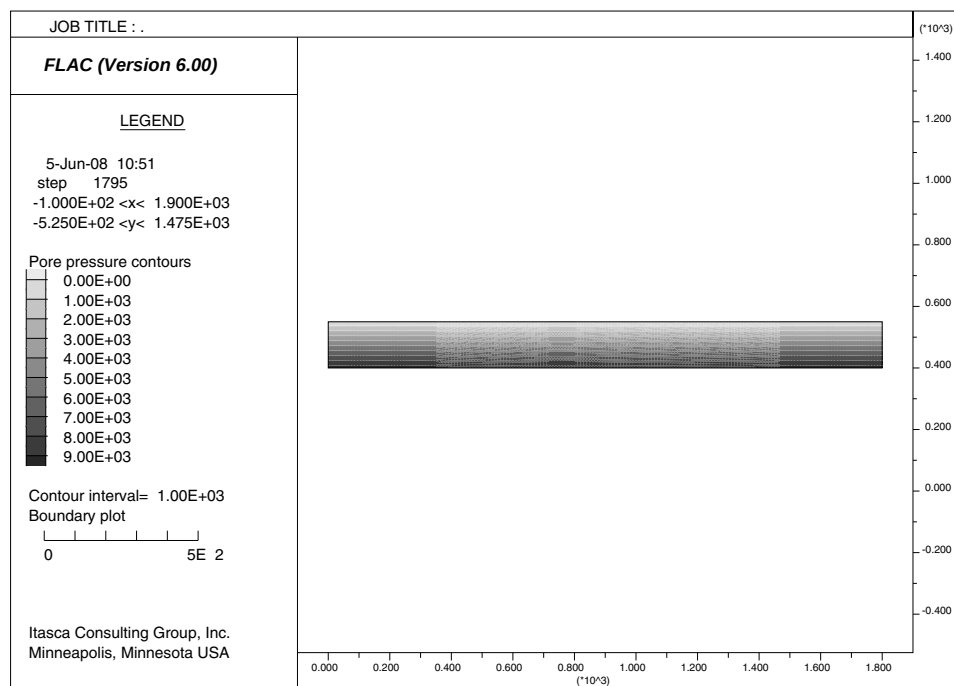


Figure 1.105 Pore pressure distribution in foundation soils

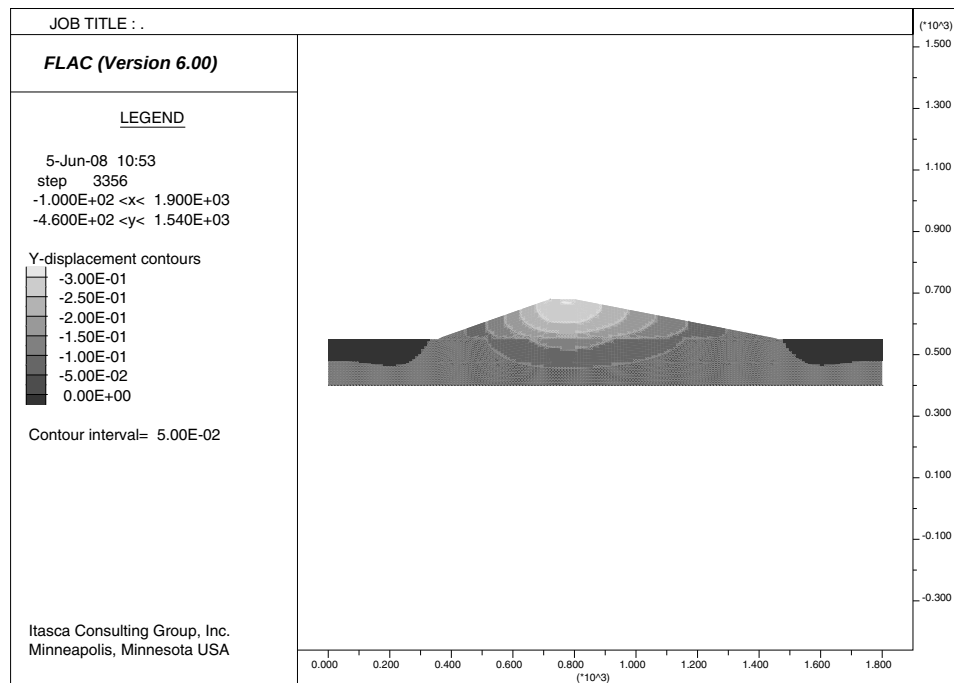


Figure 1.106 Displacements induced by embankment construction in one step

The model is run in small-strain mode up to this stage and, consequently, the gridpoint positions are not changed. This is done so that the embankment crest elevation (680 ft) does not change. If significant deformation occurs during embankment construction, making it necessary to perform this stage in large-strain mode, then the initial embankment crest elevation for the embankment zones (prior to construction) would need to be raised in order to obtain a specified elevation after construction.

State of Stress with the Reservoir Level Raised

The earthquake motion is considered to occur when the reservoir level is at full pool (i.e., at its full height at elevation 670 ft). For Stage 2 of the analysis, the pore pressure distribution through the embankment and foundation soils is calculated for the reservoir raised to this height. The **IN SITU/APPLY** tool is used to set the pore pressure distribution on the upstream side of the embankment, corresponding to the reservoir elevation at 670 ft. The mouse is dragged in this tool along the upstream boundary starting from the 670 elevation (at gridpoint $i = 70, j = 29$) and ending at the 400 elevation at the bottom-left corner of the model (at gridpoint $i = 1, j = 1$). The distribution parameters, shown in the *Apply value* dialog in Figure 1.107, produce a pore-pressure distribution along this boundary that ranges from zero at elevation 670 ft to 16,866.36 psf at elevation 400 ft.

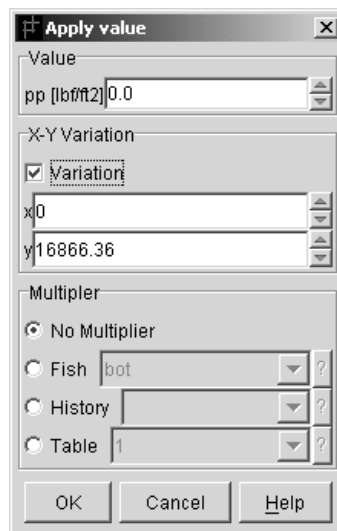


Figure 1.107 Pore-pressure distribution parameters corresponding to a reservoir elevation of 670 ft on the upstream side of the embankment

This calculation is first run in flow-only mode. The groundwater-flow calculation is turned on, and a water bulk modulus of 4.1×10^6 psf is assigned, in the **SETTINGS/GW** tool. The water modulus corresponds to water with entrained air. The fast unsaturated-flow calculation **FUNSAT** and water bulk scaling with permeability and porosity **FASTWB** are set in order to speed the calculation to steady-state flow. The mechanical calculation mode is turned off in the **SETTINGS/MECH** tool. In the **IN SITU/FIX** tool, the pore pressures are fixed at gridpoints along the downstream slope to allow flow across this surface. The porosity and permeability values are also specified for the embankment materials, in the **MATERIAL/GWPROP** tool.

[Figure 1.108](#) plots pore-pressure histories at different locations in the model, indicating that constant values are reached for the equilibrium ratio limit. [Figure 1.109](#) displays the pore-pressure distribution through the embankment and foundation at steady state. The saved state at steady-state flow is named “EDAM5.SAV.”

The static equilibrium state is now calculated for the new pore-pressure distribution. A pressure distribution is applied along the upstream slope to represent the weight of the reservoir water. This time a *mechanical* pressure is assigned in the **IN SITU/APPLY** tool. The pressure ranges from zero at elevation 670 ft (at gridpoint $i = 70, j = 29$) to 8120.8 psf at elevation 550 ft at the toe of the slope (at gridpoint $i = 1, j = 29$); the dialog is displayed in [Figure 1.110](#). The groundwater-flow calculation is turned off, and the water bulk modulus is set to zero (in the **SETTINGS/GW** tool). The mechanical calculation is turned on (in the **SETTINGS/MECH** tool). The model is now solved for this applied condition, and the resulting total vertical-stress contour plot for the model at this stage is shown in [Figure 1.111](#). We also note that the shear stresses at this stage are quite low (less than 10% of the total vertical stresses throughout most of the model) and should not adversely affect the application of hysteretic damping during the dynamic loading phase. (See Step 5 in [Section 1.5.3](#).) The saved state at this stage is named “EDAM6.SAV.”

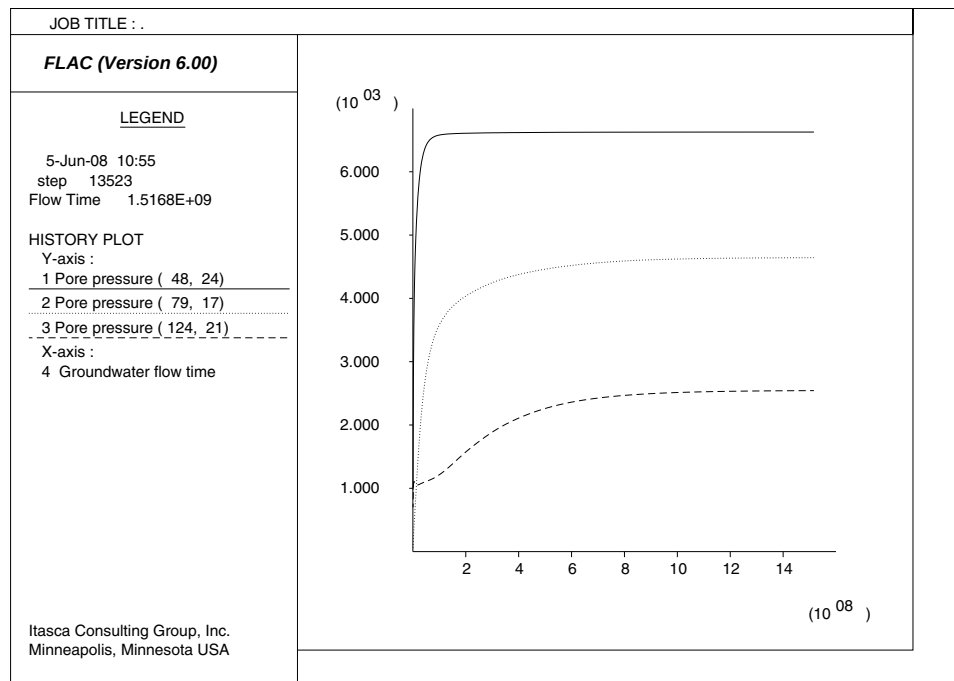


Figure 1.108 Pore-pressure histories

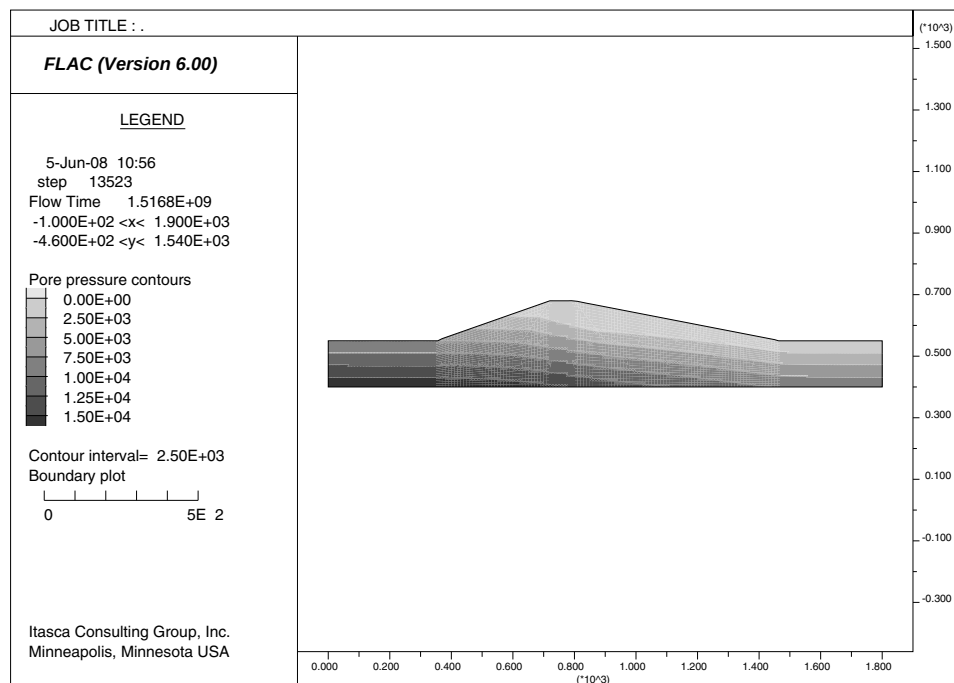


Figure 1.109 Pore-pressure distribution at steady state flow for reservoir raised to 670 ft

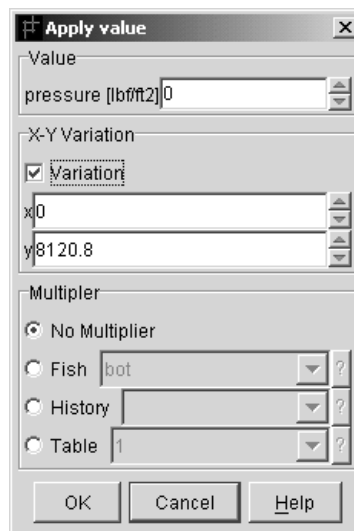


Figure 1.110 Mechanical pressure distribution parameters corresponding to a reservoir elevation of 670 ft on the upstream side of the embankment

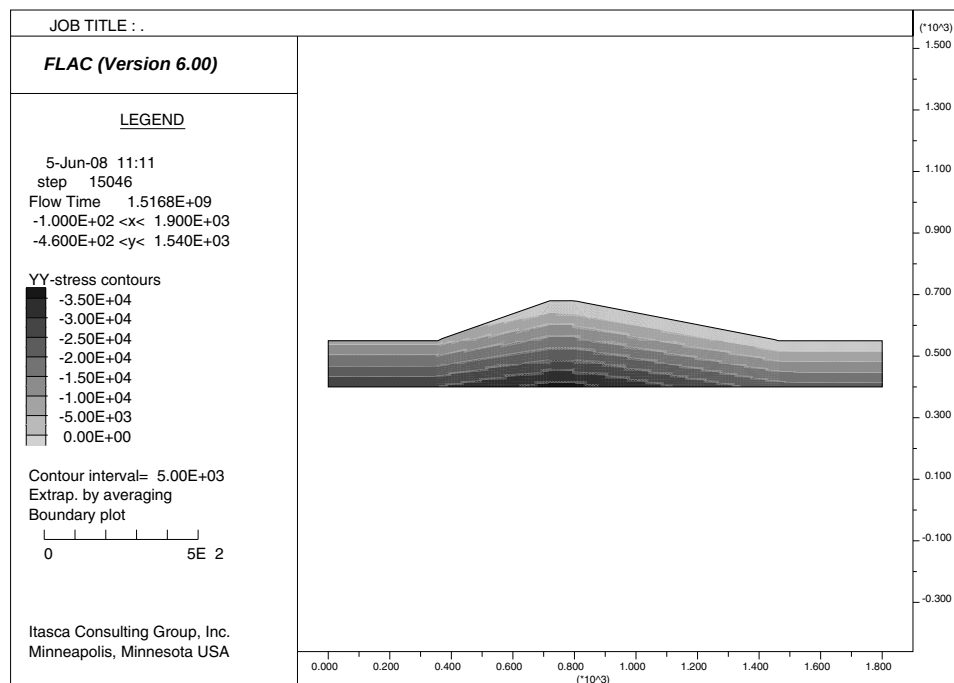


Figure 1.111 Total vertical-stress distribution at steady state flow for reservoir raised to 670 ft

1.6.1.8 Apply Dynamic Loading Conditions

For the dynamic loading stage, pore pressures can change in the materials due to dynamic volume changes induced by the seismic excitation. However, pore-pressure generation due to particle rearrangement does not occur. In order for pore pressures to change as a result of volume change, the actual value of water bulk modulus must be prescribed. The value of 4.18×10^6 psf is re-specified for the water bulk modulus. Note that the groundwater-flow mode is not active, because it is assumed that the dynamic excitation occurs over a much smaller time frame than required for pore pressures to dissipate. (*FLAC* can carry out the groundwater flow calculation in parallel with the dynamic calculation if dissipation is considered important. See [Section 1.5.2.3](#).)

The following conditions are set in this first dynamic simulation, following the steps listed in Step 6 of [Section 1.5.3](#). The dynamic calculation mode is turned on, using the SETTINGS/DNYA tool, and the large-strain mode is selected in the SETTINGS/MECH tool.

The filtered and baseline-corrected input velocity created previously and stored in table 104 is called into *FLAC* using the RUN/CALL tool and selecting the “TABLE104.DAT” file.

The displacements and velocities in the model are initialized by pressing the DISPLMT & VELOCITY button in the IN SITU/INITIAL tool. In this way, only seismic induced motions and deformations are shown in the model results. Damping is *not* prescribed for this initial dynamic simulation. Acceleration histories are recorded at several gridpoints throughout the model via the UTILITY/HISTORY tool. Also, special *FISH* functions are implemented to monitor the shear strain and excess pore pressure at selected locations, and relative displacements along the upstream slope near the crest.* Examples of these functions are listed in [Example 1.27](#). The velocity and shear strain histories are used to evaluate the dominant natural frequencies and maximum cyclic shear strains in the model, when no additional damping is prescribed.

The model state is saved at this point, with these dynamic conditions set, and named “EDAM7.SAV.” This will provide a convenient starting state for the dynamic analysis when damping is added.

The dynamic boundary conditions are now applied in the IN SITU/APPLY tool. First, the free-field boundary is set for the side boundaries by selecting the FREE-FIELD button.

Next, the dynamic input is assigned to the bottom boundary. In this model, a compliant boundary condition is assumed for the base (i.e., the foundation materials are assumed to extend to a significant depth beneath the dam). Therefore, it is necessary to apply a quiet (viscous) boundary along the bottom of the model to minimize the effect of reflected waves at the bottom.

Quiet boundary conditions are assigned in both the x - and y -directions by first selecting the XQUIET button and dragging the mouse along the bottom boundary, and then selecting the YQUIET button and repeating the procedure.

* Note that relative displacement is referenced to the base of the model. See “RELDISPX.FIS” in [Example 1.27](#).

Example 1.27 FISH functions to monitor variables during seismic loading

```

def reldispx
    reldispx = xdisp(62,29) - xdisp(62,1)
    reldispy = ydisp(62,29) - ydisp(62,1)
end
;
def strain_hist
    array arr1(4)
    while_stepping
        dum1 = fsr(77,20,arr1)
        str_77_20=str_77_20 + 2.0*arr1(4)
    end
end
;
def inipp
    ppini = pp(49,23)
end
;
def excpp
    excpp = pp(49,23) - ppini
end;

```

The dynamic wave is applied as a shear-stress boundary condition along the base in the following manner. The **STRESS/SXY** boundary-condition type is selected in the **IN SITU/APPLY** tool, and the mouse is dragged from the bottom-left corner of the model (gridpoint $i = 1, j = 1$) to the bottom-right corner ($i = 180, j = 1$). The **ASSIGN** button is pressed, which opens the *Apply value* dialog. The velocity record, in table 104, is considered a multiplier, v_s , for the applied value. The velocity record is applied by checking the **TABLE** radio button, and selecting table number 104 as the multiplier.

The applied value for **SXY** in the *Apply value* dialog is set to $-2\rho C_s$ (from [Eq. \(1.123\)](#)), in which ρ and C_s correspond to the properties for foundation soil 2. The input selections for the *Apply value* dialog are shown in [Figure 1.112](#).

The model state is saved again at this point and named “EDAM8.SAV.”

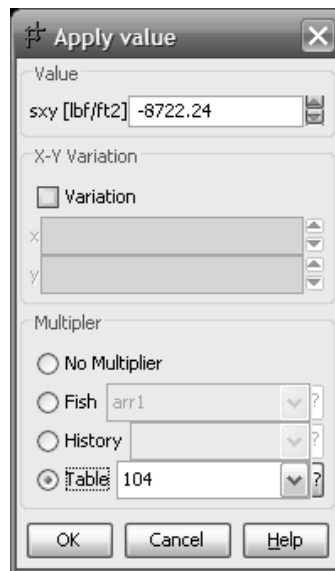


Figure 1.112 Apply shear stress boundary condition in Apply value dialog

1.6.1.9 Run Undamped Elastic-Dynamic Simulation

Before running a dynamic model with actual material strength and damping properties, an elastic simulation is made without damping, to estimate the maximum levels of cyclic strain and natural frequency ranges of the model system. Acceleration and shear stress/strain histories are recorded at several locations in the *FLAC* model for this elastic simulation. [Figure 1.113](#) plots shear stress versus shear strain in zone (77,20) located in embankment soil 2. Maximum shear strains of approximately 0.12% were observed in this region of the model. These strains are not considered to be sufficient to cause excessive reductions in shear modulus. The shear-modulus reduction factor is roughly 0.6 for this strain level, as shown in [Figure 1.86](#).

The frequency range for the natural response of the system is observed to be relatively uniform throughout the model. [Figure 1.114](#) displays a typical power spectrum for this simulation; the predominant frequency was found to be approximately 1.0 Hz.

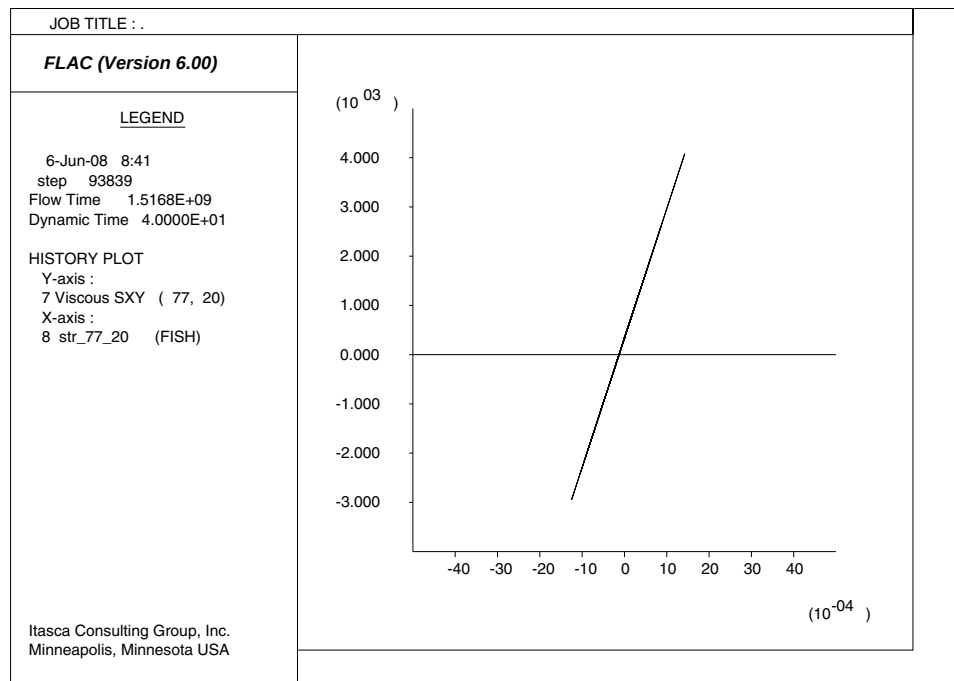


Figure 1.113 Shear stress versus shear strain in embankment soil 2 at zone (77,20) – elastic material and undamped

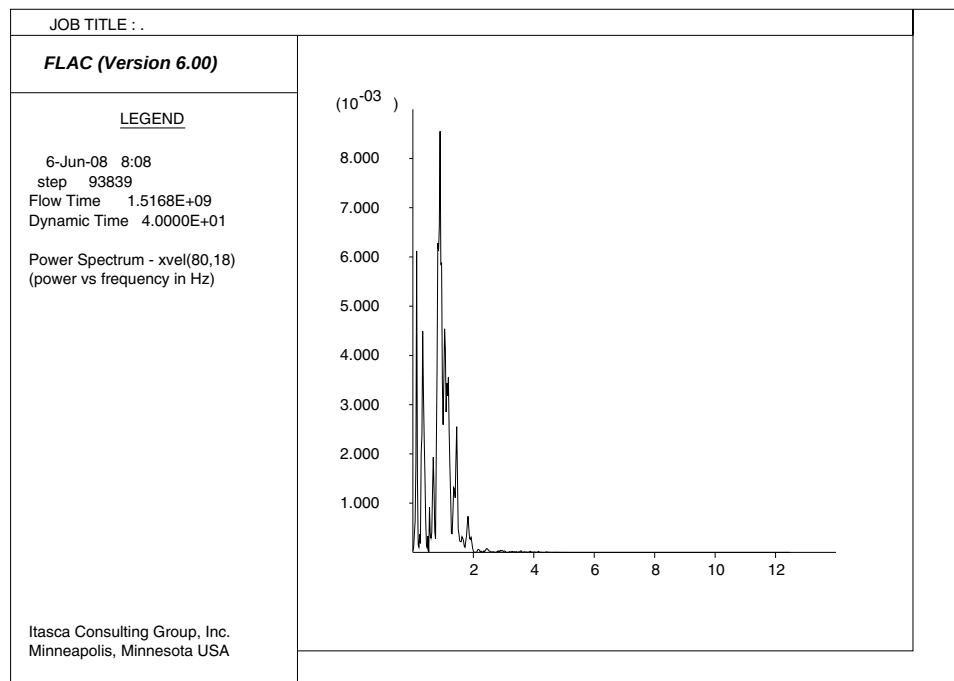


Figure 1.114 Power spectrum of x-velocity time history in embankment soil 2 at gridpoint (80,18) – elastic material and undamped

In this simulation, the dynamic input produces x -accelerations at the surface of the foundation material that are very similar to the target motion (“ACC.TARGET.HIS”), as shown in [Figure 1.115](#). The peak acceleration is approximately 11 ft/sec².

The undamped, elastic calculation is saved as “EDAM9.SAV.” The Fast Fourier analysis results are saved in “EDAM10.SAV.”

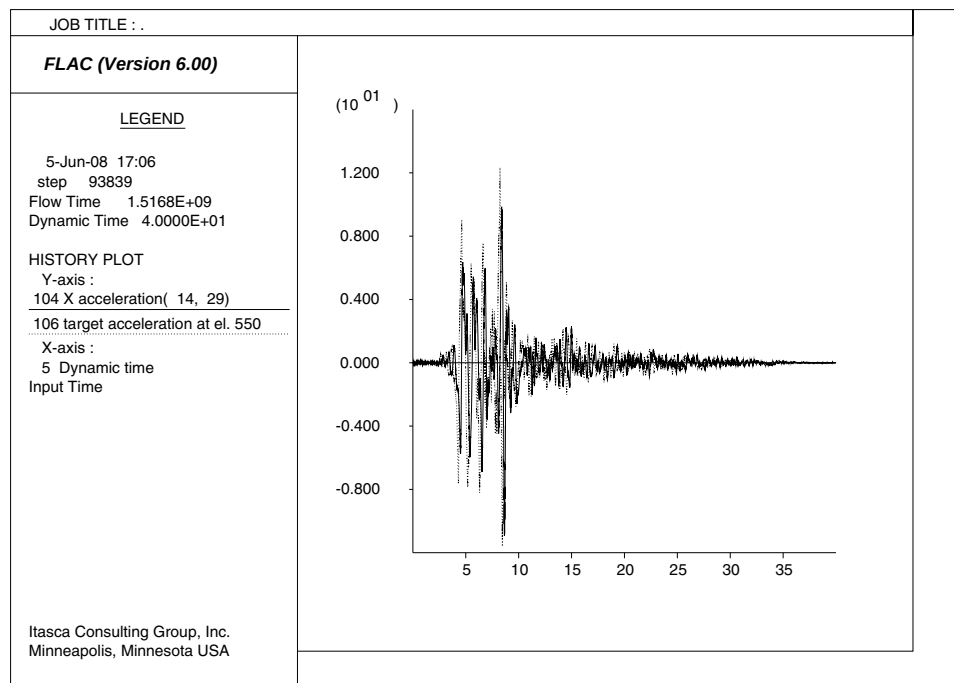


Figure 1.115 Comparison of target acceleration to x -acceleration monitored at surface of foundation soil 1 – elastic material and undamped

1.6.1.10 Run Damped Simulations with Actual Mohr-Coulomb Strength Properties

Simulation with hysteretic damping

Hysteretic damping is applied to correspond to the dynamic characteristics represented by the (G/G_{max}) and (λ) curves shown in [Figures 1.86](#) and [1.87](#). These figures also show a comparison of the (G/G_{max}) and (λ) variations to those computed using the default hysteretic model in *FLAC*. The selected parameters ($L_1 = -3.156$ and $L_2 = 1.904$) for the default model produce curves that provide a reasonable match to the data up to approximately 0.1%, as shown in these figures. This is considered appropriate for the peak strain levels as identified from the undamped run.

The damping is applied at the model state “EDAM7.SAV” by double-clicking on this state name in the *Project Tree*. Hysteretic damping is assigned in the **IN SITU/INITIAL** tool. The dialog shown in [Figure 1.116](#) is opened by selecting the **ZONES** type, checking the **HYSTERETIC DAMPING** menu item, and then **ASSIGN**, to assign the same values for all zones in the model.

Hysteretic damping does not completely damp high-frequency components, so a small amount of stiffness-proportional Rayleigh damping is also applied. A value of 0.2% at the dominant frequency (1.0 Hz) is assigned in the *Dynamic damping parameters* dialog shown in [Figure 1.117](#). Note that Rayleigh damping is applied by selecting the **GPS** type, and then **DYNAMIC DAMPING** in the **IN SITU/INITIAL** tool.

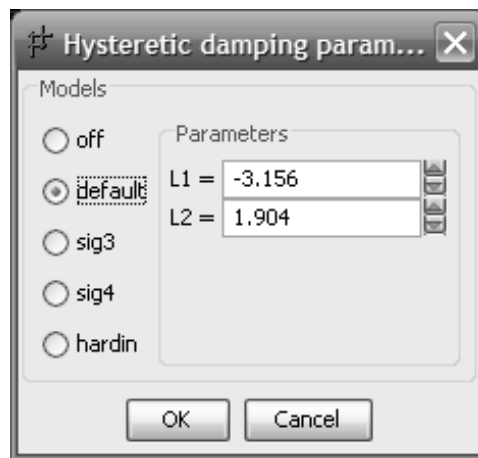


Figure 1.116 Hysteretic damping parameters

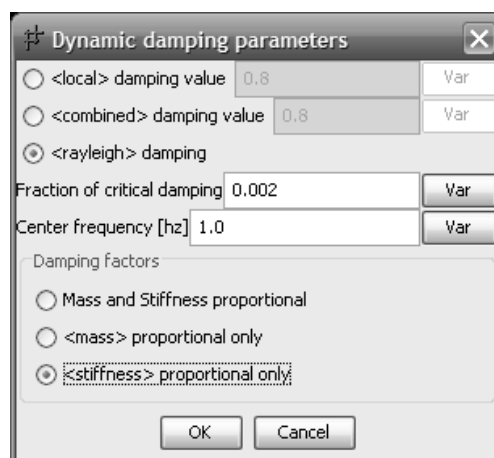


Figure 1.117 Rayleigh damping parameters used with hysteretic damping

After **EXECUTE** is pressed in the **IN SITU/INITIAL** tool, a new branch is created in the project tree to perform a calculation including the additional hysteretic damping (and the small amount of stiffness-proportional Rayleigh damping). The dynamic boundary conditions must be applied again from the **IN SITU/APPLY** tool. The free-field is applied on the side boundaries, and *sxy*-stress history and quiet boundaries are applied at the base, in the same way as for the undamped simulation. The model state is saved at this stage as “EDAM11.SAV.”

A new simulation is now made for a dynamic time of 40 seconds. Note that the dynamic timestep used for this calculation is approximately 3.97×10^{-4} seconds.

The amplitude of the *x*-acceleration at the foundation surface is somewhat smaller for this simulation than that for the target motion; in this simulation, the peak acceleration is approximately 9 ft/sec². Compare [Figure 1.118](#) to [Figure 1.85](#). Some adjustment to the input motion is required to provide a better comparison. For this exercise, the input shear stress is increased by a factor of 1.33, and the hysteretic damping simulation is repeated. The peak acceleration is now nearly 12 ft/sec² and there is a closer agreement with the target motion, as shown by comparing [Figure 1.119](#) to [Figure 1.85](#). This factor is applied for the following simulations.

Movement of the embankment after 40 seconds is primarily concentrated along the upstream slope. This is shown in the *x*-displacement contour plot (in [Figure 1.120](#)) and the shear-strain increment contour plot (in [Figure 1.121](#)). The movement of gridpoint (62,29) along the upstream slope is shown in [Figure 1.122](#). The effect of material failure and hysteresis damping is evident in the cyclic shear strain response; compare [Figure 1.123](#) to [Figure 1.113](#).

The pore pressure and effective vertical stress histories in [Figure 1.124](#), recorded at (*i* = 49, *j* = 23) near the upstream face, illustrate the minor pore-pressure change in the embankment materials during the seismic loading.

The model state is saved at this stage as “EDAM12.SAV.”

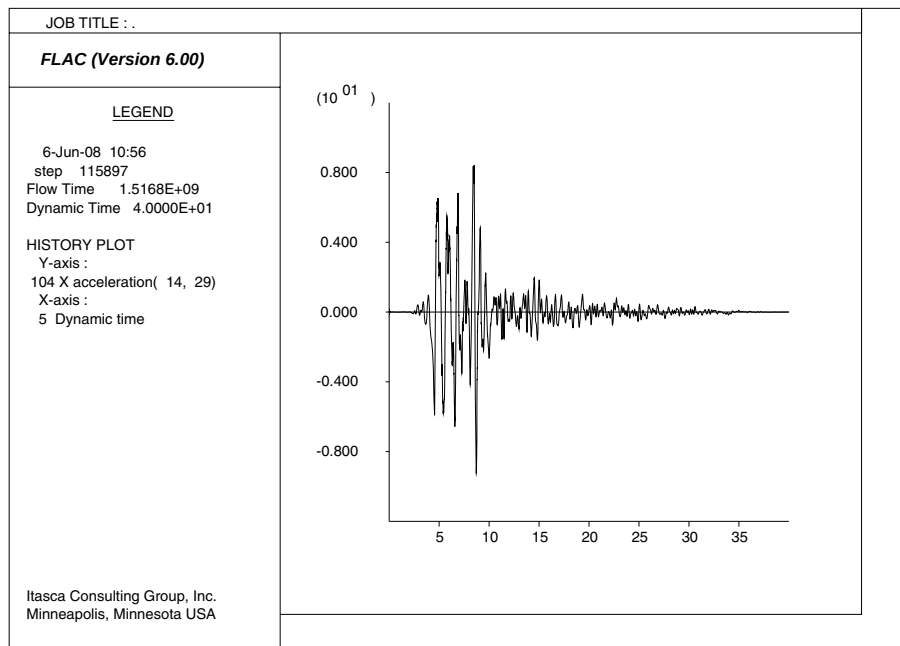


Figure 1.118 *x-acceleration monitored at surface of foundation soil 1 – Mohr-Coulomb material and hysteretic damping*

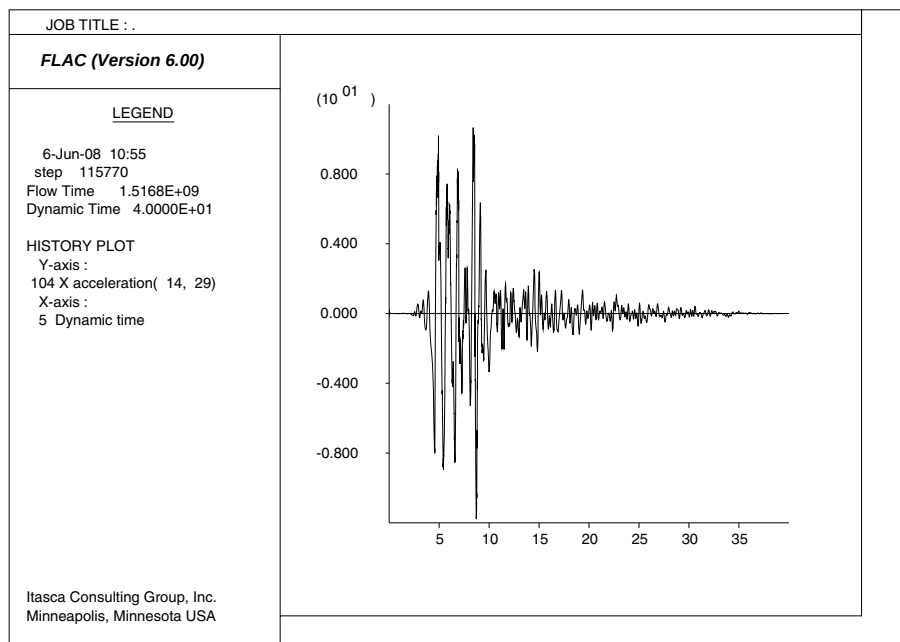


Figure 1.119 *x-acceleration monitored at surface of foundation soil 1 – Mohr-Coulomb material and hysteretic damping (input stress increased by a factor of 1.33)*

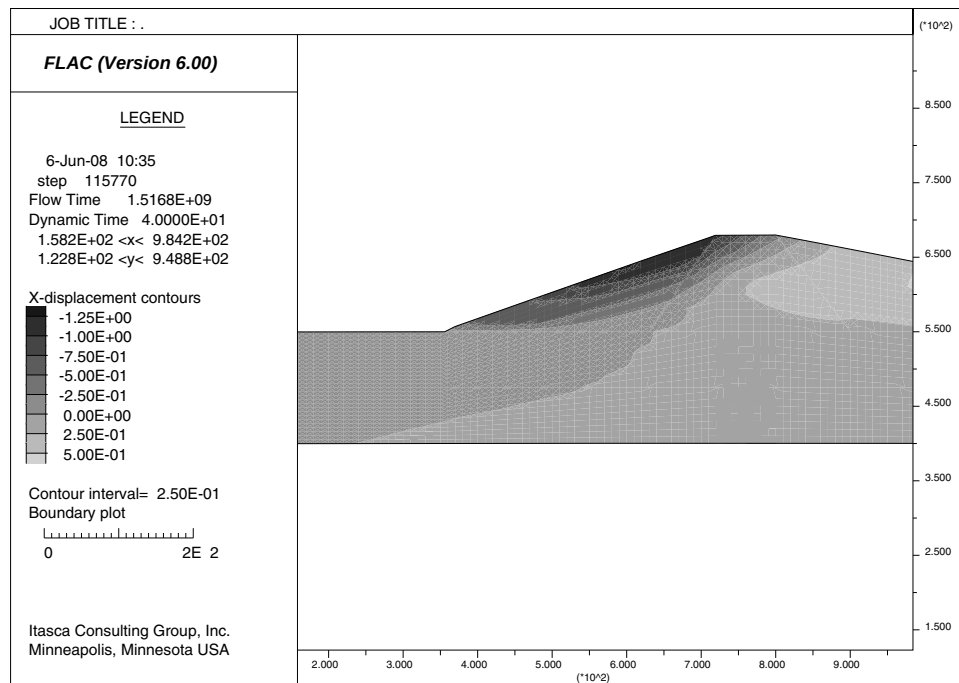


Figure 1.120 *x-displacement contours at 40 seconds*
 – Mohr-Coulomb material and hysteretic damping

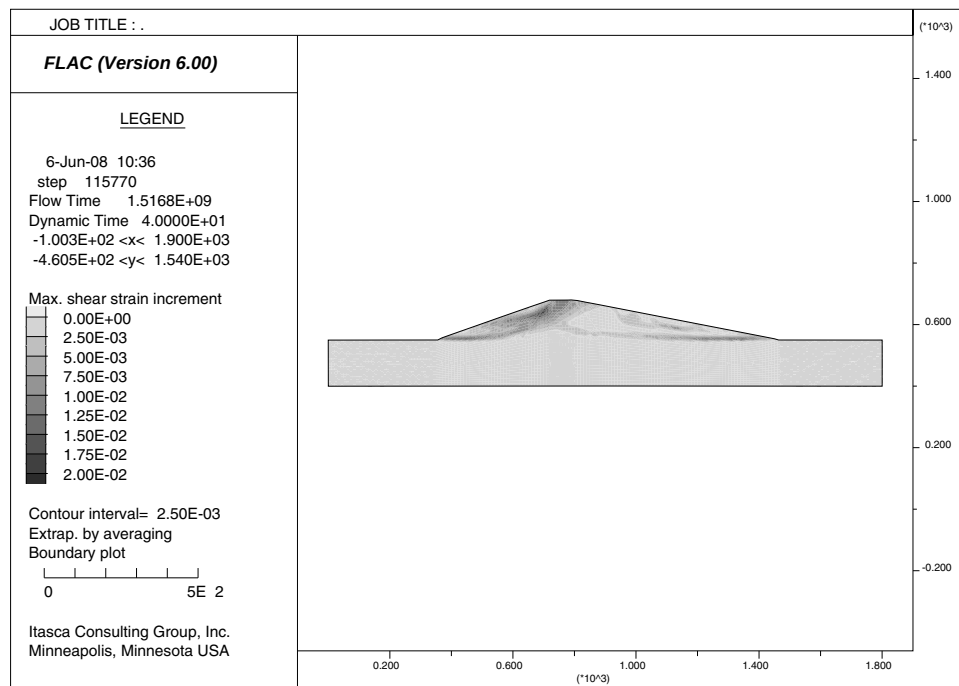


Figure 1.121 *Shear-strain increment contours at 40 seconds*
 – Mohr-Coulomb material and hysteretic damping

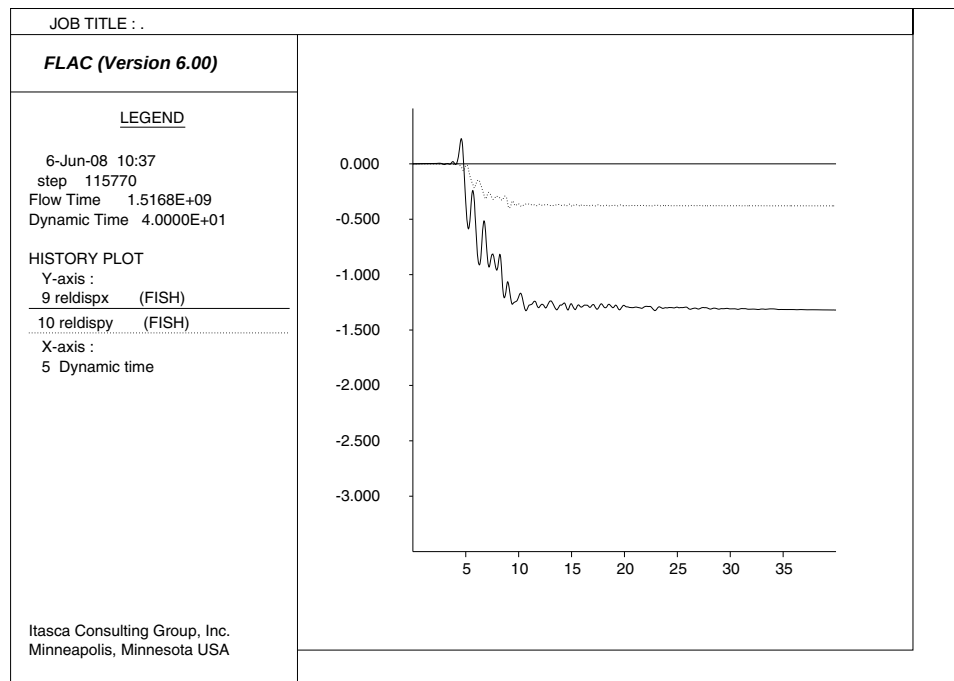


Figure 1.122 Relative displacements at gridpoint (62,29) along upstream slope – Mohr-Coulomb material and hysteretic damping

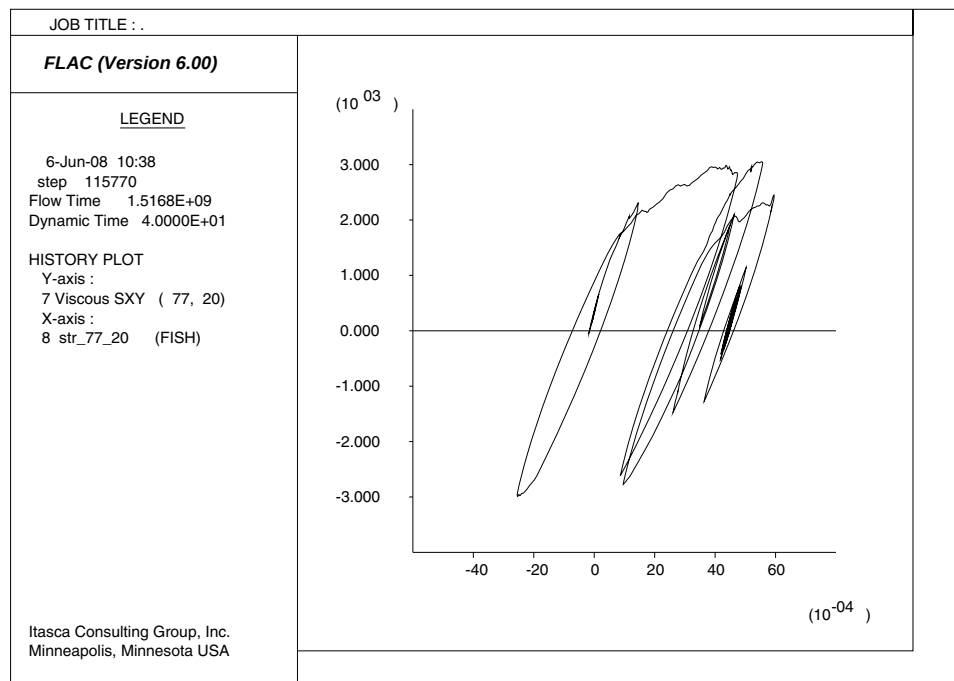
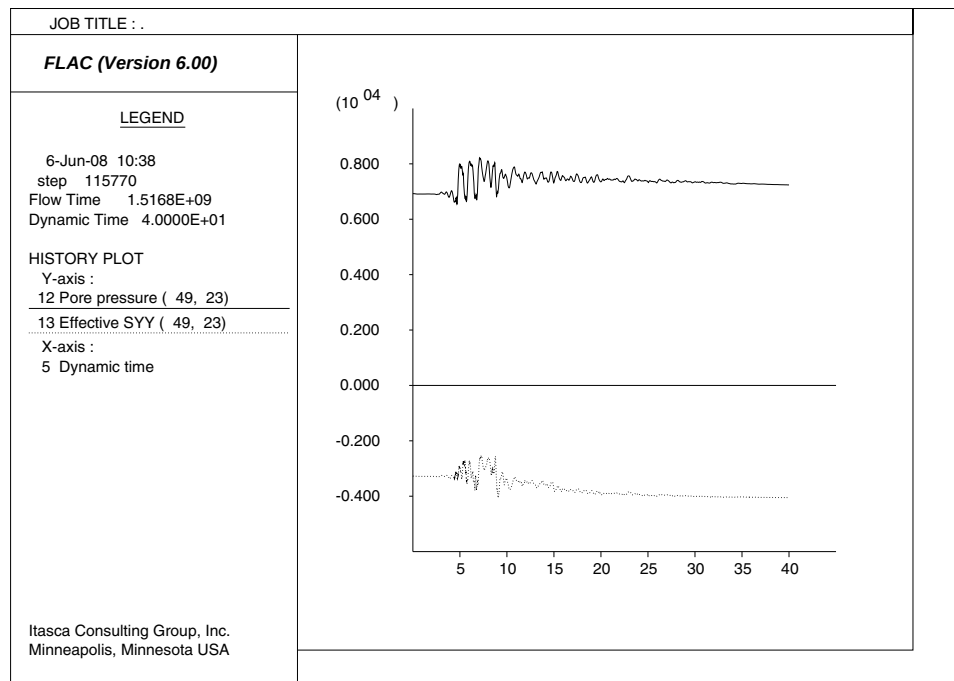


Figure 1.123 Shear stress versus shear strain in embankment soil 2 at zone (77,20) – Mohr-Coulomb material and hysteretic damping



**Figure 1.124 Pore-pressure and effective vertical stress near upstream slope
– Mohr-Coulomb material and hysteretic damping**

Simulation with Rayleigh damping

The dynamic simulation stage is now repeated using only Rayleigh damping. The parameters for the Rayleigh damping model are initially selected based upon the *SHAKE* analysis described in [Section 1.6.1.3](#). It is assumed that the parameters correspond to an equivalent uniform strain of approximately 0.08%. The initial shear modulus is reduced by a factor of 0.8 and the damping ratio is selected as 0.063. The center frequency for Rayleigh damping is 1.0 Hz, as determined from the input wave (see [Figure 1.93](#)) and the undamped analysis (for example, see [Figure 1.114](#)). The Rayleigh damping parameters are specified as shown by the dialog in [Figure 1.125](#). The *FISH* function “GREduce.FIS” is executed to reduce the elastic moduli by a factor of 0.8 (see [Example 1.28](#)).

Note that one set of Rayleigh damping parameters is assumed for all of the soils in this model. In general, different damping parameters may be needed to represent the different damping behavior of the different materials and positions within the foundation and embankment. The spatial variation in damping can be prescribed with the **INITIAL dy_damp** command.

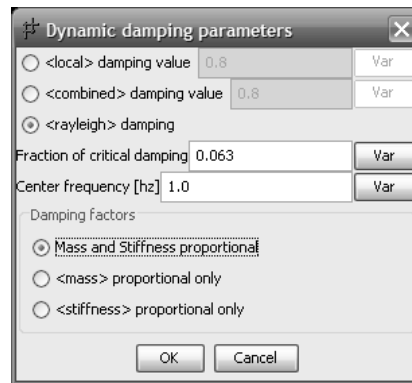


Figure 1.125 Rayleigh damping parameters for Rayleigh damping simulation

Example 1.28 GREduce.FIS – Reduce elastic moduli by modulus reduction factor

```

;Name:greduce
;Diagram:
;Input:_prat/float/0.3/Poisson's ratio
;Input:_gfac/float/0.8/modulus reduction factor
def greduce
  loop i (1,izones)
    loop j (1,jzones)
      shear_mod(i,j)=_gfac*shear_mod(i,j)
      bulk_mod(i,j)=shear_mod(i,j)*(2.*(1+_prat))/(3.*(1.-2.*_prat))
    endloop
  endloop
end

```

Also note that, for this case, with mass- and stiffness-proportional Rayleigh damping of 6.3% at the natural frequency of 1.0 Hz, the limiting timestep is approximately 3.6×10^{-5} seconds. This timestep is roughly eleven times smaller than that for hysteretic damping. The model state is saved at this stage as “EDAM13.SAV.” The Rayleigh damping run at 40 seconds is saved as “EDAM14.SAV.”

If Rayleigh damping alone is used, the results are comparable to those with hysteretic damping. [Figure 1.126](#) plots the x -displacement contours at 40 seconds for Rayleigh damping. [Figure 1.127](#) shows the shear-strain increment contours at this time. Both plots compare reasonably well with those using hysteretic damping (compare to [Figures 1.120](#) and [1.121](#)). [Figure 1.128](#) plots the relative movement at gridpoint (62,29). The displacements are slightly less than those for hysteretic damping (compare to [Figure 1.122](#)).

The effect of Rayleigh damping is also evident in the cyclic shear strain response; compare [Figure 1.129](#) to [Figures 1.113](#) and [1.123](#).

Pore pressure and effective vertical stress histories for the Rayleigh damping run are also similar to those for the hysteretic damping run (compare [Figure 1.130](#) to [Figure 1.124](#)).

As material yield is approached, neither Rayleigh damping nor hysteretic damping account for energy dissipation of extensive yielding. Irreversible strain occurs external to both schemes, and dissipation is represented by the Mohr-Coulomb model. The mass-proportional term of Rayleigh damping may inhibit yielding because rigid-body motions that occur during failure are erroneously resisted. Consequently, hysteretic damping may be expected to give larger permanent deformations in this situation, but this condition is believed to be more realistic than one using Rayleigh damping.

This comparison demonstrates the substantial benefit of hysteretic damping. The results are comparable to those using Rayleigh damping for similar damping levels, and the runtime with hysteretic damping is greatly reduced.

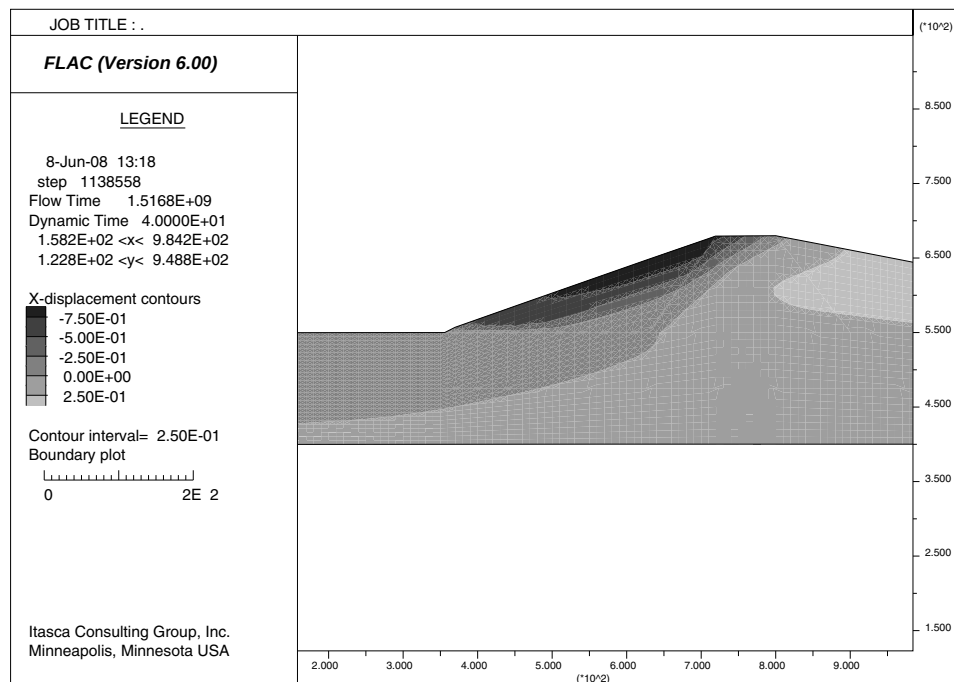
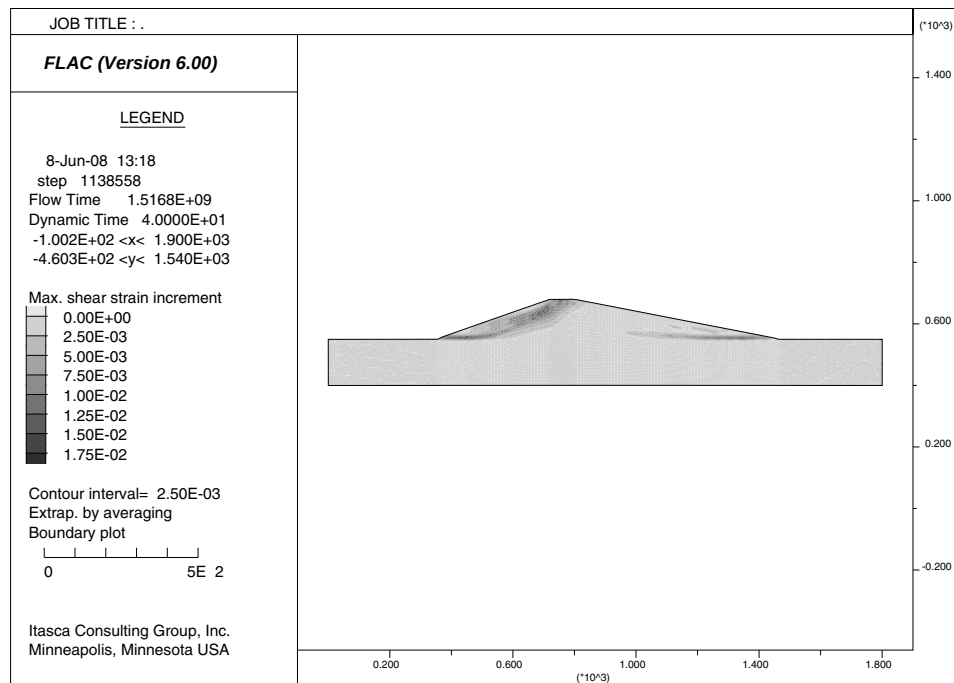
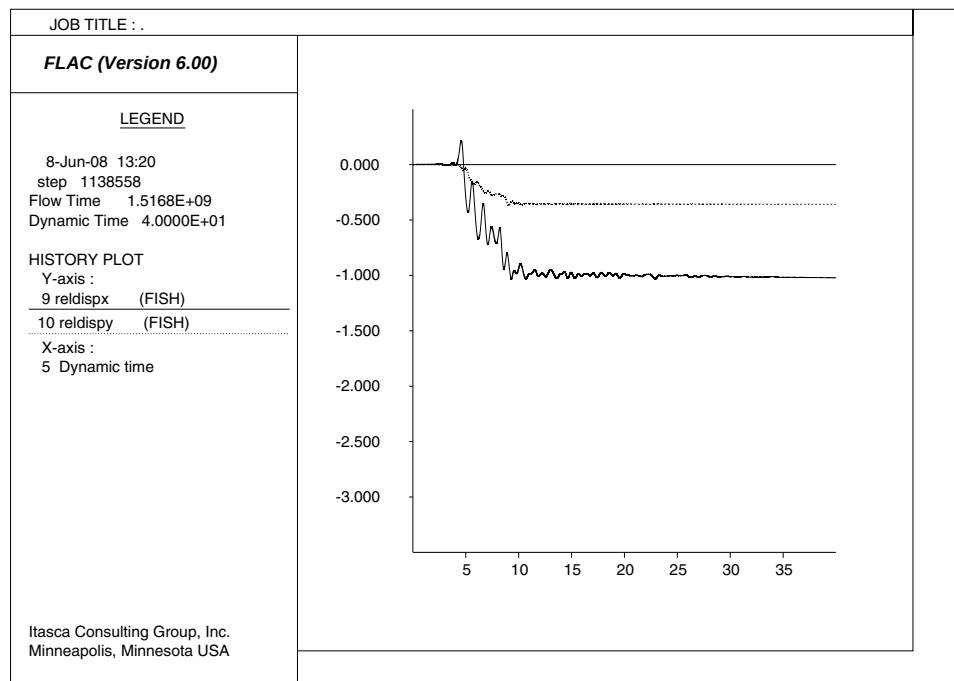


Figure 1.126 *x-displacement contours at 40 seconds
 – Mohr-Coulomb material and Rayleigh damping*



**Figure 1.127 Shear-strain increment contours at 40 seconds
 – Mohr-Coulomb material and Rayleigh damping**



**Figure 1.128 Relative displacements at gridpoint (62,29) along upstream slope
 – Mohr-Coulomb material and Rayleigh damping**

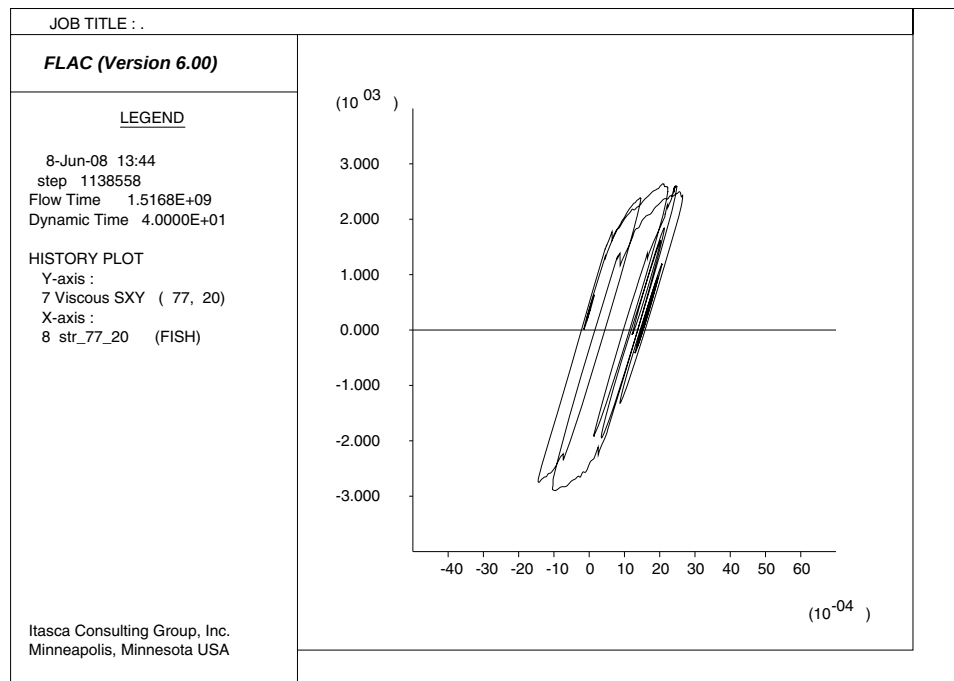


Figure 1.129 Shear stress versus shear strain in embankment soil 2 at zone (77,20) – Mohr-Coulomb material and Rayleigh damping

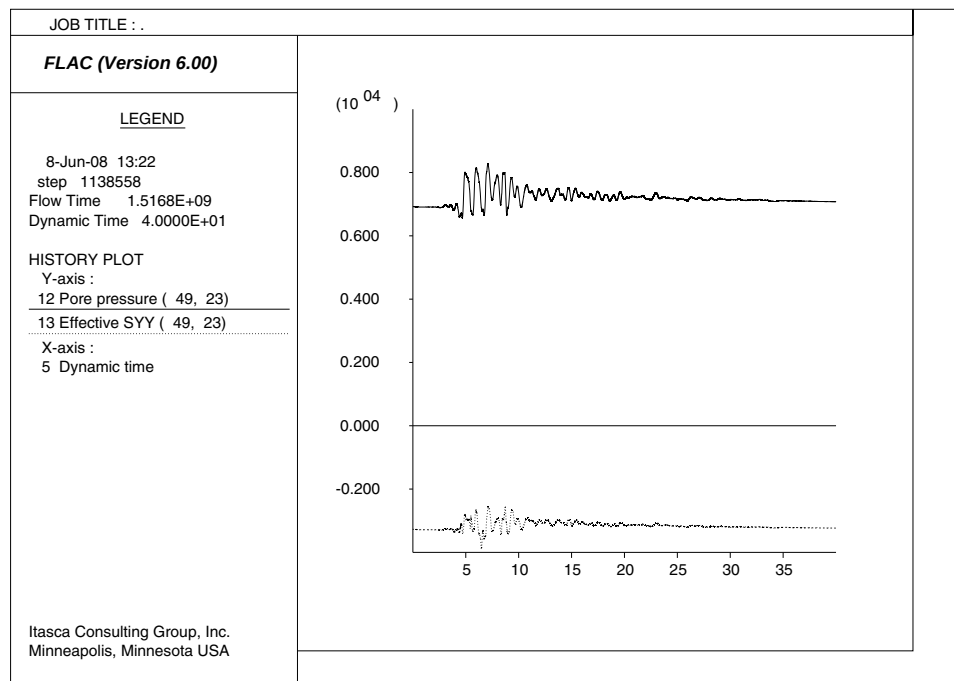


Figure 1.130 Pore-pressure and effective vertical stress near upstream slope – Mohr-Coulomb material and Rayleigh damping

1.6.1.11 Run Seismic Calculation Assuming Liquefaction

The embankment soils are now changed to liquefiable materials. The Finn/Byrne liquefaction model is prescribed for embankment soils 1 and 2, with parameters set to correspond to SPT measurements. For a normalized SPT blow count of 10, the Byrne model parameters are $C_1 = 0.4904$ and $C_2 = 0.8156$ (see [Section 1.6.1.3](#)).

The liquefaction simulation starts at the saved state “EDAM6.SAV.” The embankment soils are changed at this state by using the **MATERIAL/MODEL** tool. (Note that this tool is activated when the **INCLUDE ADVANCED CONSTITUTIVE MODELS?** box is checked in the *Model options* dialog.) The **REGION** range is selected and the **DYNAMIC** models box is checked in this tool. The **FINN** model is then assigned to each region of the embankment soils. When the mouse is clicked within one of the embankment soil regions, a dialog opens to prescribe the model properties. [Figure 1.131](#) shows the dialog with the properties selected for embankment soil 1. Note that the **BYRNE** radio button is checked in order to prescribe the appropriate parameters for the Byrne formulation. Also, the value for **LATENCY** is set to 1,000,000 at this stage. This is done to prevent the liquefaction calculation from being activated initially. The model is first checked to make sure that it is still at an equilibrium state when switching materials to the Byrne model, before commencing the dynamic simulation.

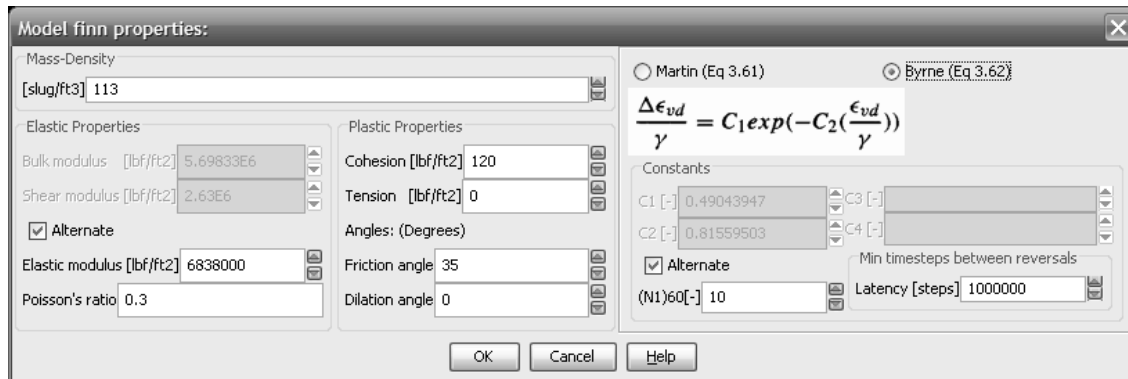


Figure 1.131 Model finn properties dialog with properties for embankment soil 1

The model is now ready for the dynamic analysis. The water bulk modulus is assigned as 4.1×10^6 psf using the **SETTINGS/GW** tool. The value for latency of the embankment soils is reduced to 50 in the **MATERIAL/PROPERTY** tool. The dynamic conditions are now set again in the same manner as in Stage 3. The model state is “EDAM15.SAV” before damping and dynamic boundaries are applied; it is “EDAM16.SAV” after they are applied.

The model is now run for a dynamic time of 40 seconds. The results in [Figures 1.132 through 1.137](#) show the effect of pore-pressure generation in the embankment soils. There is now a substantial movement along the upstream face, as shown by figures [Figures 1.132 through 1.135](#). The relative vertical settlement at gridpoint (62,29) is now approximately 14 in, and the relative shift upstream is approximately 3.6 ft, as shown in [Figure 1.135](#).

A significant increase in pore pressure (and decrease in effective stress) is calculated in the upstream region, as indicated in [Figure 1.136](#). The location of the pore pressure/effective stress measurement is at gridpoint (49,23), which is at a depth of approximately 45 ft below the upstream slope face, and 135 ft from the toe of the upstream slope.

The normalized excess pore-pressure ratio (or cyclic pore-pressure ratio), u_e / σ'_c ,* can be used to identify the region of liquefaction in the model. Contours of the cyclic pore-pressure ratio greater than 0.99 are plotted in [Figure 1.137](#). These contours show the extent of the liquefied embankment soils, primarily in the upstream region. (The excess pore-pressure ratio is calculated in *FISH* function “GETEXCESSPP.FIS,” and stored in *FISH* extra array **ex_5**, see [Example 1.29](#).)

The final state is “EDAM17.SAV.”

Example 1.29 GETEXCESSPP.FIS – Excess pore pressure ratio

```
;Name:getExcesspp
;Input:nsample/int/50/sampling interval
;Input:nstep/int/1/smpling substep
;Note:Calculates excess pore pressure ratio
;Note:ex_1 - stores init. pore pressure state (saved in savepp.fis)
;Note:ex_2 - stores init. eff. confining stress (calculated in savepp.fis)
;Note:ex_4 - calculates exc. pore pressure (pp(i,j) - ex_1(i,j))
;Note:ex_3 - calculates exc. pore pressure ratio: ex_4/ex_2
;Note:ex_5 - stores max. exc. pore press. ratio (set to zero in savepp.fis)
def getExcesspp
  whilestepping
    if nstep = nsample then
      loop i (1,izones)
        loop j (1,jzones)
          if pp(i,j) > ex_1(i,j) then
            ex_4(i,j) = pp(i,j) - ex_1(i,j)
          else
            ex_4(i,j) = 0.0
          endif
          ex_3(i,j) = abs(ex_4(i,j)/ex_2(i,j))
          ex_5(i,j)= max(ex_5(i,j),abs(ex_3(i,j)))
        endloop
      endloop
      nstep = 1
    endif
    nstep = nstep + 1
  end
```

* where u_e is the excess pore-pressure and σ'_c is the initial effective confining stress. Note that a liquefaction state is reached when $u_e / \sigma'_c = 1$.

The liquefaction run is repeated using only Rayleigh damping with the same damping and modulus-reduction parameters as the simulation with Mohr-Coulomb material (damping ratio of 0.063, center frequency of 1.0 Hz, and modulus reduction factor of 0.8). The same plots created for the liquefaction run with hysteretic damping are re-created for the run with Rayleigh damping. See [Figures 1.138 through 1.143](#). As the figures show, the results are very similar. The final state in this case is “EDAM19.SAV.”

Comments

This simple example assumes that the initial shear modulus, G_{max} (before the seismic loading), is uniform throughout each material unit. It may be more appropriate to vary G_{max} for soils as a function of the in-situ effective stress (e.g., see Kramer 1996). An initial variation can be implemented during the static loading stage.

This example also assumes that the shear strength parameters of the liquefiable soils do not change. It has been shown (e.g., Olson et al. 2000) that if the effective stress goes to zero, the shear strength reduces to a strain-mobilized (liquefied) shear strength, which implies a residual cohesion. There are several ways to incorporate a change of strength envelope in the *FLAC* model, such that residual cohesion is developed as the material liquefies. For example, a *FISH* function can be used to adjust the strength parameters as a function of change in the effective confining stress. A more rigorous approach is to modify a bilinear strength model (such as the strain-softening bilinear model, **MODEL sub1**) to include the liquefaction behavior (e.g., the Byrne model). The existing **MODEL finn** in *FLAC* incorporates the pore-pressure generation effect into the Mohr-Coulomb model. This can also be done with other models, using either the *FISH* constitutive model facility (see [Section 2.8](#) in the *FISH* volume) or the C++ DLL model facility (see [Section 3](#) in **Theory and Background**) to create a user-defined model.

If there is a potential for flow slides to occur either at liquefaction or during post-liquefaction, the automatic rezoning logic in *FLAC* can be used to simulate the development of the deformed flow-slide state. See [Section 6](#) in **Theory and Background** for further information.

Acknowledgment

This example is derived from data provided by Dr. Nason McCullough of CH2MHill. His assistance and critical review of this exercise are gratefully acknowledged.

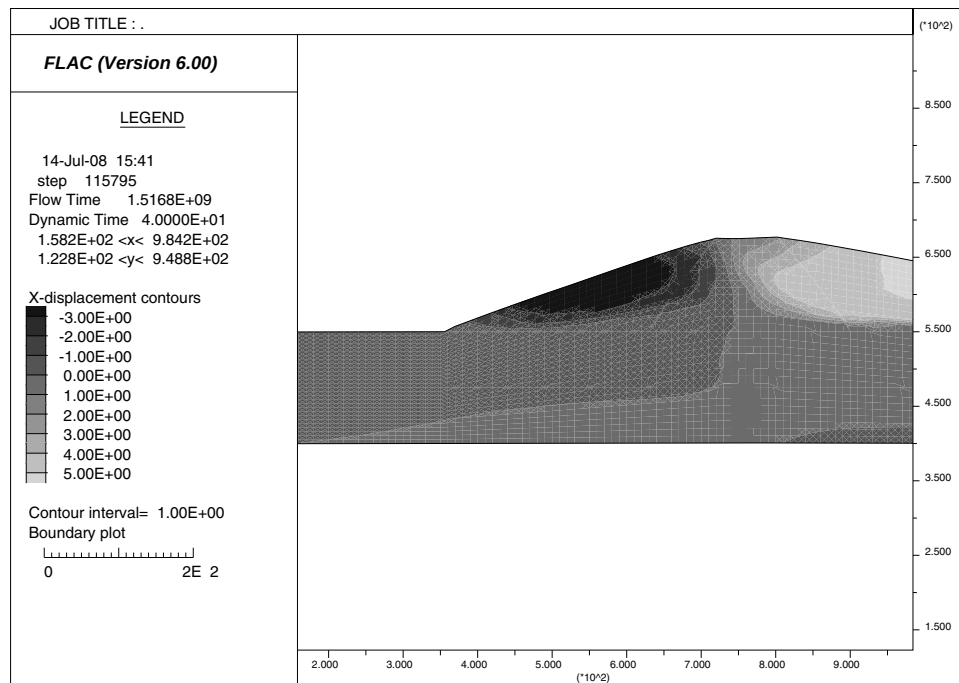


Figure 1.132 x-displacement contours at 40 seconds – Byrne (liquefaction) material and hysteretic damping

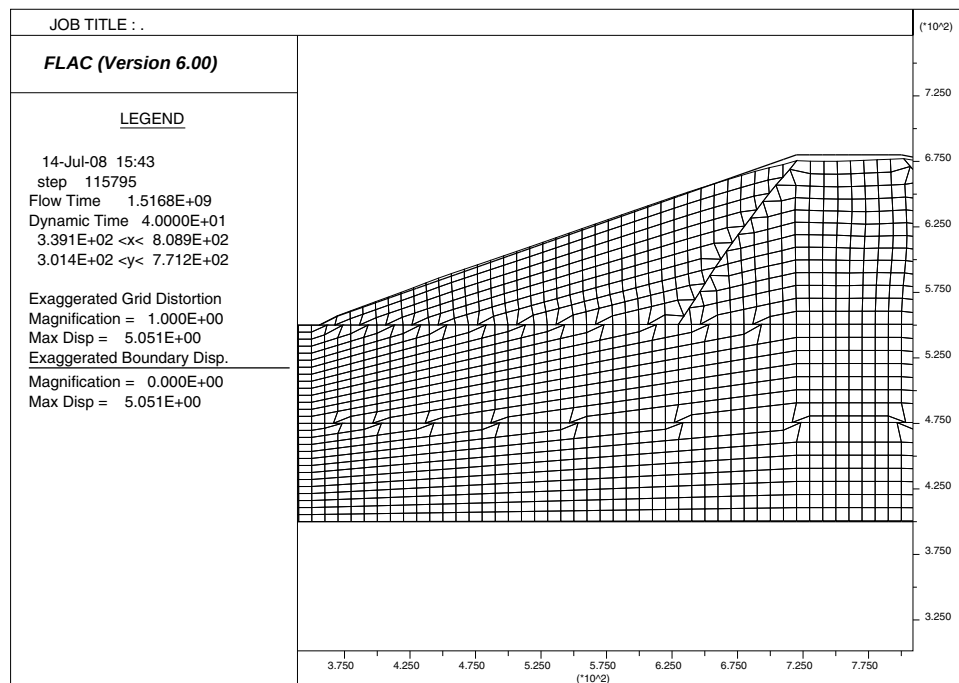


Figure 1.133 Deformed grid at 40 seconds – Byrne (liquefaction) material and hysteretic damping

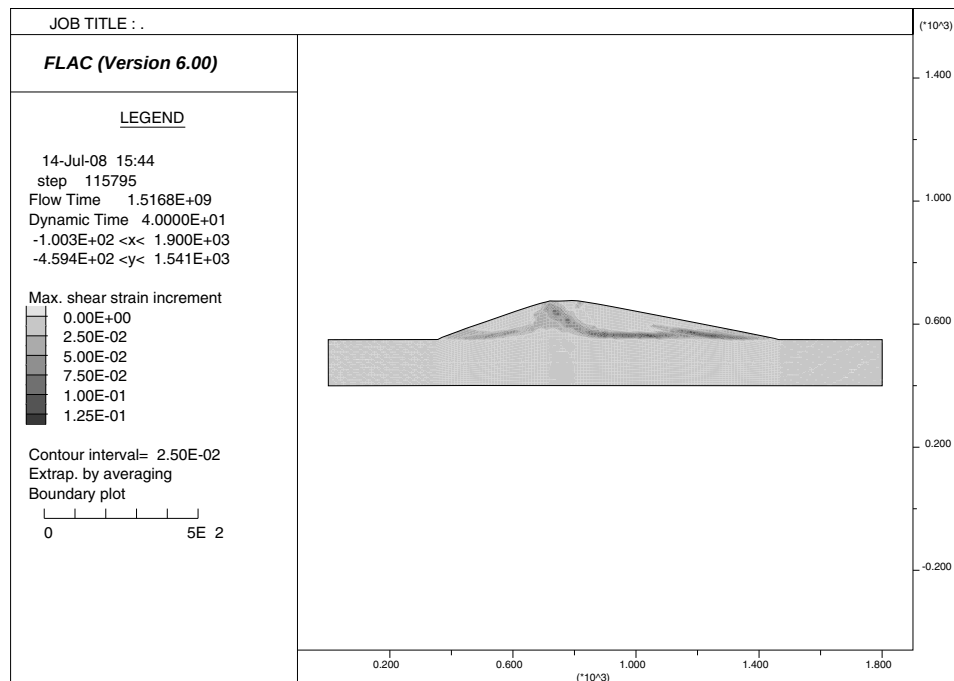


Figure 1.134 Shear-strain increment contours at 40 seconds – Byrne (liquefaction) material and hysteretic damping

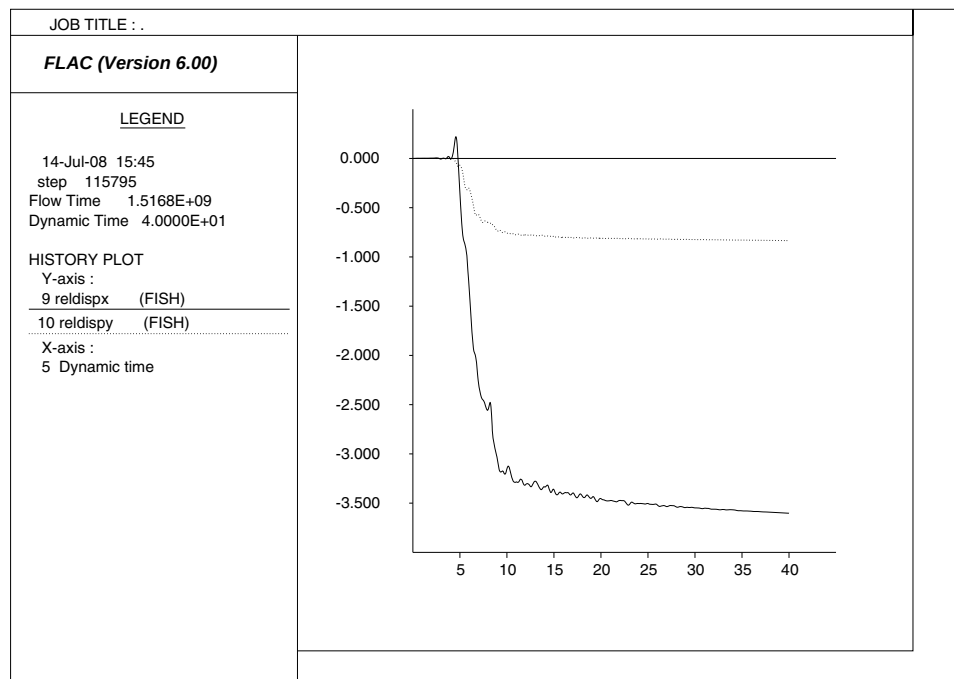


Figure 1.135 Relative displacements at gridpoint (62,29) along upstream slope – Byrne (liquefaction) material and hysteretic damping

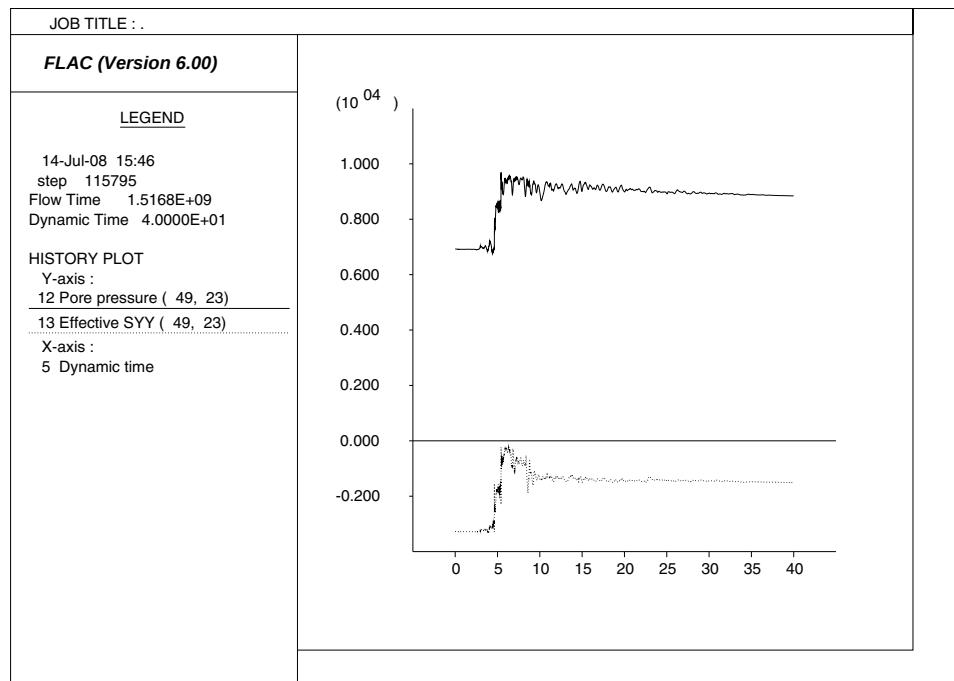


Figure 1.136 Pore-pressure and effective vertical stress near upstream slope – Byrne (liquefaction) material and hysteretic damping

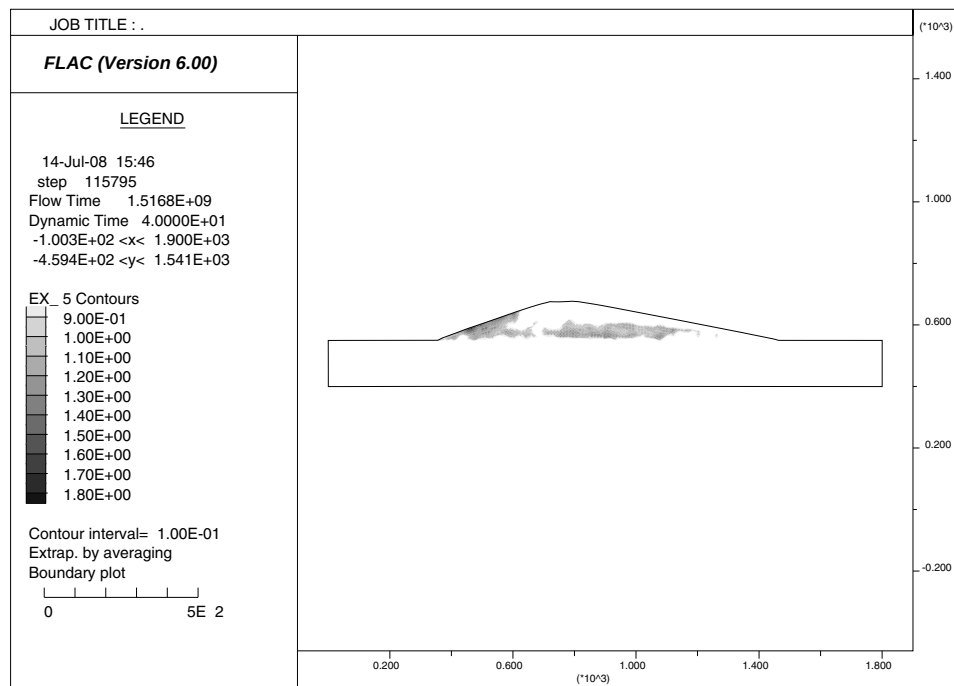


Figure 1.137 Excess pore-pressure ratio contours (values greater than 0.99) – Byrne (liquefaction) material and hysteretic damping

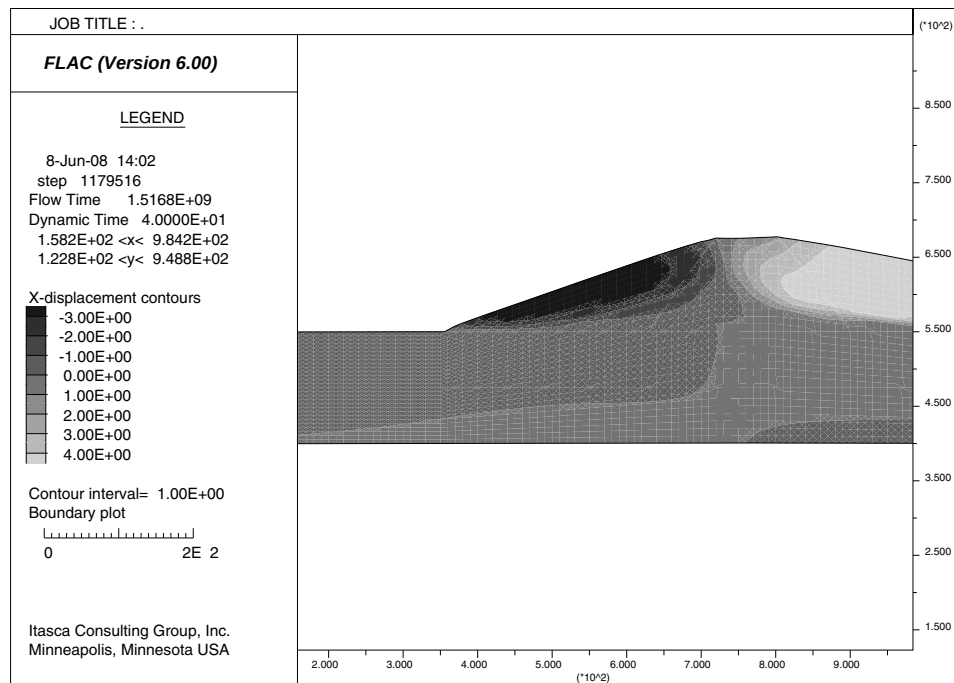


Figure 1.138 x-displacement contours at 40 seconds – Byrne (liquefaction) material and Rayleigh damping

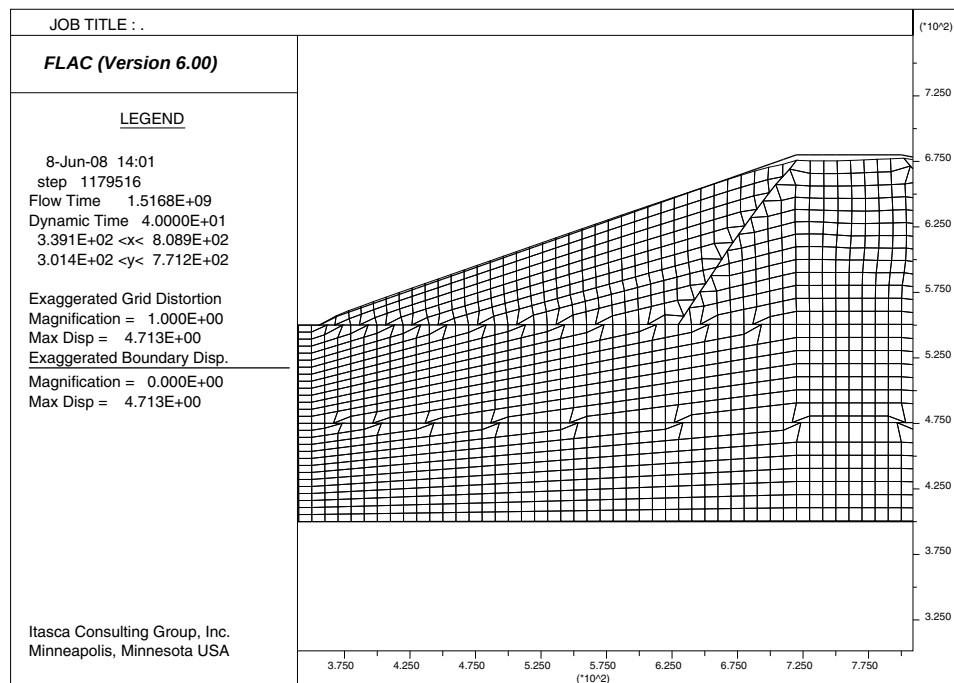


Figure 1.139 Deformed grid at 40 seconds – Byrne (liquefaction) material and Rayleigh damping

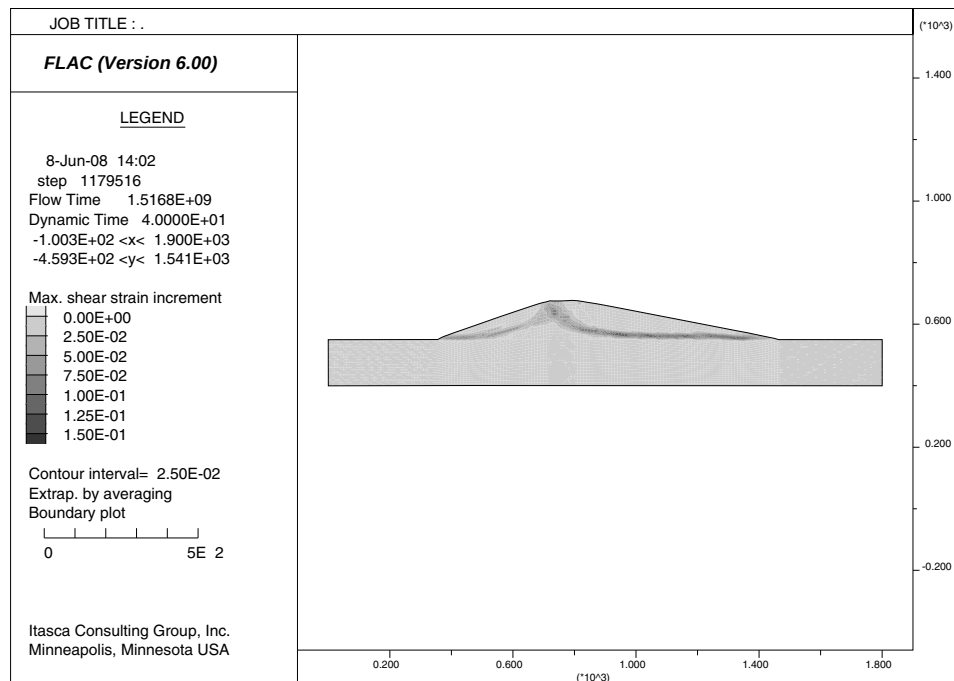


Figure 1.140 Shear-strain increment contours at 40 seconds – Byrne (liquefaction) material and Rayleigh damping

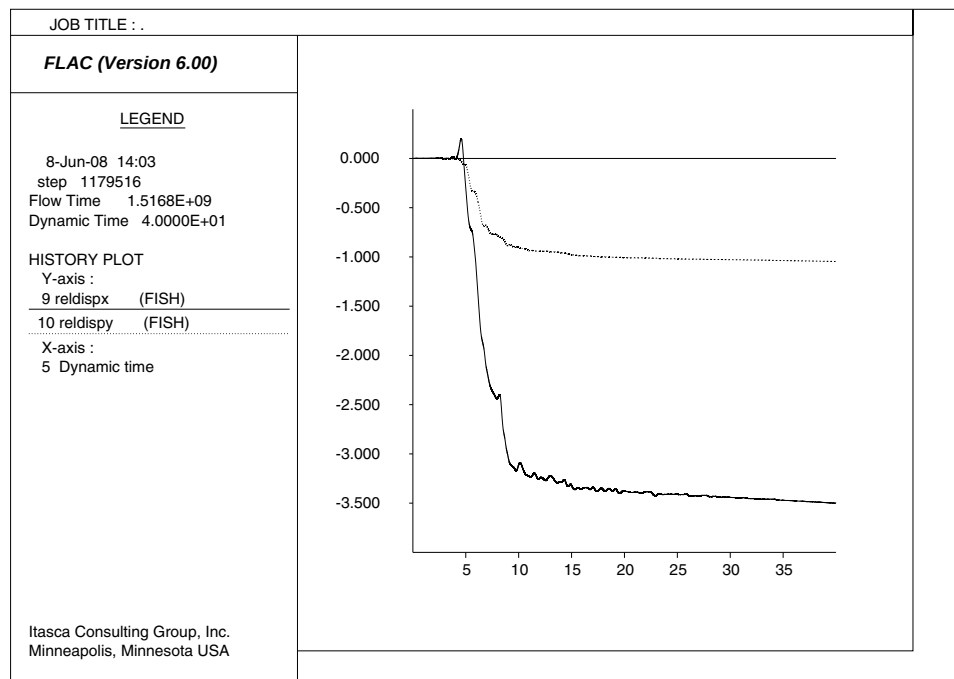


Figure 1.141 Relative displacements at gridpoint (62,29) along upstream slope – Byrne (liquefaction) material and Rayleigh damping

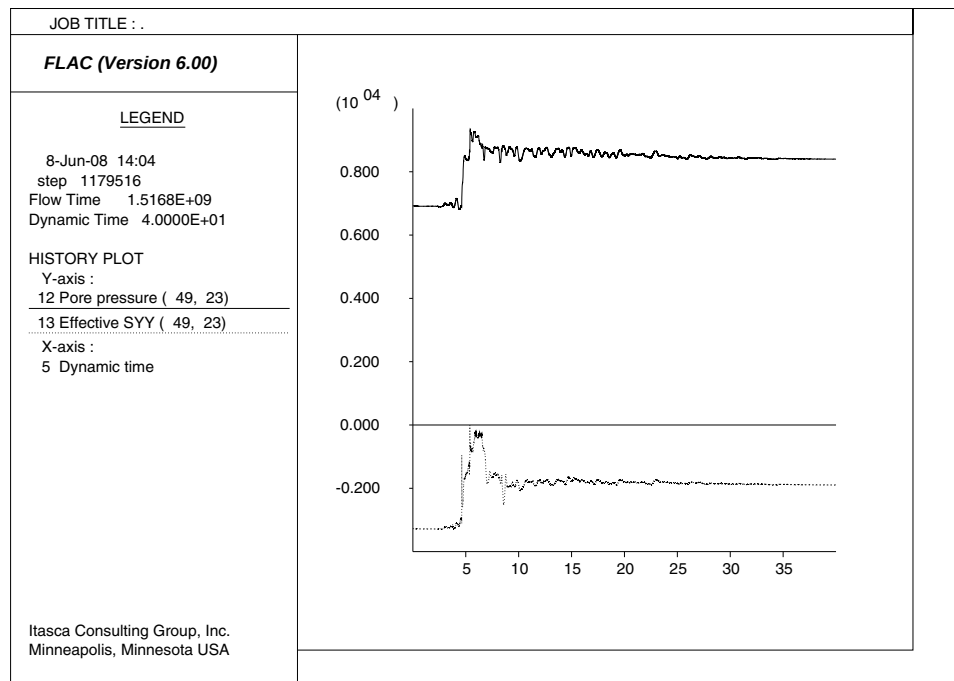


Figure 1.142 Pore-pressure and effective vertical stress near upstream slope – Byrne (liquefaction) material and Rayleigh damping

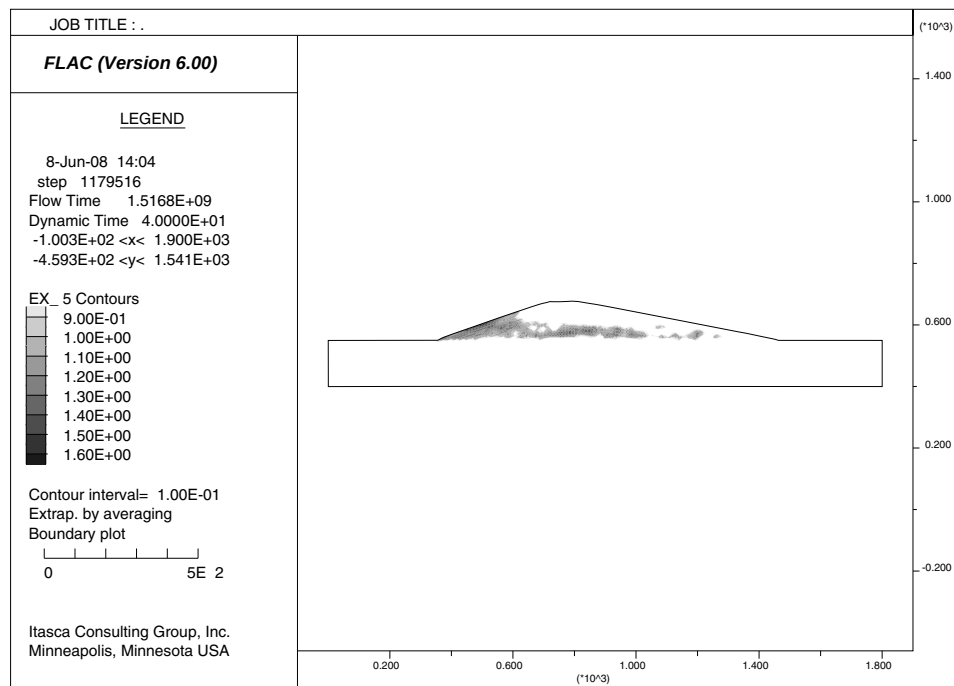


Figure 1.143 Excess pore-pressure ratio contours (values greater than 0.99) – Byrne (liquefaction) material and Rayleigh damping

Example 1.30 EARTHDAM.DAT – Seismic analysis of an embankment dam

```

;Project Record Tree export
;Title:Earth Dam
;... STATE: EDAM1 ....
;Project Record Tree export
;... STATE: EDAM1 ....
config gw ats dyn extra 10
grid 180,28
gen (0.0,400.0) (0.0,680.0) (1800.0,680.0) (1800.0,400.0) ratio 1.0,1.0 &
  i=1,181 j=1,29
model elastic
table 1 delete
table 1 0 550.0 350.0 550.0 720.0 680.0 800.0 680.0 1470 550.0 1800 550.0
set echo off
call tabtop.fis
set tid=1
gentabletop
gen line 0.0,475.0 1800.0,475.0
gen line 350.0,550.0 1470.0,550.0
gen line 720.0,680.0 630.0,550.0
gen line 800.0,680.0 900.0,550.0
save edam1.sav

;... STATE: EDAM2 ....
group 'foundation:soil 2' region 83 4
model mohr group 'foundation:soil 2'
prop density=3.88 bulk=1.06308E7 shear=4.90654E6 cohesion=160.0 &
  friction=40.0 dilation=0.0 tension=0.0 group 'foundation:soil 2'
group 'foundation:soil 1' region 94 11
model mohr group 'foundation:soil 1'
prop density=3.88 bulk=1.06308E7 shear=4.90654E6 cohesion=83.5 &
  friction=40.0 dilation=0.0 tension=0.0 group 'foundation:soil 1'
group 'embankment:soil 1' region 106 23
model mohr group 'embankment:soil 1'
prop density=3.51 bulk=5.698E6 shear=2.630e6 cohesion=120.0 friction=35.0 &
  dilation=0.0 tension=0.0 group 'embankment:soil 1'
group 'embankment:soil 1' region 60 22
model mohr group 'embankment:soil 1'
prop density=3.51 bulk=5.698E6 shear=2.630e6 cohesion=120.0 friction=35.0 &
  dilation=0.0 tension=0.0 group 'embankment:soil 1'
group 'embankment:soil 2' region 79 20
model mohr group 'embankment:soil 2'
prop density=3.73 bulk=5.698E6 shear=2.630e6 cohesion=120.0 friction=35.0 &
  dilation=0.0 tension=0.0 group 'embankment:soil 2'

```

```
prop por=0.3 perm=5.25E-9 region 74 3
prop por=0.3 perm=5.25E-8 region 73 11
save edam2.sav
```

```
;... STATE: EDAM3 ....
model null group 'embankment:soil 1'
model null group 'embankment:soil 2'
fix x y j 1
fix x i 181
fix x i 1
set gravity=32.2
set flow=off
water density=1.94
set dyn=off
set echo off
call Ininv.fis
set wth=550 k0x=0.5 k0z=0.5
ininv
history 999 unbalanced
solve elastic
save edam3.sav
```

```
;... STATE: EDAM4 ....
model mohr group 'embankment:soil 2'
prop density=3.73 bulk=5.698E6 shear=2.630e6 cohesion=120.0 friction=35.0 &
  dilation=0.0 tension=0.0 group 'embankment:soil 2'
model mohr group 'embankment:soil 1'
prop density=3.51 bulk=5.698E6 shear=2.630e6 cohesion=120.0 friction=35.0 &
  dilation=0.0 tension=0.0 group 'embankment:soil 1'
solve
save edam4.sav
```

```
;... STATE: EDAM5 ....
apply pp 0.0 var 0.0 16866.36 from 70,29 to 1,1
fix pp i 82 149 j 29
fix pp i 150 181 j 29
prop por=0.3 perm=5.25E-8 region 59 26
prop por=0.3 perm=5.25E-8 region 100 23
prop por=0.3 perm=5.25E-9 region 76 21
history 1 pp i=48, j=24
history 2 pp i=79, j=17
history 3 pp i=124, j=21
history 4 gwttime
set mechanical=off
set flow=on
water bulk 4.1e6
```

```
set funsat=on
set fastwb=on
solve
save edam5.sav

;... STATE: EDAM6 ....
apply pressure 0.0 var 0.0 8120.8 from 70,29 to 1,29
set mechanical=on
set flow=off
water bulk=0.0
solve
save edam6.sav

;*** BRANCH: MOHR-COULOMB MODEL ****
;... STATE: EDAM7 ....
water bulk=4100000.0
set dyn=on
set =large
call table104.dat
initial xdisp 0 ydisp 0
initial xvel 0 yvel 0
set echo off
call strain_ini_hist.fis
strain_ini_hist
set echo off
call strain_hist.fis
strain_hist
set echo off
call reldispx.fis
reldispx
set echo off
call inipp.fis
inipp
set echo off
call excpp.fis
excpp
history 5 dytime
history 7 vsxy i=77, j=20
history 8 str_77_20
history 9 reldispx
history 10 reldispy
history 11 excpp
history 12 pp i=49, j=23
history 13 esyy i=49, j=23
history 14 xvel i=49, j=24
history 15 xvel i=80, j=18
```

```
history 16 xvel i=121, j=24
set step=10000000
save edam7.sav

;*** BRANCH: UNDAMPED - ELASTIC ****
;... STATE: EDAM8 ....
prop coh 1e10 tens 1e10 notnull
history 105 xvel i=80, j=1
apply ffield
apply sxy -8722.24 hist table 104 from 1,1 to 181,1
apply xquiet yquiet from 1,1 to 181,1
save edam8.sav

;... STATE: EDAM9 ....
solve dytime 40.0
save edam9.sav

;... STATE: EDAM10 ....
hist write 14 vs 5 table 8
set echo off
call fft_tables.fis
set fft_inp1=8 fft_inp2=9
fft_tables
set echo off
call Fft.fis
fftransform
hist write 15 vs 5 table 10
set echo off
call fft_tables.fis
set fft_inp1=10 fft_inp2=11
fft_tables
fftransform
hist write 16 vs 5 table 12
set echo off
call fft_tables.fis
set fft_inp1=12 fft_inp2=13
fft_tables
fftransform
hist write 105 vs 5 table 110
save edam10.sav

;*** BRANCH: HYSTERETIC DAMPING ****
restore edam7.sav

;... STATE: EDAM11ORI ....
initial hyst default -3.156 1.904
```

```
set dy_damping rayleigh=0.0020 1.0 stiffness
apply ffield
apply sxy -8722.24 hist table 104 from 1,1 to 181,1
apply xquiet yquiet from 1,1 to 181,1
history 101 xaccel i=79, j=1
history 102 xaccel i=76, j=29
history 103 xaccel i=161, j=29
history 104 xaccel i=14, j=29
save edam11ori.sav

;... STATE: EDAM12ORI ....
solve dytime 40.0
save edam12ori.sav

;... STATE: EDAM12AORI ....
hist 106 read acc_target.his
save edam12aori.sav

;*** BRANCH: HYSTERETIC DAMPING-AMP FACTOR=1.33 ****
restore edam7.sav

;... STATE: EDAM11 ....
initial hyst default -3.156 1.904
set dy_damping rayleigh=0.0020 1.0 stiffness
apply ffield
;; amp = 1.33 * 8722.24
apply sxy -11600.6 hist table 104 from 1,1 to 181,1
apply xquiet yquiet from 1,1 to 181,1
history 101 xaccel i=79, j=1
history 102 xaccel i=76, j=29
history 103 xaccel i=161, j=29
history 104 xaccel i=14, j=29
save edam11.sav

;... STATE: EDAM12 ....
solve dytime 40.0
save edam12.sav

;... STATE: EDAM12A ....
hist 106 read acc_target.his
save edam12a.sav

;*** BRANCH: RAY 0.063 GMAX 0.8-AMP FACTOR=1.33 ****
restore edam7.sav

;... STATE: EDAM13 ....
```

```

set echo off
call greduce.fis
set _prat=0.30
set _gfac=0.8
greduce
solve
set dy_damping rayleigh=0.063 1.0
apply ffield
;; amp = 1.33 * 8722.24
apply sxy -11600.6 hist table 104 from 1,1 to 181,1
apply xquiet yquiet from 1,1 to 181,1
history 101 xaccel i=79, j=1
history 102 xaccel i=76, j=29
history 103 xaccel i=161, j=29
history 104 xaccel i=14, j=29
save edam13.sav

;... STATE: EDAM14 ....
solve dytime 40
save edam14.sav

;*** BRANCH: FINN MODEL (N1)60 = 10 ****
restore edam6.sav

;... STATE: EDAM15 ....
model finn region 80 21
prop density=3.73 bulk=5698000.0 shear=2630000.0 cohesion=120.0 &
  friction=35.0 ff_latency=1000000 ff_c1=0.49044 ff_c2=0.81559 ff_switch=1 &
  region 80 21
model finn region 60 25
prop density=3.51 bulk=5698000.0 shear=2630000.0 cohesion=120.0 &
  friction=35.0 ff_latency=1000000 ff_c1=0.49044 ff_c2=0.81559 ff_switch=1 &
  region 60 25
model finn region 107 25
prop density=3.51 bulk=5698000.0 shear=2630000.0 cohesion=120.0 &
  friction=35.0 ff_latency=1000000 ff_c1=0.49044 ff_c2=0.81559 ff_switch=1 &
  region 107 25
solve
water bulk=4100000.0
set dyn=on
set =large
call table104.dat
initial xdisp 0 ydisp 0
initial xvel 0 yvel 0
set echo off
call strain_ini_hist.fis

```

```
strain_ini_hist
set echo off
call strain_hist.fis
strain_hist
set echo off
call reldispx.fis
reldispx
set echo off
call inipp.fis
inipp
set echo off
call excpp.fis
excpp
history 5 dytime
history 7 vsxy i=77, j=20
history 8 str_77_20
history 9 reldispx
history 10 reldispy
history 11 excpp
history 12 pp i=49, j=23
history 13 esyy i=49, j=23
history 14 xaccel i=77, j=1
history 15 xaccel i=77, j=29
history 16 xaccel i=172, j=29
history 17 xaccel i=9, j=29
set step=10000000
prop ff_latency 50 region 78 19
prop ff_latency 50 region 60 22
prop ff_latency 50 region 103 26
save edam15.sav

;*** BRANCH: HYSTERETIC DAMPING WITH EXCESSPP AMP FACTOR=1.33 ****
;... STATE: EDAM16 ....
initial hyst default -3.156 1.904
set dy_damping rayleigh=0.0020 1.0 stiffness
apply ffield
;; amp = 1.33 * 8722.24
apply sxy -11600.6 hist table 104 from 1,1 to 181,1
apply xquiet yquiet from 1,1 to 181,1
set echo off
call savepp.fis
savepp
set echo off
call getExcesspp.fis
set nsample=50 nstep=1
getExcesspp
```

```
save edam16.sav

;... STATE: EDAM17 ....
solve dytime 40.0
save edam17.sav

;... STATE: EDAM17A ....
hist write 14 vs 5 table 114
set echo off
; directory is changed to call file in case of nested calls
set cd name C:\Program Files\Itasca\flac600\gui\fishlib\Tables
call INT.FIS
set cd back
set int_in=114 int_out=115
integrate
save edam17a.sav

;*** BRANCH: RAY 0.063 GMAX 0.8 WITH EXCESSPP AMP FACTOR=1.33 ****
restore edam15.sav

;... STATE: EDAM18 ....
set echo off
call greduce.fis
set _prat=0.30
set _gfac=0.8
greduce
solve
set dy_damping rayleigh=0.063 1.0
apply ffield
;; amp = 1.33 * 8722.24
apply sxxy -11600.6 hist table 104 from 1,1 to 181,1
apply xquiet yquiet from 1,1 to 181,1
set echo off
call savepp.fis
savepp
set echo off
call getExcesspp.fis
set nsample=50 nstep=1
getExcesspp
save edam18.sav

;... STATE: EDAM19 ....
solve dytime 40
solve
save edam19.sav
```

```
;... STATE: EDAM19A ....
hist write 14 vs 5 table 114
set echo off
; directory is changed to call file in case of nested calls
set cd name C:\Program Files\Itasca\flac600\gui\fishlib\Tables
call INT.FIS
set cd back
set int_in=114 int_out=115
integrate
save edam19a.sav

;*** plot commands ***
;plot name: pp bound
plot hold pp fill bound
;plot name: syy
plot hold syy fill bound
;plot name: ssi bound
plot hold ssi fill bound
;plot name: grid bound
plot hold grid magnify 1.0 bound magnify 0.0 white
;plot name: xdisp
plot hold xdisp fill inv bound
;plot name: str v str 77,20
plot hold history 7 line vs 8
;plot name: reldisp
plot hold history 9 line 10 line skip 100 vs 5
;plot name: pp-esyy hist
plot hold history 12 line 13 line vs 5
;plot name: excess pp
plot hold history 11 line vs 5
;plot name: disp
plot hold displacement iwhite bound bound magnify 0.0 iwhite
;plot name: xacc at crest
plot hold history 16 vs 5
;plot name: compare base vel
label table 104
input velocity
label table 110
recorded velocity
plot hold table 104 line 115 line alias 'Velocity at base (ft/sec)'
;plot name: excess pp ratio
plot hold ex_5 zone fill min 0.99 bound
;plot name: compare acc
plot hold history 104 line 106 vs 5
```

1.7 Verification Problems

Several examples to validate and demonstrate the dynamic option in *FLAC* are presented. The data files for these examples are contained in the “ITASCA\FLAC600\Dynamic” directory.

1.7.1 Natural Periods of an Elastic Column

A column of elastic material resting on a rigid base has natural periods of vibration, depending on the mode of oscillation and the confining conditions. Three cases are examined: an unconfined column; a confined column in compression; and a column in shear.

The column is loaded by applying gravity either in the x - or y -direction and observing the oscillations with zero damping. The case of confined compression is modeled by inhibiting lateral displacement along the vertical boundaries, which prevents lateral deformation of the mesh. For unconfined compression, lateral displacement is not inhibited. For the column in shear, vertical motion is inhibited to eliminate bending modes; the loading is applied laterally.

The theoretical value for natural period of oscillation, T , is given by [Eq. \(1.125\)](#),

$$T = 4L \sqrt{\frac{\rho}{E^*}} \quad (1.125)$$

where E^* is the appropriate modulus selected from [Table 1.4](#):

Table 1.4 *Moduli appropriate to various deformation modes*

Confined Compression	Unconfined Compression	Shear
$K + (4/3) G$	$4G \left[\frac{(1/3) G + K}{K + (4/3) G} \right]$ (plane strain, Young's modulus)	G
2.5714×10^4	1.4286×10^4	1.0×10^4

FLAC data files for the three cases are given in [Examples 1.31](#), [1.32](#) and [1.33](#). Material properties are given below.

Table 1.5 *Material properties*

Properties	Symbol	Value	Comment
bulk modulus	K	2.0×10^4	for compression tests
shear modulus	G	0.428562×10^4	
Poisson's ratio	ν	0.4	
bulk modulus	K	1.0×10^4	for shear tests
shear modulus	G	1.0×10^4	
Poisson's ratio	ν	0.125	
density	ρ	1.0	
applied gravity	g_y	-1.0	for compression tests
	g_x	0.1	for shear tests
column height	L	800	
column width	W	100	

The theoretical periods and calculated (*FLAC*) natural periods of oscillation averaged over several periods by the *FISH* function **crossings** are compared in [Table 1.6](#) (see [Example 1.34](#)):

Table 1.6 *Comparison of theoretical and calculated (FLAC) dynamic period T of oscillation for three modes*

	Confined Compression	Unconfined Compression	Shear
Theoretical	19.96	26.77	32.00
<i>FLAC</i>	19.95	26.77	31.99

Example 1.31 Data file for confined compression

```
con dy
gr 2 8
m e
gen -50 -400 -50 400 50 400 50 -400
prop den 1 bulk 2e4 shear 0.428562e4
fix y j=1
fix x i=1
fix x i=3
set grav 1.0
hist n=1
hist yvel i=2 j=9
hist dytime
solve dytime 200
call avper.fis
save dyn_conf.sav
```

Example 1.32 Data file for unconfined compression

```
con dy
gr 2 8
m e
gen -50 -400 -50 400 50 400 50 -400
prop den 1 bulk 2e4 shear 0.428562e4
fix y j=1
set grav 1.0
hist n=1
hist yvel i=2 j=9
hist dytime
solve dytime 200
call avper.fis
save dyn_unconf.sav
```

Example 1.33 Data file for shear

```

con dy
gr 2 8
m e
gen -50 -400 -50 400 50 400 50 -400
prop den 1 bulk 1e4 shear 1e4
fix x y j=1
fix y j=9
fix y i=1
fix y i=3
set grav 0.1 90
hist n=1
hist xvel i=2 j=9
hist dytime
solve dytime 200
call avper.fis
save dyn_shear.sav

```

Example 1.34 Listing of “AVPER.FIS”: function to compute average period

```

hist write 1 tab 1 ; Note: velocity history must be number 1
def crossings
  ndif          = 0
  dif           = 0.0
  t_cross_old   = 0.0
  sign          = 1.0
  delta_t       = dytime / step
  loop n (1,step)
    if sgn(ytable(1,n)) # sgn(sign)
      sign      = -sign
      t_cross   = (n - 1) * delta_t
      if t_cross_old # 0.0
        dif     = dif + t_cross - t_cross_old
        ndif    = ndif + 1
      endif
      t_cross_old = t_cross
    endif
  end_loop
  ii = out(' Average period = '+string(2.0*dif/ndif))
end
crossings

```

1.7.2 Comparison of *FLAC* to *SHAKE* for a Layered, Linear-Elastic Soil Deposit

The program *SHAKE* is widely used in the field of earthquake engineering for computing the seismic response of horizontally layered soil deposits. Here, we compare *FLAC* with *SHAKE* for the case of a one-dimensional layered, linear-elastic soil deposit, driven at its base by the horizontal acceleration given by [Eq. \(1.126\)](#)

$$\ddot{u}(t) = \sqrt{\beta e^{-\alpha t} t^\gamma} \sin(2\pi f t) \quad (1.126)$$

where: $\alpha = 2.2$;
 $\beta = 0.375$;
 $\gamma = 8.0$; and
 $f = 3 \text{ Hz}$.

This input acceleration wave, plotted in [Figure 1.144](#), shows a maximum horizontal acceleration of 0.2 g reached after 3.75 seconds. The wave form selected for this comparison test does not require a baseline correction (see [Section 1.4.1.2](#)); the final velocity and displacement are both zero. Also, this form does not contain high-frequency components that could cause numerical distortion of the wave (see [Section 1.4.2](#)).

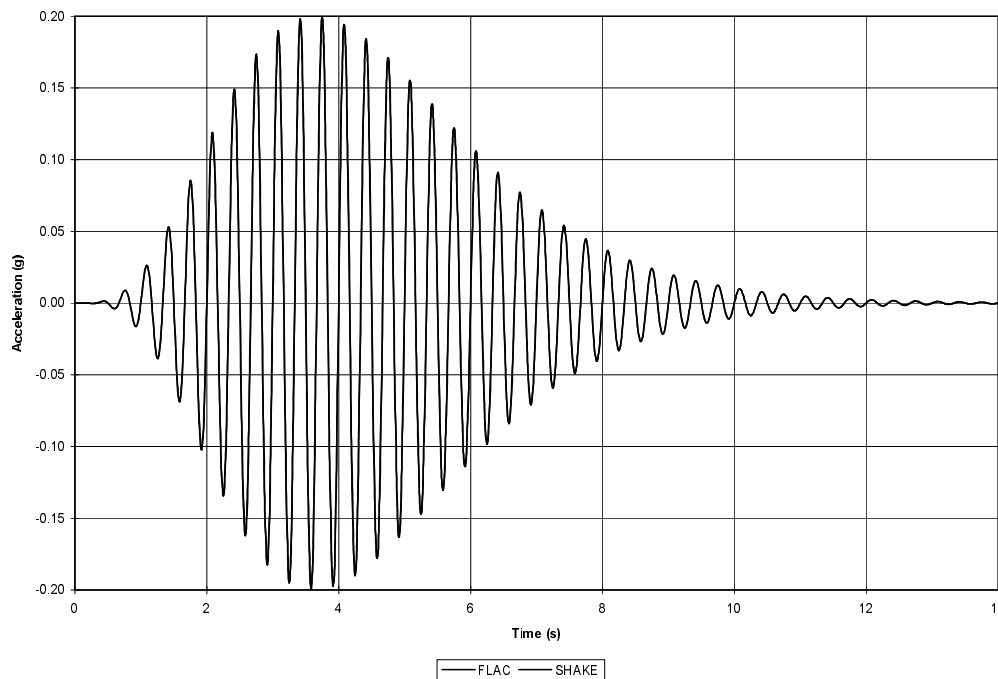


Figure 1.144 Input acceleration at bottom of model

The version of *SHAKE* used in the comparison is *SHAKE-91* (Idriss and Sun 1992). *SHAKE-91* is a modified version of *SHAKE* (originally published in 1971 by the Earthquake Engineering Research Center at the University of California in Berkeley, California, *SHAKE-91* can be downloaded at <http://nisee.berkeley.edu/software/>). *SHAKE-91* computes the response of a semi-infinite horizontally layered soil deposit overlying a uniform half-space subjected to vertically propagating shear waves. The program performs a linear analysis in the frequency domain; an iterative procedure accounts for some of the nonlinear effects in the soil. In this verification problem, we assume that the soil is linear. (A comparison assuming nonlinear elastic material is given in [Section 1.7.3](#)).

FLAC is compared to *SHAKE-91* for the following problem conditions. A layered soil deposit is 160 feet thick and contains two materials, as shown in [Figure 1.145](#). The stiffer layer (material 2) is 40 feet thick, starts at a depth of 40 feet, and is sandwiched between the softer layers (material 1).

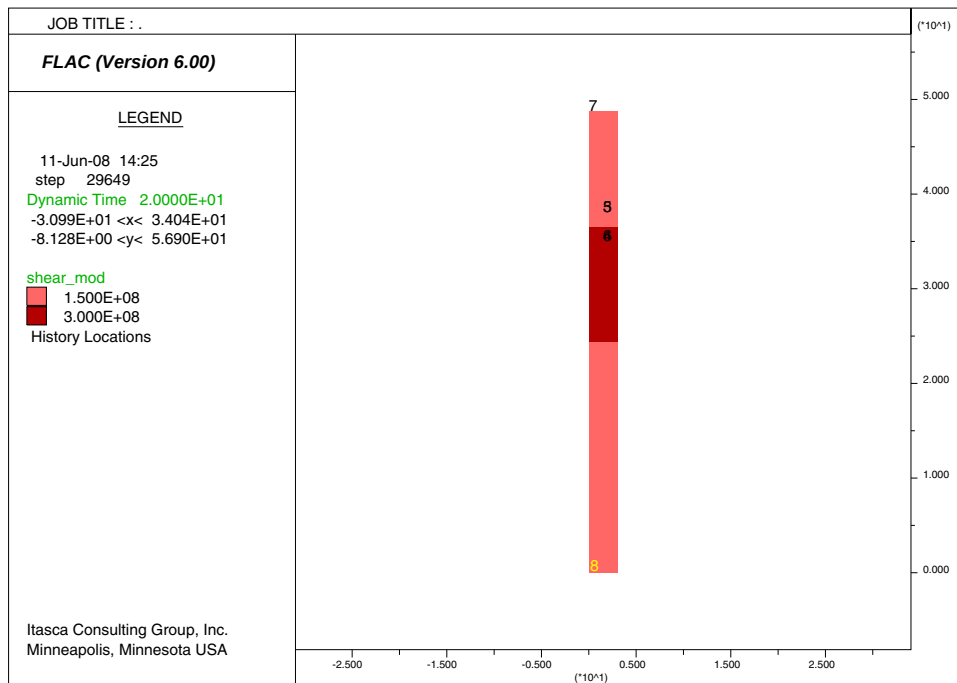


Figure 1.145 One-dimensional model containing two materials (history locations are also shown)

The soil is treated as a linear elastic material, with the following properties:

material	1	2
shear modulus (MPa)	150	300
density (kg/m ³)	1800	2000
fraction of critical damping	10%	10%

By assuming that shear modulus and damping are strain-independent, the same properties are used in *FLAC* and *SHAKE-91*. The data file for the analysis with *SHAKE-91* is shown in [Example 1.35](#). The file for the *FLAC* analysis is given in [Example 1.36](#).

The *FLAC* model consists of 16 square zones, each with a length of 10 feet (3.05 m); the zone length is well within 1/10 of the longest wavelength, to provide accurate wave transmission. Vertical movement is prevented at the sides of the model. Rayleigh damping is specified at 10%, operating at a center frequency of 3 Hz.

[Figure 1.146](#) shows the horizontal acceleration at the top of the model (gridpoint 17 in *FLAC*, and sub-layer 1 in *SHAKE-91*) as a function of time. Both records are very similar: the maximum acceleration calculated by *FLAC* is 0.160 g; the maximum acceleration calculated by *SHAKE-91* is 0.156 g (a 2.6% difference).

[Figures 1.147](#) and [1.148](#) show the evolution of shear stress and shear strain at a depth of 35 feet (within material 1). The *FLAC* results have been obtained through a history of σ_{xy} at zone 13 and the *FISH* function **shrstr13**. The *SHAKE-91* results have been obtained at the top of sub-layer 5, using analysis option 7 in the code. The results from both codes are again very similar, with a difference of less than 4%. Note that the stress histories do not contain the viscous component contributed by Rayleigh damping.

[Figure 1.149](#) shows two shear stress vs strain curves calculated by *SHAKE-91*: one at a depth of 35 feet (in material 1), and the other at a depth of 45 feet (in material 2). They have been obtained through option 7 at the top of sub-layers 5 and 7. In both cases, the relation between stress and strain is linear, with a slope equal to the shear modulus.

[Figure 1.150](#) shows a viscous shear stress (σ_{xy}^v) vs strain plot, calculated by *FLAC* at the same locations as in the previous figure. While the average slope is again equal to the shear modulus, this plot shows hysteresis loops due to the viscous damping. If we were to plot *FLAC* shear stress (σ_{xy}) versus shear strain, we would obtain a linear relation.

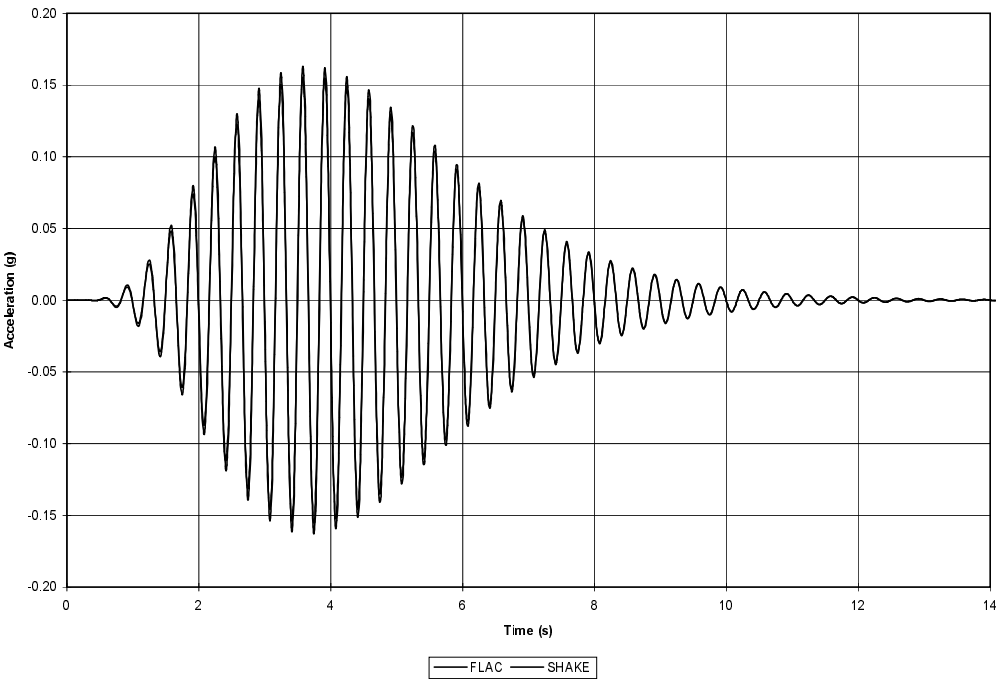


Figure 1.146 Horizontal acceleration at top of model

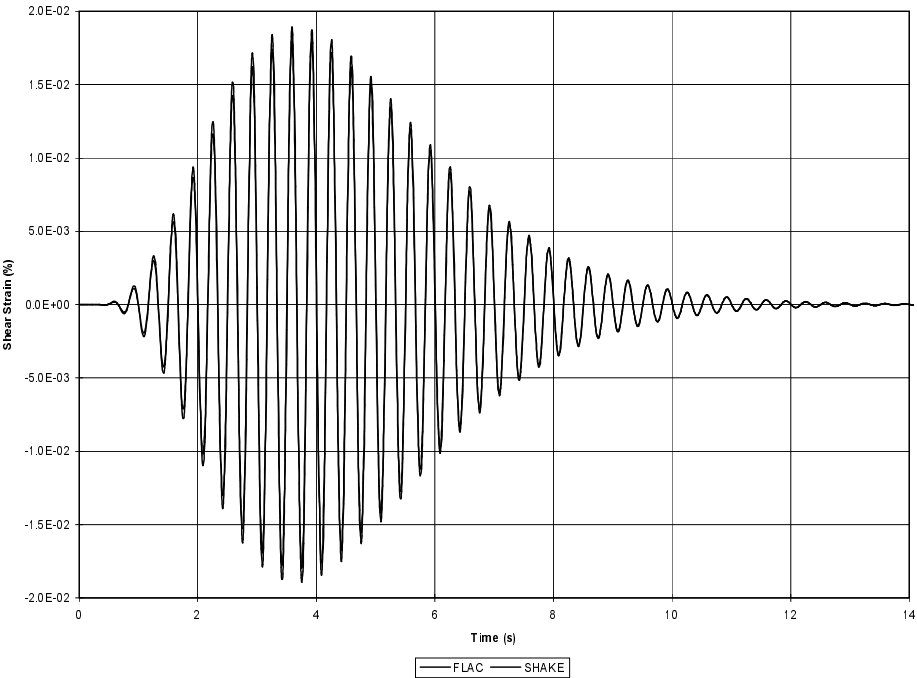


Figure 1.147 Shear strain history at 35 ft depth in model

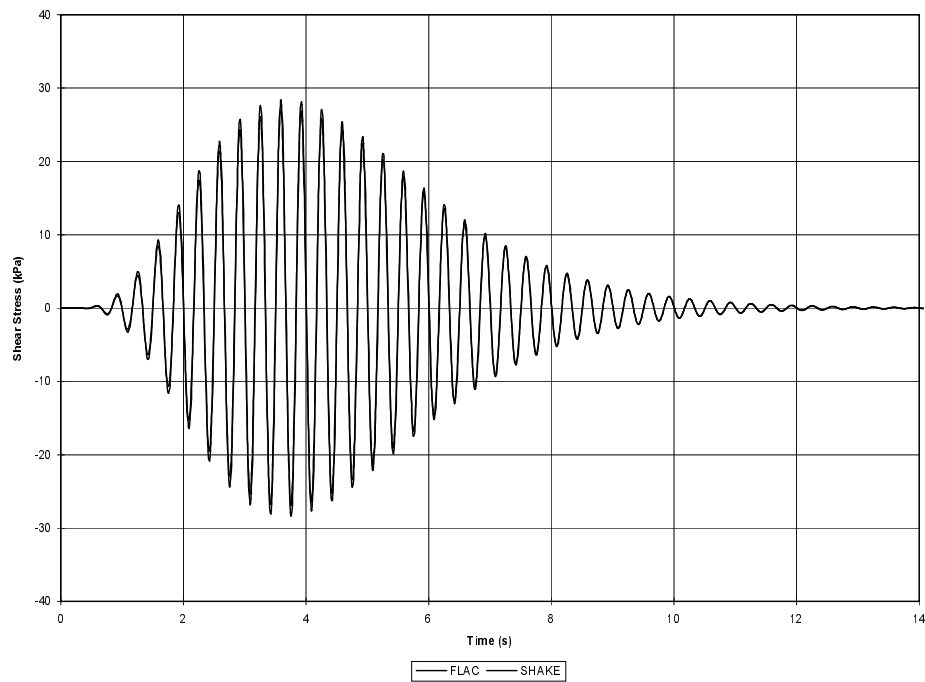


Figure 1.148 Shear stress history at 35 ft depth in model

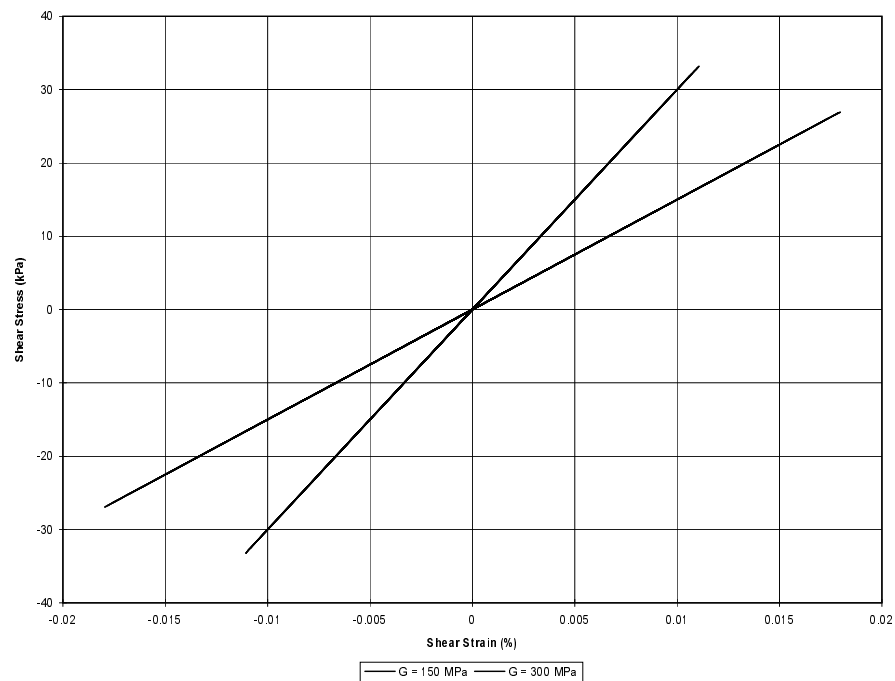


Figure 1.149 Shear stress versus shear strain in material 1 and material 2 (SHAKE-91 results)

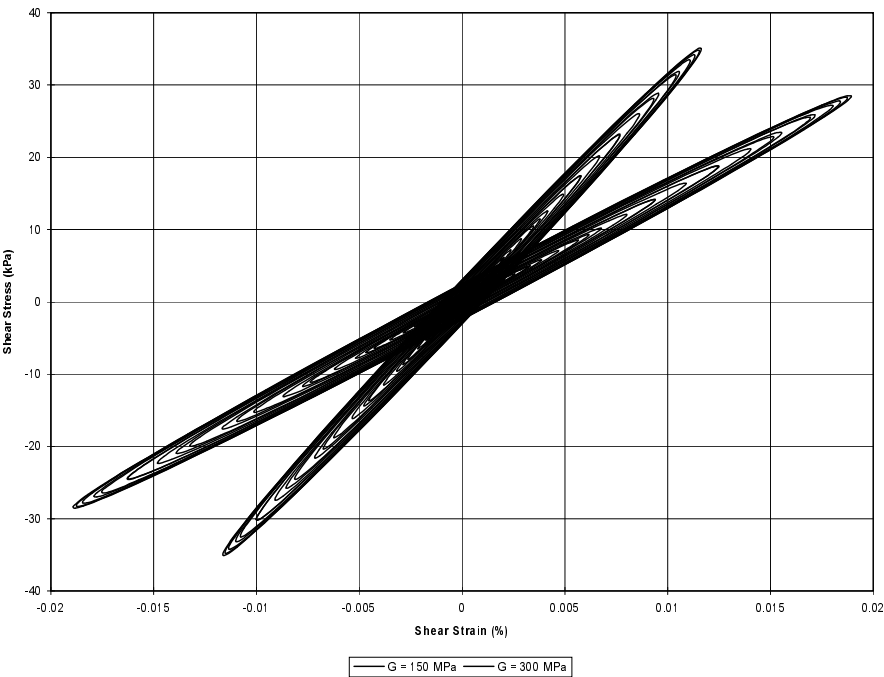


Figure 1.150 Viscous shear stress versus shear strain in material 1 and material 2 (FLAC results)

Example 1.35 SHAKE-91 model of layered soil deposits

option 1 - dynamic soil properties -							
1							
3							
11	material #1 modulus						
0.0001	0.0003	0.001	0.003	0.01	0.03	0.1	0.3
1.	3.	10.					
1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.000	1.000	1.000					
11	damping for material #1						
0.0001	0.0003	0.001	0.003	0.01	0.03	0.1	0.3
1.	3.16	10.					
10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00
10.00	10.00	10.00					
11	material #2 modulus						
0.0001	0.0003	0.001	0.003	0.01	0.03	0.1	0.3
1.	3.	10.					
1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.000	1.000	1.000					
11	damping for material #2						
0.0001	0.0003	0.001	0.003	0.01	0.03	0.1	0.3

```

1.          3.          10.
10.00      10.00      10.00      10.00      10.00      10.00      10.00      10.00
10.00      10.00      10.00
      8      material #3 modulus
.0001      0.0003      0.001      0.003      0.01      0.03      0.1      1.0
1.000      1.000      1.000      1.000      1.000      1.000      1.000      1.000
      5      damping for material #3
.0001      0.001      0.01      0.1      1.
10.00      10.00      10.00      10.00      10.00
      3      1      2      3
Option 2 -- Soil Profile
      2
      1 19      Example -- 160-ft layer with 2 materials
      1 2          10.00 3130.086      .100      .11225
      2 2          10.00 3130.086      .100      .11225
      3 2          10.00 3130.086      .100      .11225
      4 2          5.00 3130.086      .100      .11225
      5 2          5.00 3130.086      .100      .11225
      6 1          5.00 6260.173      .100      .12473
      7 1          5.00 6260.173      .100      .12473
      8 1          10.00 6260.173      .100      .12473
      9 1          10.00 6260.173      .100      .12473
     10 1          10.00 6260.173      .100      .12473
     11 2          10.00 3130.086      .100      .11225
     12 2          10.00 3130.086      .100      .11225
     13 2          10.00 3130.086      .100      .11225
     14 2          10.00 3130.086      .100      .11225
     15 2          10.00 3130.086      .100      .11225
     16 2          10.00 3130.086      .100      .11225
     17 2          10.00 3130.086      .100      .11225
     18 2          10.00 3130.086      .100      .11225
     19 3                      .100      .140      4000.
Option 3 -- input motion:
      3
2048 2048      .01      inp3.acc                      (8f10.6)
      1.0                      100. 0 8
Option 4 -- sublayer for input motion (1):
      4
     19      1
Option 5 -- number of iterations & ratio of avg strain to max strain
      5
      1 2      1.00
Option 6 -- sublayers for which accn time histories are computed and saved:
      6
      1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
      0 1 1 1 1 1 1 1 1 1 1 1 1 1 1

```

```

1 0 0 0 0 1 0 0 0 0 1 0 0 0 0
Option 6 -- sublayers for which accn time histories are computed and saved:
6
16 17 17 19 19
1 1 1 1 0
0 0 0 0 1
option 7 -- sublayer shear stress or strain are computed and saved:
7
5 1 1 0 2048 -- stress in level 5
5 0 1 0 2048 -- strain in level 5
option 7 -- sublayer shear stress or strain are computed and saved:
7
7 1 1 0 2048 -- stress in level 7
7 0 1 0 2048 -- strain in level 7
execution will stop when program encounters 0
0

```

Example 1.36 FLAC model of layered soil deposits

```

config dynamic ex 4
;-----
;Grid generation and model properties
;-----
grid 1,16
model elastic
prop bulk 150e6 she 150e6 den 1800
prop bulk 300e6 she 300e6 den 2000 j 9 12
ini x mul 3.048
ini y mul 3.048
;-----
; FISH function to calculate shear strain in zones 12 and 13
;-----
def shrstr12
shrstr12=(xdisp(1,13)-xdisp(1,12))/(y(1,13)-y(1,12))
shrstr13=(xdisp(1,14)-xdisp(1,13))/(y(1,14)-y(1,13))
end
;-----
; Histories
;-----
hist unbal
his dytime
his sxy i=1 j=13
his sxy i=1 j=12
his vsxy i=1 j=13

```

```

his vsxy i=1 j=12
his xacc i 1 j 17
his xacc i=1 j=1
his shrstr12
his shrstr13
;-----
;Boundary Conditions
;-----
fix y
;-----
;FISH function to generate input wave
;-----
def acc_p
  omega=6*pi
  alfa=2.2
  beta=3.75e-1
  gamma=8.0
  acc_p=sqrt(beta*exp(-alfa*dytime)*dytime^gamma)*sin(omega*dytime)
end
;-----
;Application of acceleration
;-----
apply xacc 1.0 his acc_p j=1
apply yacc 0.0 j=1 ; this command prevents rocking along gridpoint j=1
set dy_damp=rayleigh 0.1 3.0
set clock 100000000 step 100000000
set dynamic on
solve dytime 20.0
save shake.sav
;Branch 2:dyn_24.sav
set hisfile visc_m1.his
hist write 5 vs 10
set hisfile visc_m2.his
hist write 4 vs 9
ret

```

1.7.3 Comparison of *FLAC* to *SHAKE* for a Layered, Nonlinear, Elastic Soil Deposit

In this section, we compare a simulation of wave propagation through a geological profile using *FLAC*, first with hysteretic damping (described in Section 1.4.3.4) and then with Rayleigh damping (described in Section 1.4.3.1) to that using *SHAKE*. The *FLAC* and *SHAKE* models simulate the following problem: a horizontally layered soil deposit overlying a rigid bedrock is subjected to a horizontal acceleration base motion. The soil deposit is 150 feet (45.72 m) deep and contains 10 different soil types. The soils are treated as nonlinear elastic materials, by assuming that shear moduli and damping are strain-dependent. Table 1.7 summarizes the properties and locations of the soil layers in the deposit. The dynamic characteristics of these soils are governed by two sets of modulus reduction factor ($G/G_{\max}$) and damping ratio (λ) versus shear strain curves: the first set for sand, and the second set for clay.* These curves are shown in Figures 1.151 through 1.154, and are denoted by the “SHAKE91” legend.

Table 1.7 Soil deposit profile and properties

Soil	1	2	3	4	5	6	7	8	9	10
Shear Modulus (MPa)	186	150	168	186	225	327	379	435	495	627
Density (kg/m ³)	2000	2000	2000	2000	2000	2082	2082	2082	2082	2082
Dynamic Property (set)	1	1	1	2	2	1	1	1	1	1
Location (feet)	1-5	5-20	20-30	30-50	50-70	70-90	90-110	110-130	130-140	140-150

This example is derived from the data file provided with the downloaded *SHAKE-91* code (see <http://nisee.berkeley.edu/software/>). The *SHAKE-91* data file is listed in Example 1.37; it is modified such that the deepest layer is given a large wave speed, to correspond to rigid bedrock. It should be noted that, to simulate a rigid base (Figure 1.3), we increase the stiffness (shear modulus) of the bedrock from 3.33 GPa to 2080 GPa in the *SHAKE-91* data file, for the purpose of comparison.

The base acceleration input is a set of seismic data recorded in the Loma Prieta Earthquake, which is also downloadable (see <http://nisee.berkeley.edu/software/>) with the *SHAKE-91* code (i.e., “DIAM.ACC”). The input accelerogram is shown in Figure 1.155. The record has a peak acceleration of approximately 0.11 g, duration of 40 seconds, and dominant frequency of 2.47 Hz.

* More variations of $G/G_{\max}$ and λ for soils are available in the literature mentioned in the *SHAKE-91* manual (Idriss and Sun, 1992) and duplicated here for easy reference (e.g., Hardin and Drnevich 1970, Seed and Idriss 1970, Seed et al. 1986, Sun et al. 1988, and Vucetic and Dobry 1991).

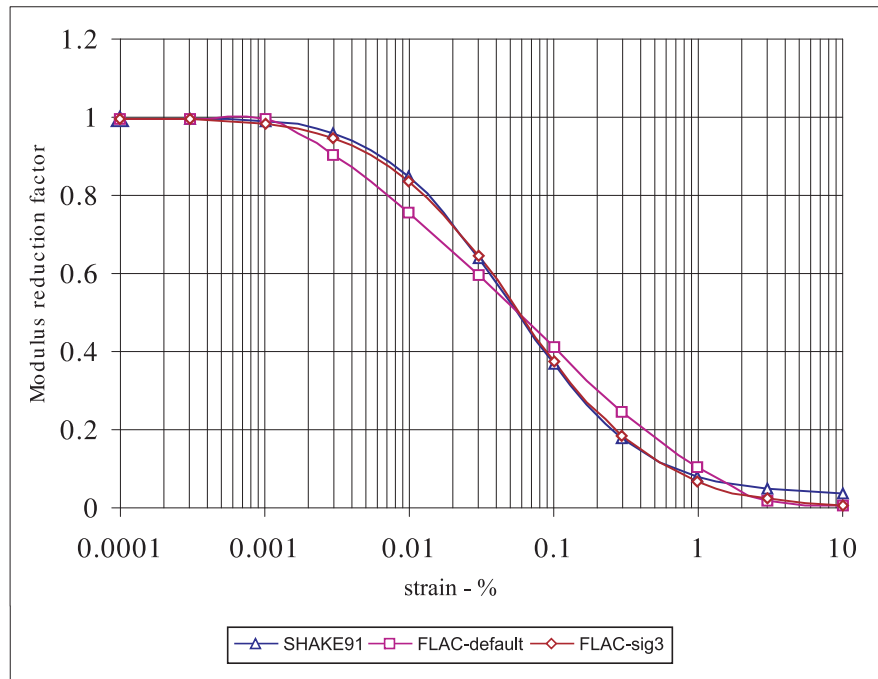


Figure 1.151 *Modulus reduction curve for dynamic property set 1 – sand*

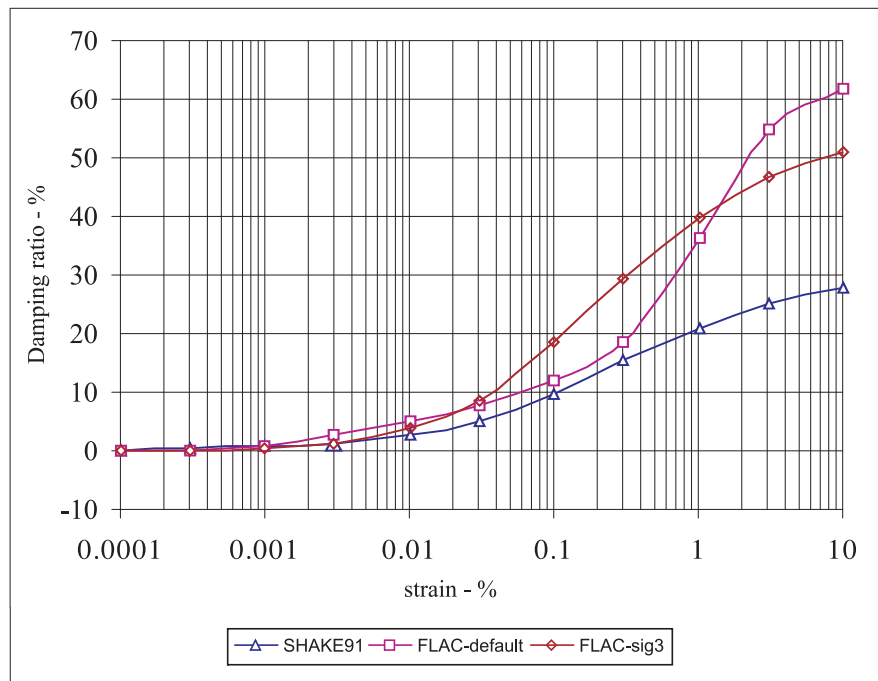


Figure 1.152 *Damping ratio curve for dynamic property set 1 – sand*

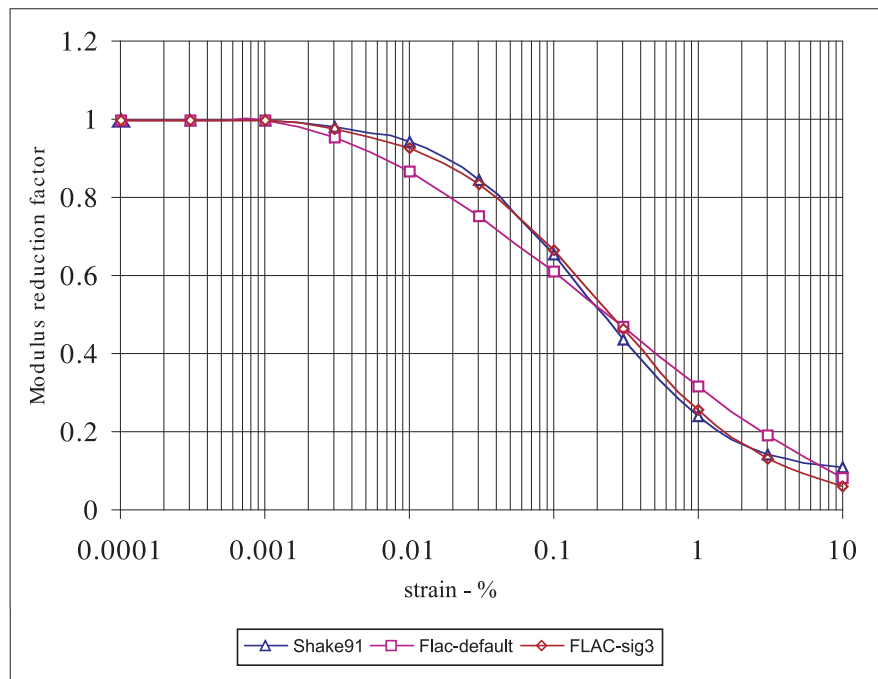


Figure 1.153 *Modulus reduction curve for dynamic property set 2 – clay*

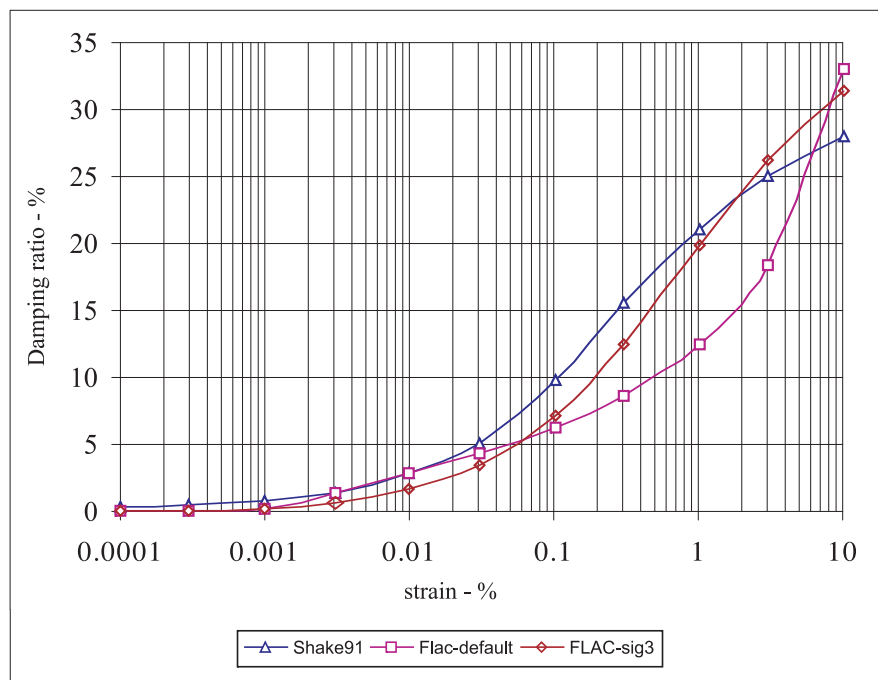


Figure 1.154 *Damping ratio curve for dynamic property set 2 – clay*

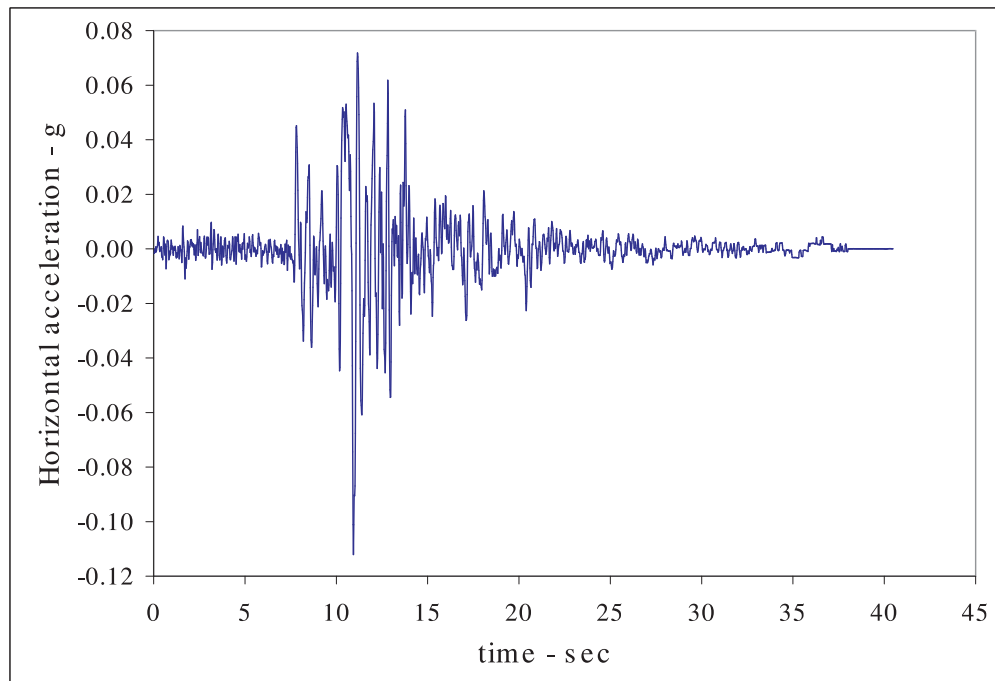


Figure 1.155 Input accelerogram

SHAKE model – The free-field response of the layered soil deposit to the base-motion acceleration is calculated using *SHAKE-91*. An iterative solution is performed until the secant shear moduli and the damping ratios throughout the column converge to within a given tolerance of the values corresponding to 50% of the maximum shear strain. Depth profiles of the maximum cyclic stress ratio, *CSR* (ratio of maximum cyclic shear stress to initial vertical effective stress) and acceleration amplification ratio (ratio of maximum acceleration to the maximum acceleration at the base) are computed from the solution. Calculations are made for four different input accelerations, with the record scaled to a maximum base acceleration of 0.001 g, 0.01 g, 0.10 g and 1.0 g.* [Figures 1.156](#) and [1.157](#) present profiles of *CSR* with depth for the input record scaled to 0.001g and 0.1g, respectively. [Figure 1.158](#) plots the acceleration amplification ratios versus the four maximum base accelerations.

* The scaling is accomplished through the built-in functionality in *SHAKE-91*, using either of two scaling parameters in *OPTION 3*.

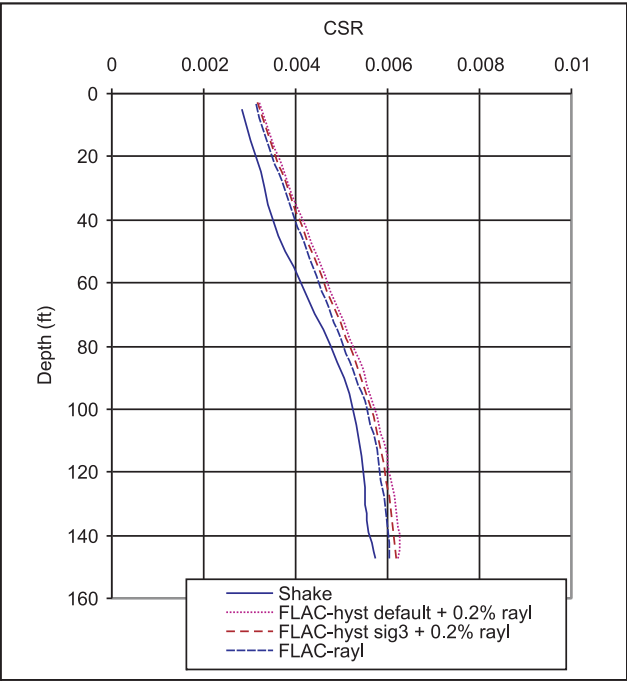


Figure 1.156 CSR profile for input motion 0.001 g

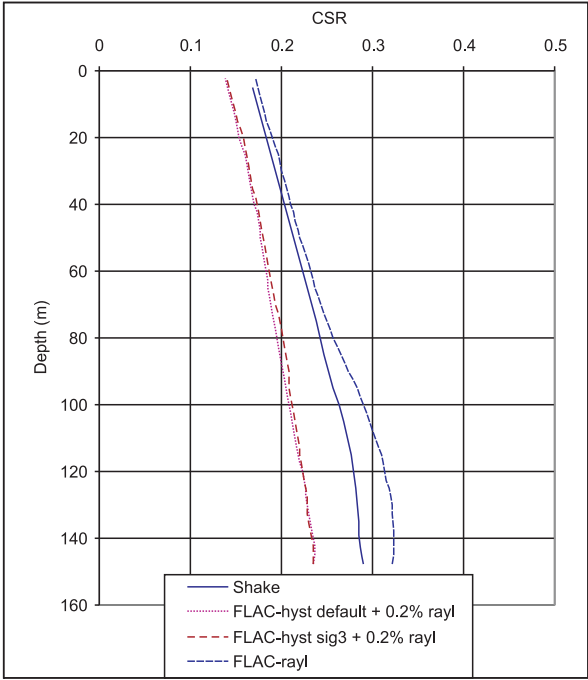


Figure 1.157 CSR profile for input motion 0.1 g

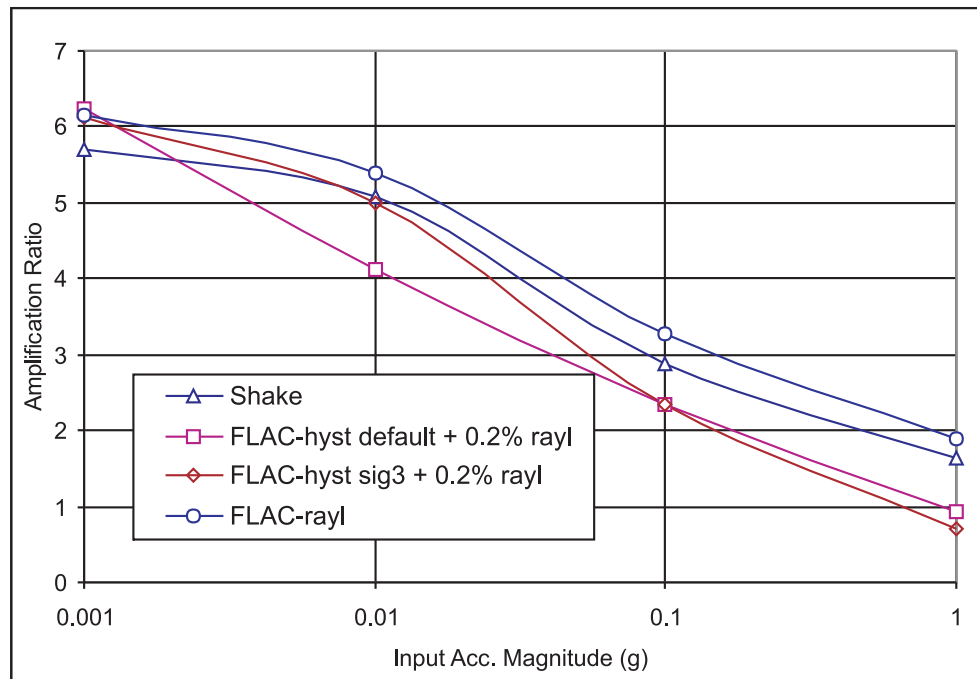


Figure 1.158 Acceleration amplification ratio versus base motion magnitude

FLAC model with hysteretic damping – A 1D column *FLAC* model composed of 30 5-feet (1.524 m) square elastic zones is assigned the soil properties listed in [Table 1.7](#). Model conditions are prescribed to simulate the free-field motion of a layered soil deposit with a rigid base. The *FLAC* data file is listed in [Example 1.38](#). The horizontal acceleration record is applied as an input boundary condition at the base of the model. *FLAC* takes the scaled base motion from *SHAKE-91*. For example, note that the history record “DIAM-FLAC-0001.ACC,” used in [Example 1.38](#), is scaled with a maximum value of 0.0001 g. Vertical movement is prevented in the model.

Hysteretic damping is added to this model by fitting a tangent-modulus function curve to the ($G/G_{\max}$) versus strain data, as shown in [Figures 1.151](#) and [1.153](#). Different fitting functions are available in *FLAC* (see [Section 1.4.3.5](#)); in this example, the default (two-parameter) and sig3 (three-parameter sigmoidal) functions are used. The parameters of the tangent-modulus functions are determined by best-fitting (e.g., least squares regression analysis) the laboratory modulus-reduction curves, assuming that the modulus reduction factor versus shear strain follows either the default or sig3 relations. The best-fit values for dynamic property set 1 – sand are listed in [Table 1.1](#) and for dynamic property set 2 – clay in [Table 1.2](#). A comparison between the tangent-modulus function curves and the laboratory curves is shown in [Figures 1.151](#) and [1.153](#).

Damping ratio curves that correspond to the tangent-modulus function curves are then calculated.* Both shear modulus reduction factor ($G/G_{\max}$) versus strain and damping ratio (λ) versus strain should be monitored, and parameters adjusted, if necessary, to provide a reasonable fit to both curves over a specified strain range. As shown in Figures 1.151 through 1.154, fairly reasonable fits for both modulus-reduction factor and damping-ratio curves are obtained with the default and sig3 hysteretic damping functions over a strain range of 0.0001% to 0.1% for both the sand and the clay.

The fitting curves of the *FLAC* default model (Figures 1.151 through 1.154) show that, for small strain (corresponding to small input acceleration), the behavior is approximately linear (i.e., both shear modulus and damping ratio are constants for both *FLAC* and *SHAKE-91*). Both codes are thus expected to give similar results in this circumstance. Here we compare accelerograms and response spectra at the top of the model for very low input acceleration. Figure 1.159 shows the horizontal acceleration at the top of the model (gridpoint 31 in *FLAC* and sub-layer 1 in *SHAKE-91*) as a function of time with maximum input acceleration amplitude of 0.0001 g. Both records are very similar; the maximum acceleration calculated by *FLAC* is 0.000592 g, while the maximum acceleration calculated by *SHAKE-91* is 0.000590 g (0.4% difference).

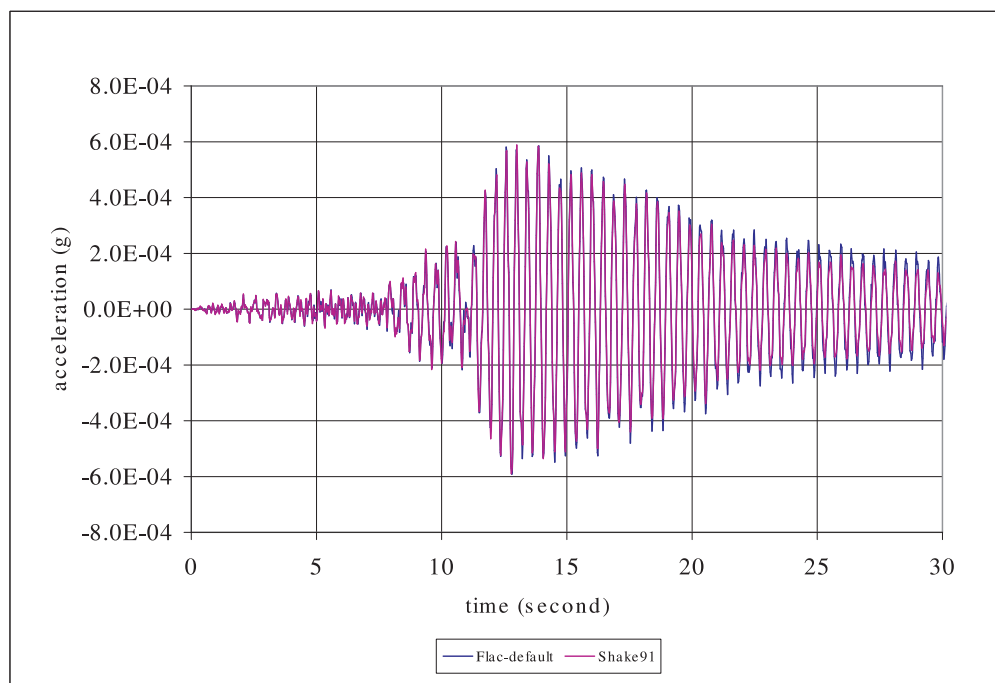


Figure 1.159 Accelerograms at the top of the model with small input

* The data file “MODRED.DAT,” provided in the “\Dynamic” directory, can be used to perform a series of simple shear tests on a single-zone *FLAC* model in order to develop damping ratio curves that correspond to the tangent-modulus functions.

Figures 1.160 through 1.163 provide pseudo-acceleration and pseudo-velocity spectra calibrated in *SHAKE-91* and *FLAC* when the maximum input acceleration amplitude is small (0.0001 g). In *SHAKE-91*, the response spectra are calculated using OPTION 9. In *FLAC*, response spectra are computed using a *FISH* function, “SPEC.FIS.” Here the damping ratio, minimum period and maximum period of interest are 5%, 0.01 and 10, respectively. From these plots it can be seen clearly that the *FLAC* and *SHAKE-91* results correspond quite closely for small-amplitude input motion.

Depth profiles of cyclic stress ratio (*CSR*) and acceleration amplification ratio are computed from the *FLAC* simulations for comparison to *SHAKE-91* results (see Figures 1.156 through 1.158). It was observed that at very low cyclic strain levels (corresponding to 0.001 g base acceleration), hysteretic damping provides almost no energy dissipation, so a small amount of stiffness-proportional Rayleigh damping (0.2%) was included to avoid low-level oscillations. The dynamic timestep for these *FLAC* runs is 5.0×10^{-4} sec.

FLAC model with Rayleigh damping – The 1D column model is also run using Rayleigh damping in place of hysteretic damping. In this example, one set of average Rayleigh damping coefficients is assigned for each input acceleration. The parameters for the Rayleigh damping model are selected to be compatible with the maximum strain levels calculated from the *SHAKE-91* simulation. Maximum shear strains at every layer in the *SHAKE-91* model are averaged over the entire model, and an equivalent uniform strain (the equivalent uniform strain is taken as 50% of the maximum strain) is then used to find appropriate shear modulus-reduction factors and damping ratios for the Rayleigh damping runs, calculated as average values from the clay and sand curves shown in Figures 1.151 through 1.154. The values are summarized in Table 1.8. The table also lists the resulting dynamic timestep for the selected Rayleigh damping parameters. The center frequency for Rayleigh damping is set to the dominant frequency of the input record, 2.47 Hz.

Table 1.8 Parameters for Rayleigh damping runs

Maximum Acceleration Amplitude (g)	Equivalent Shear Strain (%)	Modulus Reduction Factor	Damping Ratio (%)	Timestep (sec)
0.001	0.00027	1.00	0.40	4.97E-04
0.010	0.0024	0.97	1.22	3.58E-04
0.100	0.019	0.76	3.84	1.83E-04
1.000	0.3	0.30	15.00	6.50E-04

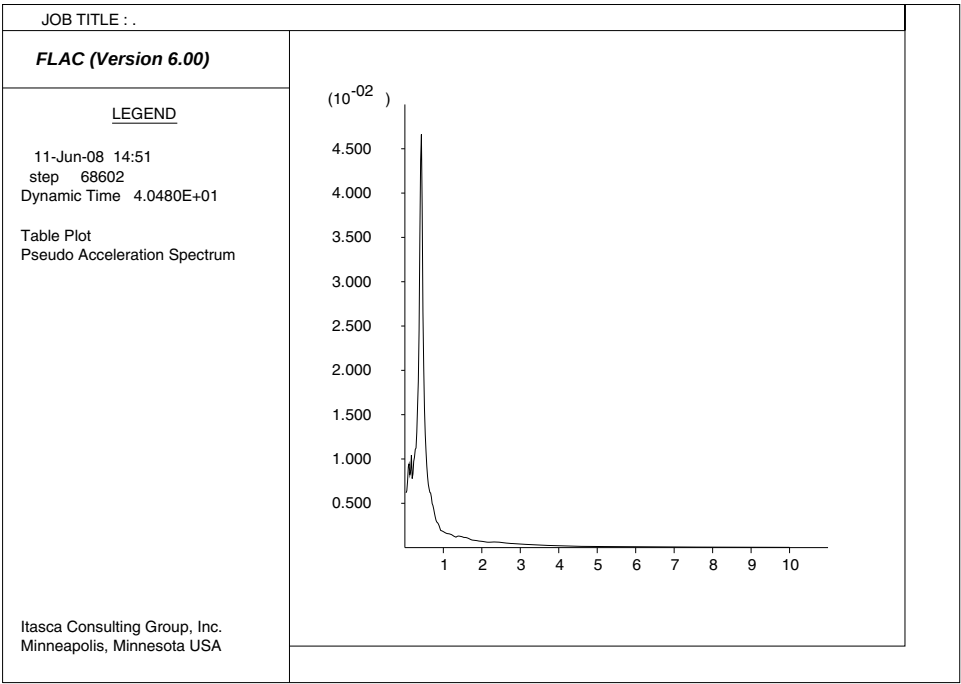


Figure 1.160 *Pseudo-acceleration spectrum at the top of the model (FLAC de-fault) – m/s²*

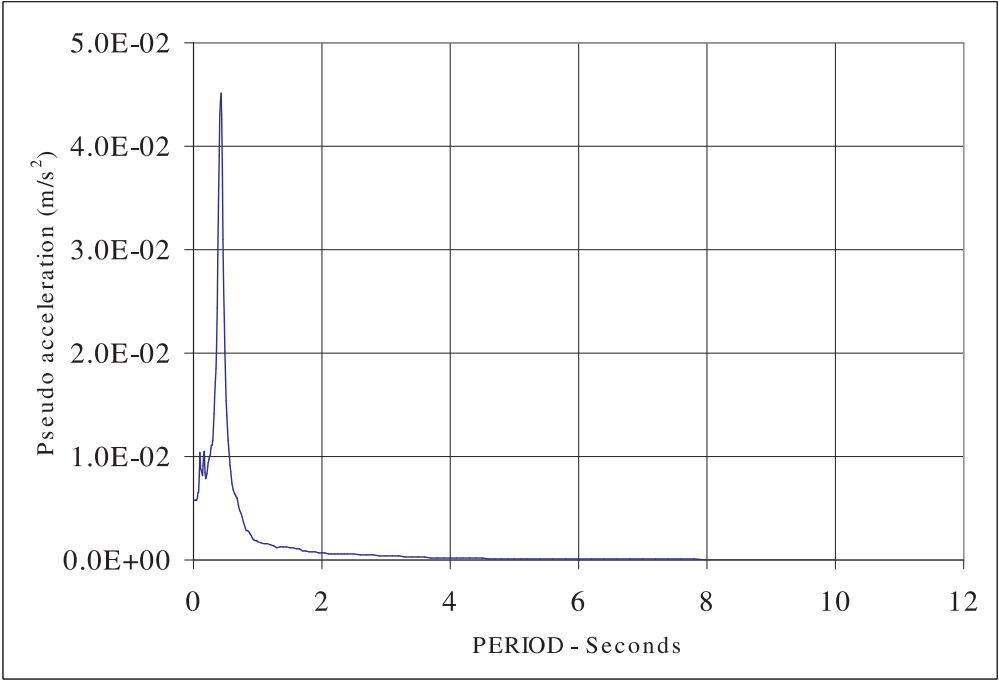


Figure 1.161 *Pseudo-acceleration spectrum at the top of the model (SHAKE-91) – m/s²*

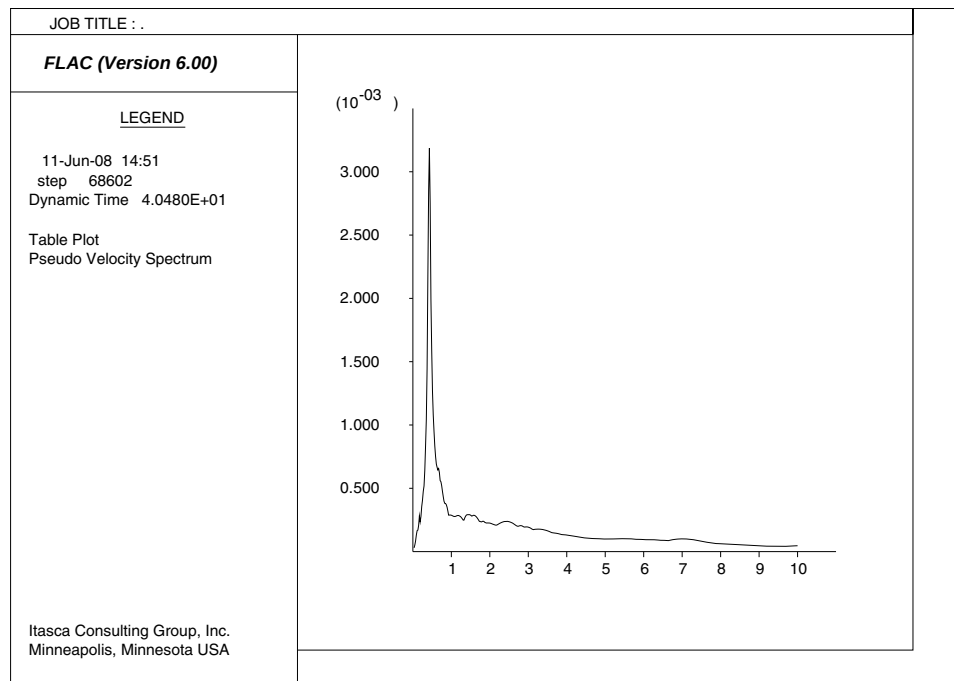


Figure 1.162 Pseudo-velocity spectrum at the top of the model (FLAC default)
– m/s

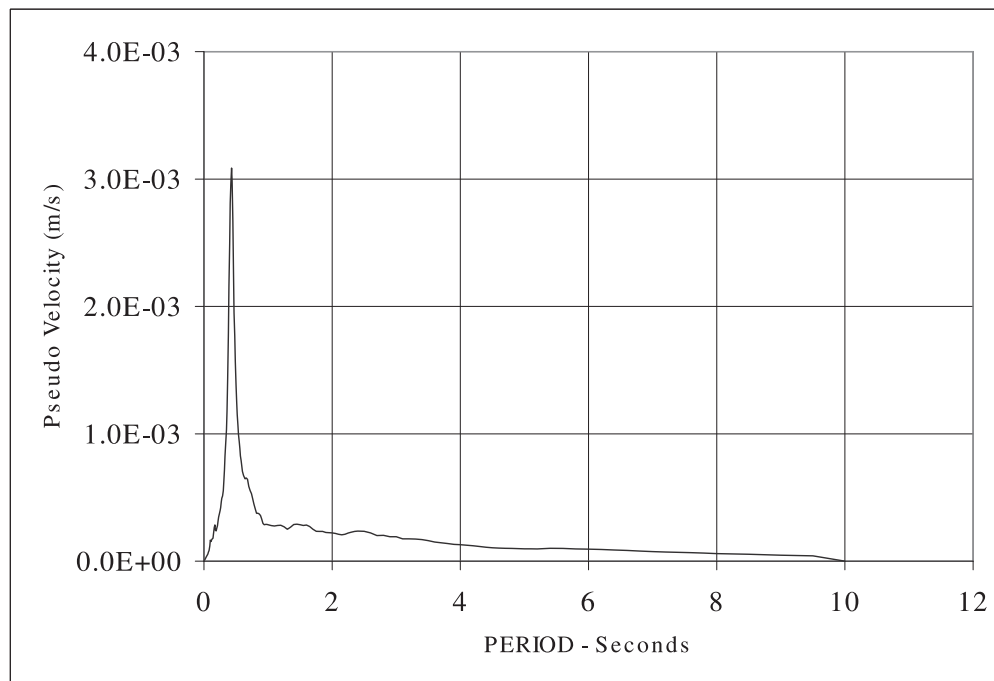


Figure 1.163 Pseudo-velocity spectrum (SHAKE-91) – m/s

Depth profiles of *CSR* and acceleration amplification are also created for the Rayleigh damping runs and are plotted in [Figures 1.156 through 1.158](#) for comparison. The *CSR* profiles and the acceleration amplifications are similar for all simulations. The results indicate that at low input acceleration magnitude (0.001 g) the amplification is slightly lower for the *SHAKE* runs than the *FLAC* runs with either hysteretic damping or Rayleigh damping. This corresponds to maximum strains of less than 0.0003%. At higher acceleration magnitudes, *SHAKE* predicts somewhat higher amplification than *FLAC* with hysteretic damping, but lower amplification than *FLAC* with Rayleigh damping. This suggests that at higher amplitudes, hysteretic damping absorbs somewhat more energy during wave transmission than both Rayleigh damping and the equivalent linear method.

The highest equivalent strain level in these models is approximately 0.3% (at 1.0 g). For this layered system, cyclic strain levels will be different in different locations at different times. Using hysteretic damping, these different strain levels produce realistically different damping levels in time and space. Constant and uniform Rayleigh damping parameters can only produce an average response. It is possible to adjust the Rayleigh damping parameters throughout the model to account for spatial variations, using an iterative, strain-compatible scheme, as in the equivalent linear method, in order to produce more realistic damping levels. The 1D wave propagation simulation was repeated for selected cases, with spatial variation in Rayleigh damping corresponding to the damping levels derived from *SHAKE*. These results were essentially the same as those from the Rayleigh damping simulations using average values.

The *FLAC* simulations with Rayleigh damping are slower than those with hysteretic damping because the stiffness-proportional component of Rayleigh damping causes a reduction in the critical timestep in an explicit solution scheme. The speed of the hysteretic damping runs range from 9% faster (at 0.001 g) to 7 times faster (at 1.0 g) than the Rayleigh damping runs.

Example 1.37 SHAKE-91 model of layered nonlinear soil deposits

```

option 1 -- dynamic soil properties -- (max is thirteen):
  1
  3
  11      #1 modulus for clay (Seed \& Sun 1989) upper range
0.0001    0.0003    0.001    0.003    0.01    0.03    0.1    0.3
1.        3.        10.
1.000    1.000    1.000    0.981    0.941    0.847    0.656    0.438
0.238    0.144    0.110
  11      damping for clay (Idriss 1990) --
0.0001    0.0003    0.001    0.003    0.01    0.03    0.1    0.3
1.        3.16     10.
21.      25.      28.
  11      #2 modulus for sand (Seed \& Idriss 1970) -- upper range
0.0001    0.0003    0.001    0.003    0.01    0.03    0.1    0.3
1.        3.        10.
1.000    1.000    0.990    0.960    0.850    0.640    0.370    0.180
0.080    0.050    0.035
  11      damping for sand (Idriss 1990) -- (about LRng from SI 1970)
0.0001    0.0003    0.001    0.003    0.01    0.03    0.1    0.3
1.        3.        10.
0.24     0.42     0.8     1.4     2.8     5.1     9.8     15.5
21.      25.      28.
  8      #3 ATTENUATION OF ROCK AVERAGE
.0001     0.0003    0.001    0.003    0.01    0.03    0.1    1.0
1.000     1.000    0.9875   0.9525   0.900    0.810    0.725    0.550
  5      DAMPING IN ROCK
.0001     0.001     0.01     0.1     1.
0.4       0.8       1.5       3.0       4.6
  3  1    2    3
Option 2 -- Soil Profile
  2
  1  17      Example -- 150-ft layer; input:Diam @ .1g
  1  2          5.00          .050          .125          1000.
  2  2          5.00          .050          .125          900.
  3  2          10.00         .050          .125          900.
  4  2          10.00         .050          .125          950.
  5  1          10.00         .050          .125          1000.
  6  1          10.00         .050          .125          1000.
  7  1          10.00         .050          .125          1100.
  8  1          10.00         .050          .125          1100.
  9  2          10.00         .050          .130          1300.
 10  2          10.00         .050          .130          1300.
 11  2          10.00         .050          .130          1400.

```

```

12      2      10.00      .050      .130      1400.
13      2      10.00      .050      .130      1500.
14      2      10.00      .050      .130      1500.
15      2      10.00      .050      .130      1600.
16      2      10.00      .050      .130      1800.
17      3      .010      .140      100000.
Option 3 -- input motion:
3
1900 4096 .02      diam.acc      (8f10.6)
      .0001      25.      3      8
Option 4 -- sublayer for input motion within (1) or outcropping (0):
4
17      0
Option 5 -- number of iterations \& ratio of avg strain to max strain
5
0      8      0.50
Option 6 -- sublayers for which accn time histories are computed \& saved:
6
1      2      3      4      5      6      7      8      9      10      11      12      13      14      15
0      1      1      1      1      1      1      1      1      1      1      1      1      1      1
1      0      0      0      0      1      0      0      0      1      0      0      0      1      0
Option 6 -- sublayers for which accn time histories are computed \& saved:
6
16      17      17
1      1      0
0      1      0
Option 7 -- sublayer at which shear stress or strain are computed \& saved:
7
4      1      1      0 1800      -- stress in level 4
4      0      1      0 1800      -- strain in level 4
Option 7 -- sublayer at which shear stress or strain are computed \& saved:
7
8      1      1      0 1800      -- stress in level 8
8      0      1      0 1800      -- strain in level 8
Option 9 -- compute & save response spectrum:
9
1      0
1      0      981.0
0.05
option 10 -- compute & save amplification spectrum:
10
17      0      1      0      0.125      -- surface/rock outcrop
execution will stop when program encounters 0
0

```

Example 1.38 FLAC model of layered nonlinear soil deposits

```

config dynamic
;-----
;Grid generation and model properties
;-----
grid 1,30
model elastic
prop bulk 300e6 she 186e6 den 2000 ; 1-5 ft
prop bulk 200e6 she 150e6 den 2000 j 27 29 ;5-20 ft
prop bulk 200e6 she 168e6 den 2000 j 25 26 ;20-30 ft
prop bulk 270e6 she 186e6 den 2000 j 21 24 ;30-50 ft
prop bulk 350e6 she 225e6 den 2000 j 17 20 ;50-70 ft
prop bulk 480e6 she 327e6 den 2082 j 13 16 ;70-90 ft
prop bulk 550e6 she 379e6 den 2082 j 9 12 ;90-110 ft
prop bulk 600e6 she 435e6 den 2082 j 5 8 ;110-130 ft
prop bulk 750e6 she 495e6 den 2082 j 3 4 ;130-140 ft
prop bulk 900e6 she 627e6 den 2082 j 1 2 ;140-150 ft
;mul a factor to have depth of 150 ft, in order to compare with Shake
ini x mul 1.524
ini y mul 1.524
;-----
; Histories
;-----
hist 1 unbal
his 2 dytime
his 231 xacc i 1 j 31 ;top accn 0'
his 224 xacc i 1 j 23 ;accn at 40'
his 201 xacc i=1 j=1 ;btm accn at 150'
;-----
;Boundary Conditions
;-----
fix y
;-----
;Application of acceleration
;-----
his read 100 Diam-flac-0001.acc
apply xacc 9.81 his 100 j=1 ; convert to actual accn value
apply yacc 0.0 j=1 ; this command prevents rocking along gridpoint j=1
ini dy_damp hyst default -3.325 0.823 j 1 16
ini dy_damp hyst default -3.156 1.904 j 17 24
ini dy_damp hyst default -3.325 0.823 j 25 30
set dynamic on
set dydt 0.002
;hist nstep 100
solve dytime 40.48

```

```
his write 231 vs 2 table 231 ; top accn hist to table, to gen. res spectra  
call spec.fis
```

```
def compuspec  
  ; setup values for fish function spectra  
  dmp=0.05 ; damping ratio  
  pmin=0.01 ; minimum period  
  pmax=10.0 ; maximum period  
  acc_in=231 ; input acc table  
  sd_out = 501 ; relative displacement table  
  sv_out = 502 ; pseudo-velocity table  
  sa_out = 503 ; pseudo-acc table  
  n_point = 500 ; # of computation points  
  spectra  
end  
compuspec  
set hisfile inp-flac-0001.his  
his write 231, 224, 201 vs 2 ;accn  
save inp-flac-0001.sav
```

1.7.4 Slip Induced by Harmonic Shear Wave

This problem concerns the effects of a planar discontinuity on the propagation of an incident shear wave. Two homogeneous, isotropic, semi-infinite elastic media, separated by a planar discontinuity with a limited shear strength, are shown in Figure 1.164. A normally incident, plane harmonic, shear wave will cause slip at the discontinuity, resulting in frictional energy dissipation. Thus, the energy will be reflected, transmitted and absorbed at the discontinuity. The problem is modeled with *FLAC*, and the results are used to determine the coefficients of transmission, reflection and absorption. These coefficients were compared with ones given by an analytical solution (Miller 1978).

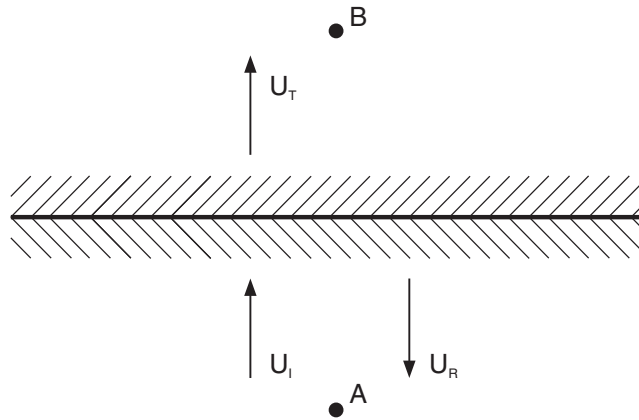


Figure 1.164 Transmission and reflection of incident harmonic wave at a discontinuity

The coefficients of reflection (R), transmission (T) and absorption (A) given by Miller (1978) are

$$R = \sqrt{\frac{E_R}{E_I}} \quad (1.127)$$

$$T = \sqrt{\frac{E_T}{E_I}} \quad (1.128)$$

$$A = \sqrt{1 - R^2 - T^2} \quad (1.129)$$

where E_I , E_T and E_R represent the energy flux per unit area per cycle of oscillation associated with the incident, transmitted and reflected waves, respectively. The coefficient A is a measure of the energy absorbed at the discontinuity. The energy flux, E_I , is given by

$$E_I = \int_{t_1}^{t_1+T} \sigma_s v_s dt \quad (1.130)$$

where: $T = (2\pi)/\omega$ = the period for the incident wave;

σ_s = shear stress;

v_s = particle velocity in the x -direction; and

ω = frequency of incident wave (radian/sec).

For elastic media,

$$\sigma_s = \rho c v_s \quad (1.131)$$

Hence,

$$E_I = \rho c \int_{t_1}^{t_1+T} v_s^2 dt \quad (1.132)$$

in which c is the velocity of the propagating shear wave.

The energy flux of the incident wave E_I is evaluated at point A (see [Figure 1.164](#)) for no slip at the discontinuity. The energy flux of the transmitted wave E_T is evaluated at point B for the case of slip at the discontinuity. The energy flux of the reflected wave E_R is calculated by determining the difference of velocities in two cases: slip and no slip.

[Figure 1.165](#) shows the numerical model, which consists of a 4×31 grid and an interface, EF, which has high stiffness and is used to simulate the discontinuity. The conditions used are as follows:

Boundary Conditions

- Non-reflecting viscous boundaries are located at GH and CD.
- Vertical motion is prevented along lateral boundaries GC and DH.

Loading Conditions

- Shear stresses corresponding to the incident wave are applied along CD.
- The maximum stress of the incident wave is 1 MPa and the frequency is 1 Hz.

Material Conditions

- elastic media

$$\rho = 2.65 \times 10^3 \text{ kg/m}^3$$

$$K = 16,667 \text{ MPa}$$

$$G = 10,000 \text{ MPa}$$

- interface

$$K_n = K_s = 10,000 \text{ MPa/m}$$

$$C = \text{cohesion} = 2.5 \text{ MPa for no-slip} = 0.5, 0.1, 0.02 \text{ MPa for slip case}$$

Note that the magnitude of the incident wave must be doubled in the numerical model to account for the simultaneous presence of the non-reflecting boundary (see [Section 1.4.1.1](#)).

[Example 1.39](#) provides a data file that makes four complete simulations of the problem: the first simulation is for a fully elastic case; and the remaining simulations correspond to the various values of cohesion. Computed values for R , T and A are written to the log file “FLAC.LOG” if the model is run in command-driven mode. If run in *GHIC* mode, the computed values are displayed in the *Console* resource pane.

Example 1.39 Verification of dynamic slip – four complete simulations

```

config dyna
g 4 31
model elastic
set ncw 100
; null zones for the interface (joint)
model null j=16
; grid generation
gen -200 -200 -200 0 -120 0 -120 -200 j=1,16
gen -200 0 -200 200 -120 200 -120 0 j=17,32
; interface
int 1 aside from 1 16 to 5 16 bside from 1 17 to 5 17
def setup
    mat_shear = 10000.0
    mat_dens = 0.00265
    freq = 1.0
    tload = 10.0
    w = 2.0 * pi * freq
end
def fsin
    if dytime <= tload
        fsin = sin(w*dytime)
    else
        fsin = 0.0
    end_if

```

```

end
def common
  command
    hist reset
    set dytime = 0.0
    ini xvel = 0.0 yvel = 0.0 xdis = 0.0 ydis = 0.0
    ini sxx = 0.0 syy = 0.0 szz = 0.0 sxy = 0.0
    apply remove i 1,5 j 1,16
    apply remove i 1,5 j 17,32
    apply xquiet yquiet j=1
    apply xquiet yquiet j=32
    apply sxy 2 his fsin j=1
    his nstep 10
    his unbal
    his dytime
    his xvel i=2 j=1
    his xvel i=2 j=32
    his xdisp i=2 j=1
    his xdisp i=2 j=32
    his sxy i=1 j=1
    his sxy i=1 j=31
    his fsin
    solve dytime 5
  end_command
end
def energy ; - compute energy coefficients for slipping-joint example -
;
; table 1 - x-velocity at point A for elastic joint case
; table 2 - x-velocity at point A for slipping joint case
; table 3 - x-velocity at point B for slipping joint case
; Ei - energy flux for incident wave
; Et - energy flux for transmitted wave
; Er - energy flux for reflected wave
; AAA - a measure of energy absorbed at the interface
; items - no. of items in tables
;
Cs      = sqrt(mat_shear / mat_dens)
factor = mat_dens * Cs
Ei      = 0.0
Et      = 0.0
Er      = 0.0
t_n_1   = 0.0
nac     = 0
AAA     = 0.0
TTT     = 0.0
RRR     = 0.0

```

```

loop i (1,items)
  t_n = xtable(1,i)
  d_t = t_n - t_n_1
  t_n_1 = t_n
  Vsa_e = ytable(1,i)
  Vsa_s = ytable(2,i)
  Vsb_s = ytable(3,i)
  Ei = Ei + factor * Vsa_e * Vsa_e * d_t
  Et = Et + factor * Vsb_s * Vsb_s * d_t
  Er = Er + factor * (Vsa_s-Vsa_e) * (Vsa_s-Vsa_e) * d_t
  if i > i_mean
    nac = nac + 1
    RRR = RRR + sqrt(Er/Ei)
    TTT = TTT + sqrt(Et/Ei)
  endif
endloop
RRR = RRR / float(nac)
TTT = TTT / float(nac)
AAA = AAA + sqrt(1.0-RRR*RRR-TTT*TTT)
command
  set log on
end_command
ii = out(' R = '+string(RRR))
ii = out(' T = '+string(TTT))
ii = out(' A = '+string(AAA))
command
  set log off
end_command
end
setup
prop den=0.00265 bulk=16667 shear=10000
int 1 kn=10000 ks=10000 coh=2.5 fric=0.0
fix y
set clock 1000000 step 10000000
set dynamic on
common
his write 3 vs 2 tab 1 ; save elas incident wave
save dinte.sav
int 1 coh 0.5
common
his write 3 vs 2 tab 2
his write 4 vs 2 tab 3
set items 701 i_mean=600
energy
save dintp5.sav
int 1 coh 0.1

```

```

common
his write 3 vs 2 tab 2
his write 4 vs 2 tab 3
energy
save dintp1.sav
int 1 coh 0.02
common
his write 3 vs 2 tab 2
his write 4 vs 2 tab 3
energy
save dintp02.sav

```

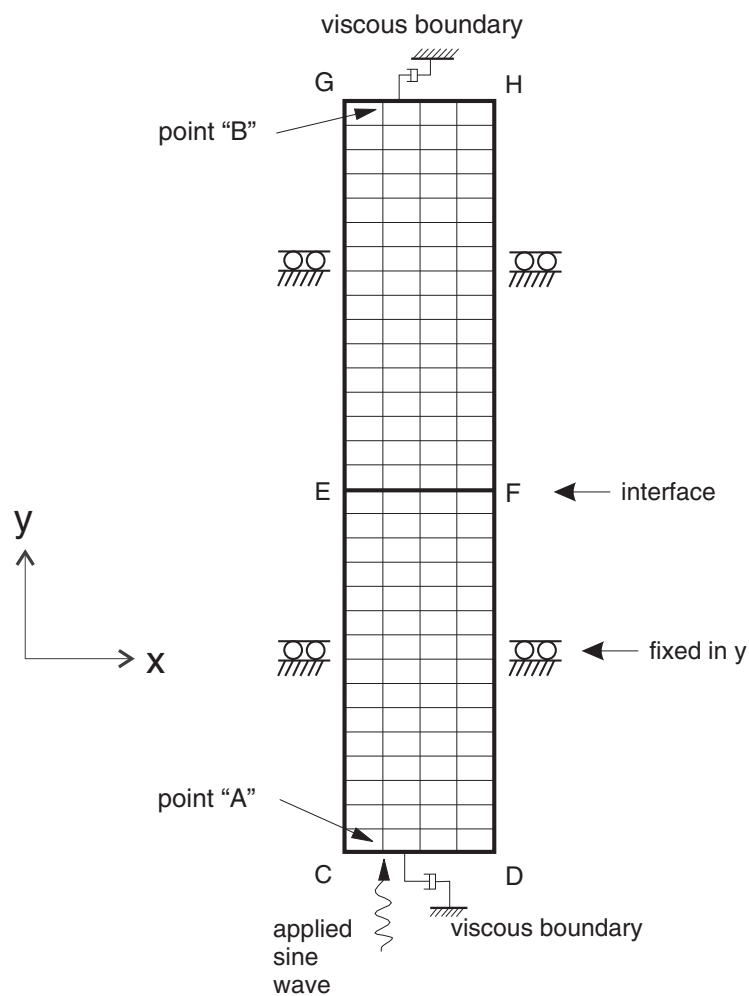


Figure 1.165 Problem geometry and boundary conditions for the problem of slip induced by harmonic shear wave

The initial assumption of an elastic discontinuity is achieved by assigning a high cohesion (2.5 MPa, in this case) to the interface. Figure 1.166 shows the time variation of shear stress near points A and B. From the amplitude of the stress history at A and B, it is clear that there was perfect transmission of the wave across the discontinuity. It is also clear from Figure 1.166 that the viscous boundary condition provides perfect absorption of normally incident waves. Following the execution of the elastic case, the velocity history at point A is saved in table 1, to be used later for calculating E_I , used in the equations for energy coefficients.

The cohesion of the discontinuity is then set, successively, to 0.5, 0.1 and 0.02 MPa to permit slip to occur. The recorded shear stresses at points A and B for the three cases are shown in Figures 1.167, 1.168 and 1.169, respectively. The peak stress at point A is the superposition of the incident wave and the wave reflected from the slipping discontinuity. It can be seen in Figures 1.167 through 1.169 that the shear stress of point B is limited by the discontinuity strength.

After each inelastic simulation, the velocity histories at points A and B are saved in tables 2 and 3, and the energy flux and coefficients R , T and A are computed by the *FISH* function **energy** and written to the log file. All conditions are then reset to zero and requested histories are deleted, in preparation for the next simulation; this is done in function **common**. It was determined that at least five cycles of the input wave were necessary before the computed coefficients settled down to steady-state values. Even then, there is a periodic fluctuation in the values. In order to obtain mean values, the coefficient values were averaged over the final 100 timesteps: the *FISH* variable **i_mean** controls the step number at which this averaging process starts. Figure 1.170 compares the numerical results with the exact solution for the coefficients for three values of the dimensionless parameter

$$\frac{\omega \gamma U}{\tau_s}$$

where: τ_s = discontinuity cohesion;

U = displacement amplitude of the incident wave;

$\gamma = \sqrt{\rho G}$; and

ω = frequency of incident wave (1 Hz).

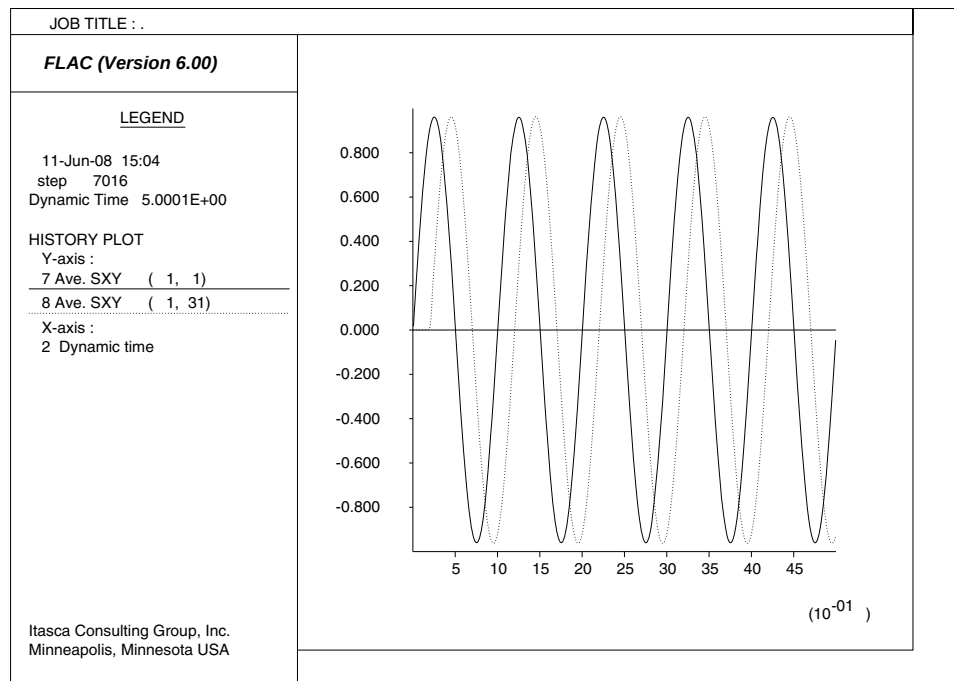


Figure 1.166 Time variation of shear stress at points A and B for elastic discontinuity (cohesion = 2.5 MPa)

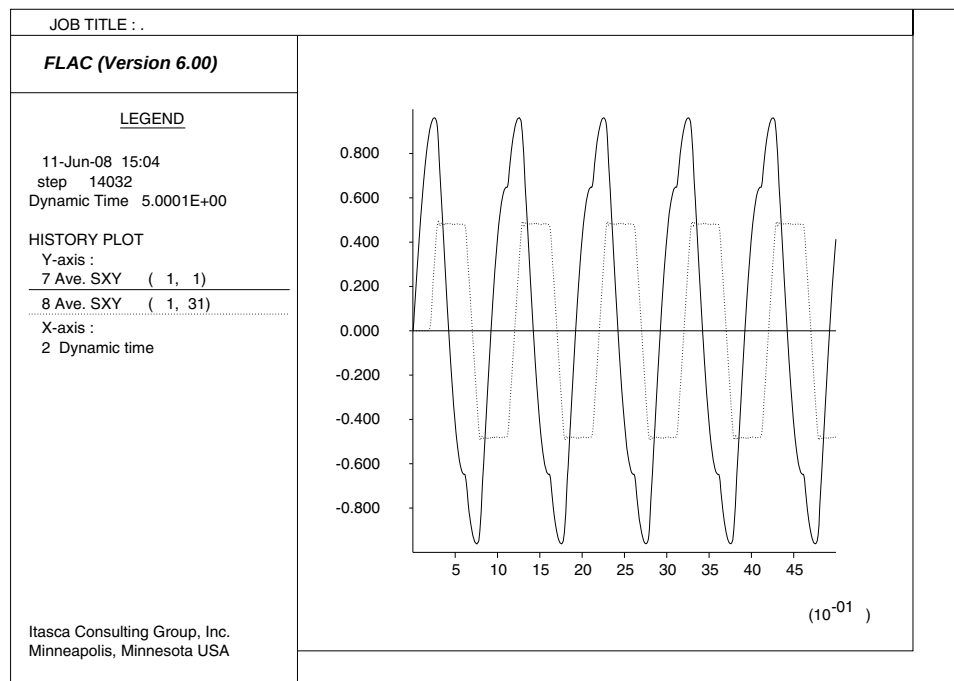


Figure 1.167 Time variation of shear stress at points A and B for slipping discontinuity (cohesion = 0.5 MPa)

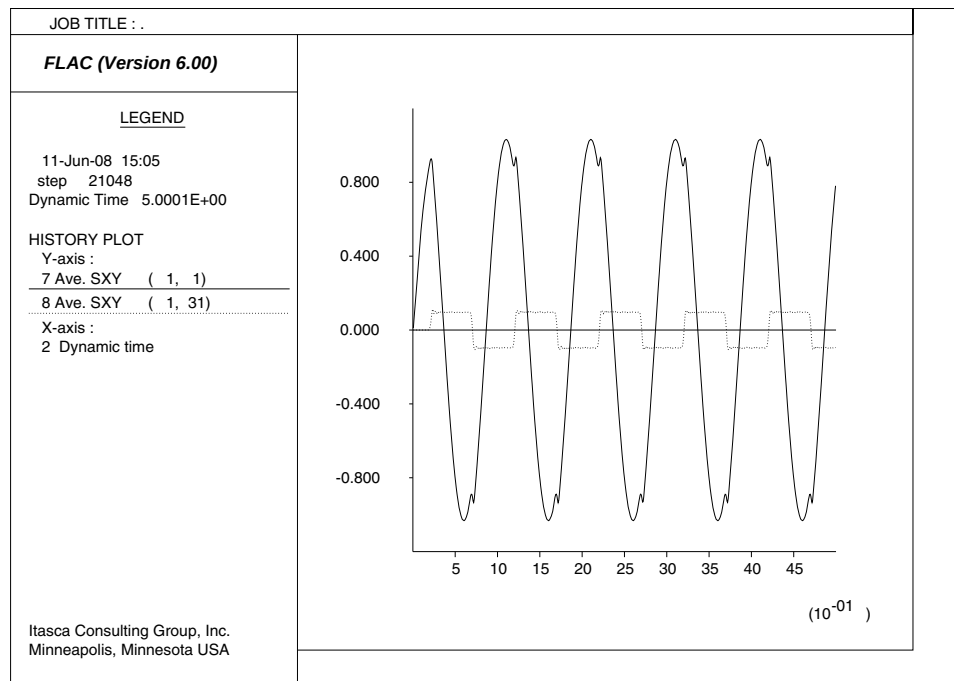


Figure 1.168 Time variation of shear stress at points A and B for slipping discontinuity (cohesion = 0.1 MPa)

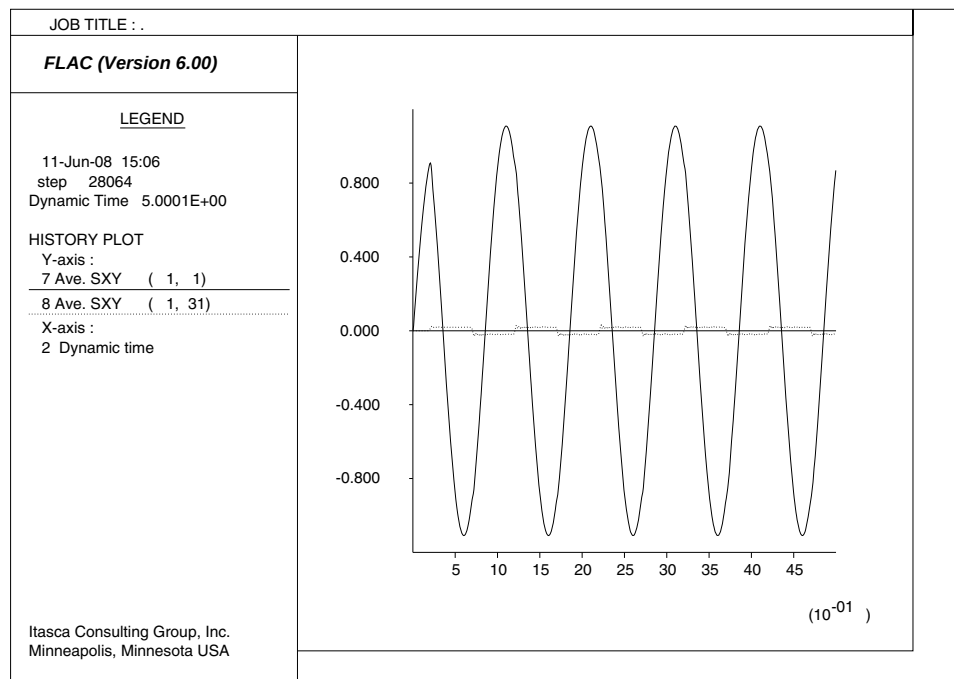


Figure 1.169 Time variation of shear stress at points A and B for slipping discontinuity (cohesion = 0.02 MPa)

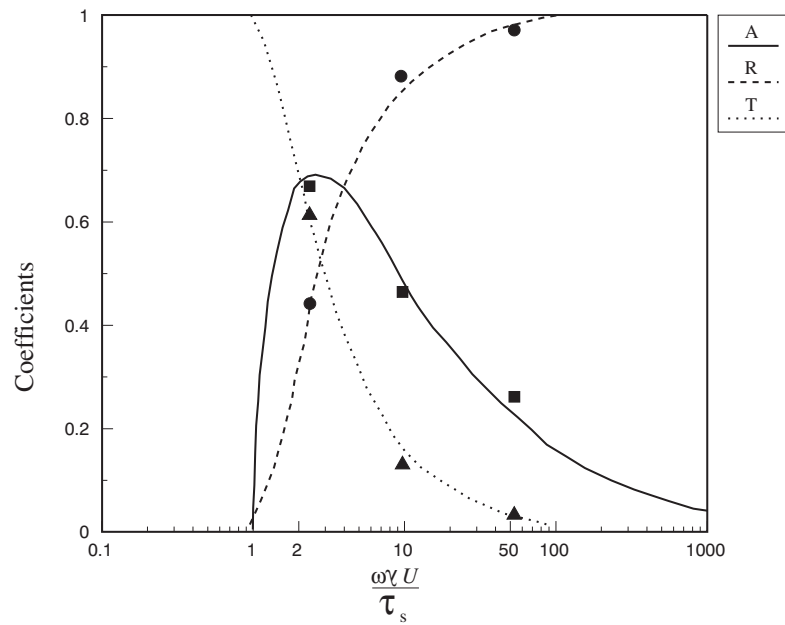


Figure 1.170 Comparison of transmission, reflection and absorption coefficients (analytical solution from Miller 1978)

The displacement amplitude for the incident wave (U) was obtained by monitoring the horizontal displacement at point A for non-slipping discontinuities. As can be seen, the *FLAC* results agree well with the analytical solution.

1.7.5 Hollow Sphere Subject to an Internal Blast

This problem concerns the propagation of a spherical wave due to an impulsive pressure (explosion) in a sphere. In unbounded (i.e., infinite) media, two types of waves can exist: compression and shear waves. In this problem, the axisymmetric nature of the problem eliminates the shear wave. Therefore, only the solution for the compression wave needs to be sought. The problem provides a test of the dynamic capabilities of *FLAC*, and is applicable to impact and explosion modeling.

The analytical solution (assuming that the material is elastic and isotropic) for this problem was derived by Blake (1952). The solution is based on the following governing equation:

$$\frac{\partial^2 \phi}{\partial t^2} = C_p^2 \nabla^2 \phi \quad (1.133)$$

where: C_p = compressional wave velocity;

t = time;

ϕ = a potential function; and

∇^2 = Laplacian operator.

Let $p(t)$ be an impulse which jumps from zero to p_0 at $t = 0$, and then decays exponentially with time constant α^{-1} . Thus, the pressure function can be defined by

$$\begin{aligned} p(t) &= p_0 e^{-\alpha t} & \text{for } t \geq 0 \\ p(t) &= 0 & \text{for } t < 0 \end{aligned} \quad (1.134)$$

A step function of the pressure ($\alpha = 0$) will be used for this problem. For such a pressure function, the potential function which satisfies the governing equation is

$$\phi_{\alpha=0} = \frac{p_0 a^3 K}{\rho C_p^2 r} \left[-1 + \sqrt{\frac{4K}{4K-1}} \exp(-\alpha_0 \tau) \cos\left(\omega_0 \tau - \tan^{-1} \frac{1}{\sqrt{4K-1}} \right) \right] \quad (1.135)$$

where: a = radius of the sphere;

$K = \frac{1-\nu}{2(1-2\nu)}$;

ν = Poisson's ratio;

r = radial coordinate;

$\alpha_0 = \frac{C_p}{2aK}$ = radiation damping constant;

$\tau = t - \frac{r-a}{C_p}$; and

$\omega_0 = \frac{c}{2aK} \sqrt{4K-1}$ = natural frequency.

The radial displacement can be found by differentiating the potential function with respect to radial distance:

$$\begin{aligned}
 u_r = \frac{\partial \phi}{\partial r} = & -\frac{p_0 a^3 K}{\rho C_p^2 r^2} \left[-1 + \sqrt{2-2\nu} \exp(-\alpha_0 \tau) \cos \left(\omega_0 \tau - \tan^{-1} \frac{1}{\sqrt{4K-1}} \right) \right] \\
 & + \frac{p_0 a^3 K}{\rho C_p^2 r} \left[\frac{\alpha_0}{C_p} \sqrt{2-2\nu} \exp(-\alpha_0 \tau) \cos \left(\omega_0 \tau - \tan^{-1} \frac{1}{\sqrt{4K-1}} \right) \right. \\
 & \left. + \frac{\omega_0}{C_p} \sqrt{2-2\nu} \exp(-\alpha_0 \tau) \sin \left(\omega_0 \tau - \tan^{-1} \frac{1}{\sqrt{4K-1}} \right) \right] \quad (1.136)
 \end{aligned}$$

A sphere embedded in an infinite, isotropic medium can be simulated by an axisymmetric condition. [Figures 1.171](#) and [1.172](#) show two different grids used for the simulation. One has a circular boundary; the other has a rectangular boundary. The *FISH* functions “HDONUT.FIS” and “HHOLE.FIS” are called to generate appropriate boundaries. The radius of the sphere is assumed to be 10 m, and the outer boundary is located at a distance ten times the radius.

Horizontal movement is prevented at the axis of symmetry. A viscous boundary condition is imposed on the outer boundary to absorb the wave. The material properties used for the problem are

$$\begin{aligned}
 \text{shear modulus } (G) & 1 \times 10^{10} \text{ Pa} \\
 \text{bulk modulus } (K) & 1.665 \times 10^{10} \text{ Pa} \\
 \text{density } (\rho) & 1675 \text{ kg/m}^3
 \end{aligned}$$

A pressure equal to 1000 Pa is applied at the inner boundary to simulate the blasting.

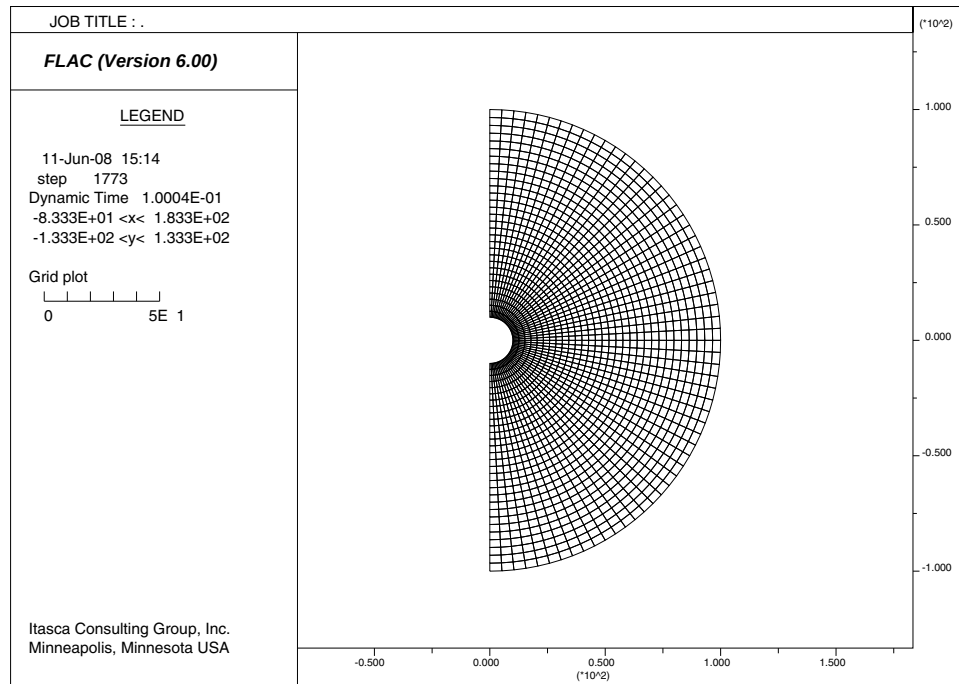


Figure 1.171 Grid with circular boundary

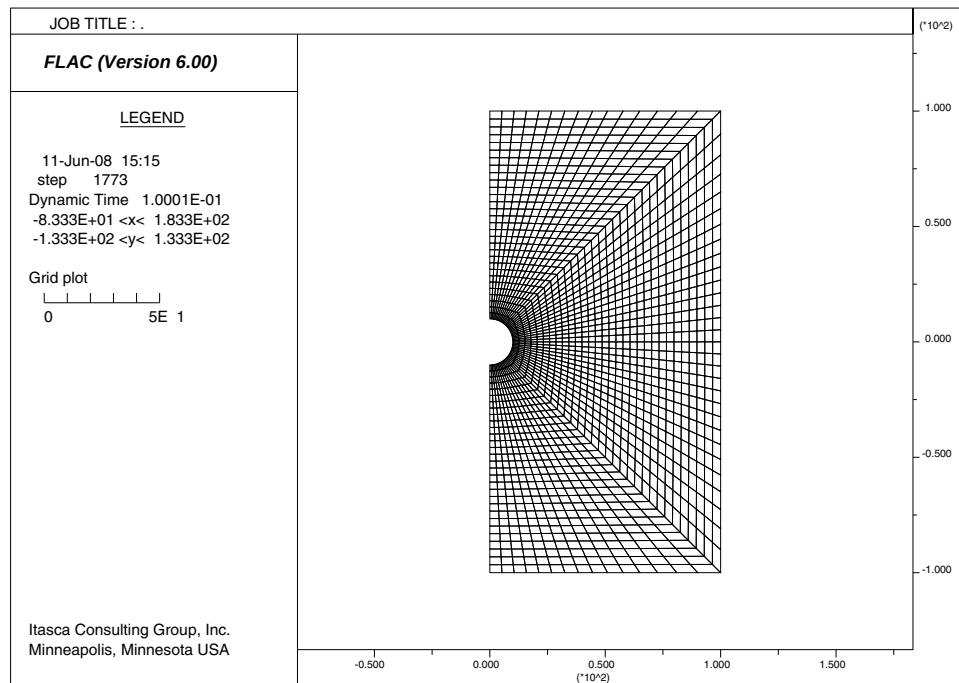


Figure 1.172 Grid with rectangular boundary

The radial displacement histories recorded up to 0.1 second at $r = 2.051a$, $3.424a$ and $4.867a$ are given in [Figures 1.173](#) and [1.174](#) for circular and rectangular outer boundaries, respectively. The delay of the response at locations far from the sphere can be noted in both cases. In both cases, *FLAC* is able to capture the response at peak and steady states. The fluctuation at late time is due to the fact that the radiated wave is not absorbed completely by the viscous boundary.

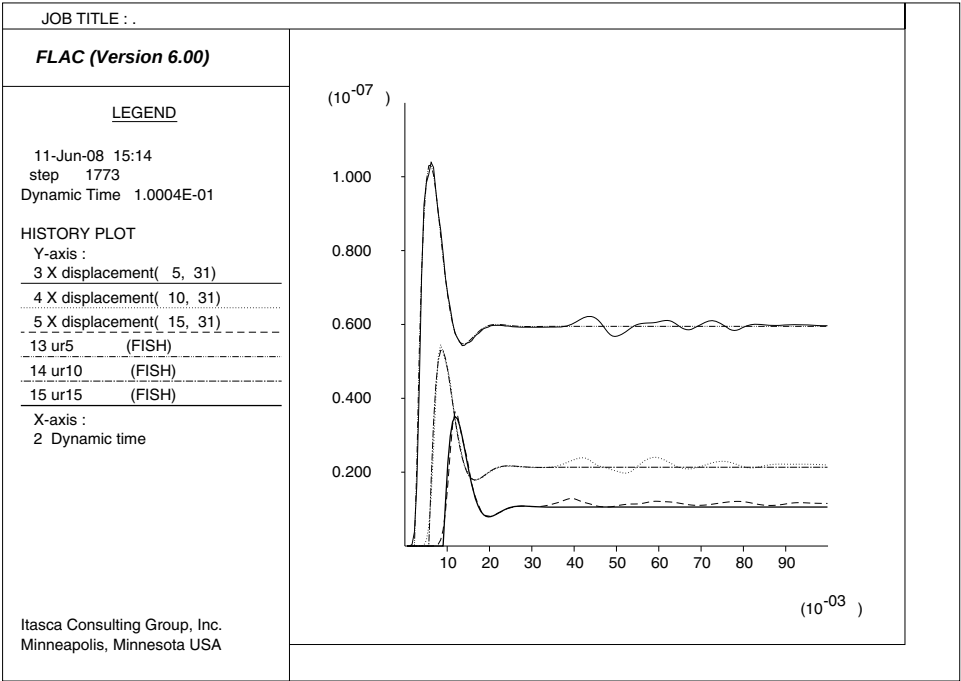


Figure 1.173 *Radial displacement histories at $r = 2.051a$, $3.424a$ and $4.867a$ (circular boundary)*

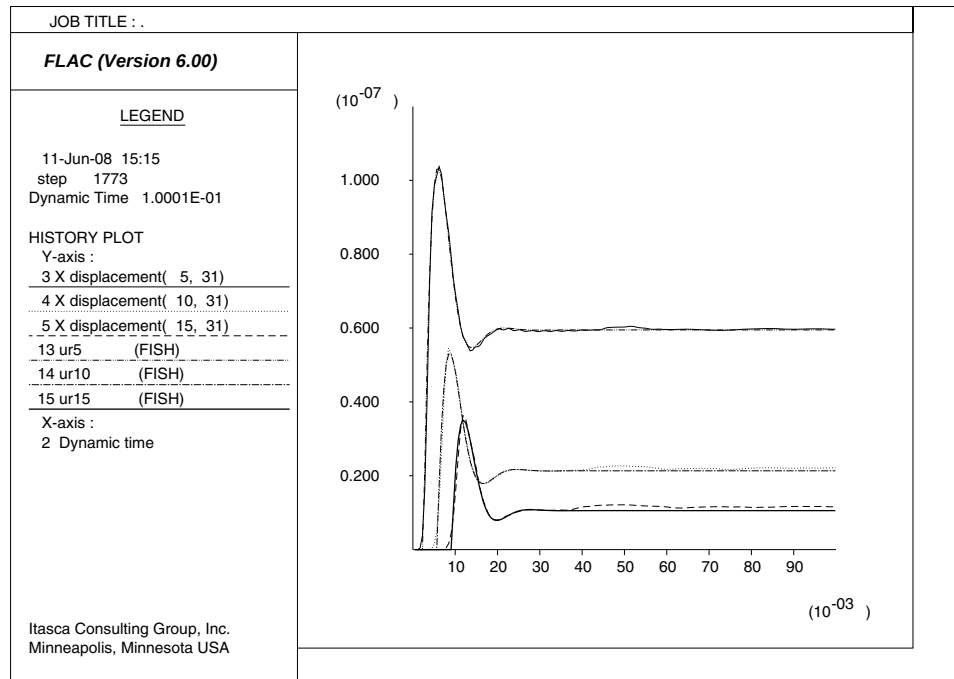


Figure 1.174 Radial displacement histories at $r = 2.051a$, $3.424a$ and $4.867a$ (rectangular boundary)

Example 1.40 Hollow sphere subject to internal blasting

```

config dyn axi
grid 30 60
model elastic
; -----
; Poisson's ratio 0.25
; -----
prop shear=1e10 bulk=1.665e10 dens=1675
; create cavity
; -----
; circular boundary
; -----
call hdonut.fis
; -----
; rectangular boundary
; -----
; call hhole.fis
set rmin 10.0 rmul 10 gratio 1.01
hdonut
; hhole
; -----

```

```

; Boundary Condition
; -----
apply pressure 1000 i=1
apply xquiet yquiet i=31
fix x j=1
fix x j=61
; -----
; Histories
; -----
his nstep 10
his unbal
his dytime
his xdisp i=5 j=31
his xdisp i=10 j=31
his xdisp i=15 j=31
his xvel i=5 j=31
his xvel i=10 j=31
his xvel i=15 j=31
his sig1 i=5 j=31
his sig1 i=10 j=31
his sig1 i=15 j=31
; -----
; Analytical Solutions
; -----
def Anal
  jp=31
  ip=5
  ur
  ur5 = ur
  ip=10
  ur
  ur10 = ur
  ip=15
  ur
  ur15 = ur
end
def const
  p0=1000.
  mu=0.25
  zou = density(1,1)
  Cp=sqrt(bulk_mod(1,1)+4.0*shear_mod(1,1)/3.0)/sqrt(zou)
  W0=(Cp/rmin)*sqrt(1.0-2.0*mu)/(1.0-mu)
  alpha0=(Cp/rmin)*(1.0-2.0*mu)/(1.0-mu)
  cap_k = (1.0-mu)/(2.*(1.-2.*mu))
  c1 = p0*rmin*rmin*rmin*cap_k/(zou*Cp*Cp)
  c2 = atan(1./sqrt(4.0*cap_k-1.0))

```

```

      c3 = sqrt(2.0-2.0*mu)
      c4 = alpha0/Cp
      c5 = W0/Cp
    end
  const
  def ur
    r = sqrt(x(ip,jp)*x(ip,jp)+y(ip,jp)*y(ip,jp))
    tau = dytime - (r-rmin)/Cp
    if tau >= 0.0 then
      temp1 = -(c1/(r*r))*( -1.0+c3*exp(-alpha0*tau)*cos(W0*tau-c2) )
      temp2 = (c1/r)      *(   c4*c3*exp(-alpha0*tau)*cos(W0*tau-c2) )
      temp3 = (c1/r)      *(   c5*c3*exp(-alpha0*tau)*sin(W0*tau-c2) )
      ur     = temp1 + temp2 + temp3
    else
      ur = 0.0
    end_if
  end
  his Anal
  his ur5
  his ur10
  his ur15
  set step 1000000 clock 100000000
  set large
  set dy_damp=rayl 0.01 50 stiff
  solve dytime 0.1
  save sphere1.sav
  new
  config dyn axi
  grid 30 60
  model elastic
  ; -----
  ; Poisson's ratio 0.25
  ; -----
  prop shear=1e10 bulk=1.665e10 dens=1675
  ; create cavity
  ; -----
  ; circular boundary
  ; -----
  ;call hdonut.fis
  ; -----
  ; rectangular boundary
  ; -----
  call hhole.fis
  set rmin 10.0 rmul 10 gratio 1.01
  ;hdonut
  hhole

```

```

; -----
;   Boundary Condition
; -----
apply pressure 1000 i=1
apply xquiet yquiet i=31
fix x j=1
fix x j=61
; -----
;   Histories
; -----
his nstep 10
his unbal
his dytime
his xdisp i=5 j=31
his xdisp i=10 j=31
his xdisp i=15 j=31
his xvel i=5 j=31
his xvel i=10 j=31
his xvel i=15 j=31
his sig1 i=5 j=31
his sig1 i=10 j=31
his sig1 i=15 j=31
; -----
; Analytical Solutions
; -----
def Anal
  jp=31
  ip=5
  ur
  ur5 = ur
  ip=10
  ur
  ur10 = ur
  ip=15
  ur
  ur15 = ur
end
def const
  p0=1000.
  mu=0.25
  zou = density(1,1)
  Cp=sqrt(bulk_mod(1,1)+4.0*shear_mod(1,1)/3.0)/sqrt(zou)
  W0=(Cp/rmin)*sqrt(1.0-2.0*mu)/(1.0-mu)
  alpha0=(Cp/rmin)*(1.0-2.0*mu)/(1.0-mu)
  cap_k = (1.0-mu)/(2.*(1.-2.*mu))
  c1 = p0*rmin*rmin*rmin*cap_k/(zou*Cp*Cp)

```

```

c2 = atan(1./sqrt(4.0*cap_k-1.0))
c3 = sqrt(2.0-2.0*mu)
c4 = alpha0/Cp
c5 = W0/Cp
end
const
def ur
  r = sqrt(x(ip,jp)*x(ip,jp)+y(ip,jp)*y(ip,jp))
  tau = dytime - (r-rmin)/Cp
  if tau >= 0.0 then
    temp1 = -(c1/(r*r))*( -1.0+c3*exp(-alpha0*tau)*cos(W0*tau-c2) )
    temp2 = (c1/r)      *( c4*c3*exp(-alpha0*tau)*cos(W0*tau-c2) )
    temp3 = (c1/r)      *( c5*c3*exp(-alpha0*tau)*sin(W0*tau-c2) )
    ur     = temp1 + temp2 + temp3
  else
    ur = 0.0
  end_if
end
his Anal
his ur5
his ur10
his ur15
set step 1000000 clock 100000000
set large
set dy_damp=rayl 0.01 50 stiff
solve dytime 0.1
save sphere2.sav

```

Example 1.41 Create one-half donut mesh – “HDONUT.FIS”

```

;
; FISH routine to create one-half donut mesh
; each grid point is defined by its polar coordinates ALFA and RO
; RMAXIT = the maximum distance from the center for each ALFA
; RMIN    = radius of the excavation
; RMUL    = number of radii to the boundary
; GRATIO  = grid's ratio

def hdonut
figp=igp
fjgp=jgp
loop j (1,jgp)
  alfa= -pi/2.0 + (j-1)*pi/(jgp-1)
  rmaxit=rmin*rmul
  loop i (1,igp)

```

```

        ro=rmin+(rmaxit-rmin)*(gratio^(i-1)-1)/(gratio^(igp-1)-1)
        x(i,j)=ro*cos(alfa)
        y(i,j)=ro*sin(alfa)
    end_loop
end_loop
end

```

Example 1.42 Create one-half hole mesh – “HHOLE.FIS”

```

; FISH routine to create a one-half hole mesh
; each grid point is defined by its polar coordinates ALFA and RO
; RMAXIT = the maximum distance from the center for each ALFA
; RMIN    = radius of the excavation
; RMUL    = number of radii to the boundary
; GRATIO  = grid's ratio
;
;
def hhole
loop j (1,jgp)
    alfa= -0.5*pi + (j-1)*pi/(jgp-1)
    if abs(alfa) <= .25*pi then
        rmaxit=rmin*rmul/cos(alfa)
    else
        rmaxit=rmin*rmul/sin(alfa)
    end_if
    rmaxit=abs(rmaxit)
    loop i (1,igp)
        ro=rmin+(rmaxit-rmin)*(gratio^(i-1)-1)/(gratio^(igp-1)-1)
        x(i,j)=ro*cos(alfa)
        y(i,j)=ro*sin(alfa)
    end_loop
end_loop
end

```

1.7.6 Vertical Vibration of a Machine Foundation

The design of a machine foundation includes the estimation of the anticipated translational and rotational motions of the machine-foundation-soil system. This example demonstrates the calculation of the vertical response of a machine foundation consisting of a rigid, massive, strip foundation resting on a soil and excited by an oscillating machine force.

A foundation vibrating under the action of a time-varying force, P , transmits to the soil a force, R , causing a vertical uniform displacement, δ , of the soil beneath the foundation (see [Figure 1.175](#)). The dynamic equilibrium of the mass is expressed as

$$R + M\ddot{\delta} = P \quad (1.137)$$

in which M is the total mass of the foundation and the machinery.

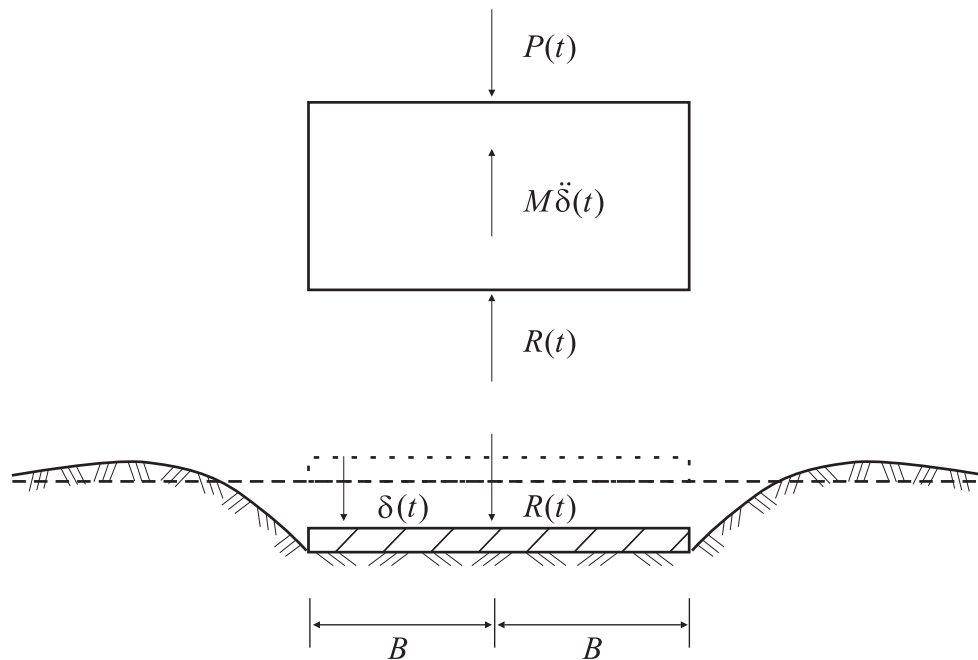


Figure 1.175 Forces acting on a machine foundation (Gazetas and Roesset 1979)

The analytical solution to this problem, assuming an elastic soil behavior, is provided by Gazetas and Roesset (1979). Their solution is for the case of a sinusoidal vertical loading of the form

$$P = P_o \sin \omega t \quad (1.138)$$

in which P_o is the force amplitude and ω is the operational frequency of the machine in radians per second.

For a harmonic exciting force, P , a dimensionless displacement (compliance) function, F_v , is defined, to relate the soil reaction force, R , to the soil displacement, δ :

$$\delta = \frac{R}{G} F_v \sin(\omega t + \psi) \quad (1.139)$$

G is the shear modulus of the soil, and F_v is the compliance function of the operational frequency, ω , and phase angle, ψ .

F_v is a complex number and can be written as

$$F_v = f_{1,v} + i f_{2,v} \quad (1.140)$$

in which $f_{1,v}$ is the real part representing the recoverable component of deformation, and $f_{2,v}$ is the imaginary part expressing the energy dissipated by the propagating waves and soil hysteresis.

The amplitude of motion, δ_o , can be expressed in terms of the amplitude of the machine force, P_o . The expression is given by Gazetas and Roesset in dimensionless form to be

$$\bar{\delta}_o = \frac{\delta_o G}{P_o} \left[\frac{f_{1,v}^2 + f_{2,v}^2}{(1 - b a_o^2 f_{1,v}^2)^2 + (b a_o^2 f_{2,v}^2)^2} \right]^{1/2} \quad (1.141)$$

in which the dimensionless mass, b , and frequency ratio, a_o , are defined as

$$b = \frac{M}{\rho B^2} ; \quad a_o = \frac{\omega B}{V_s} \quad (1.142)$$

where ρ = density, V_s = s -wave velocity of the soil, B = half-width of the strip foundation, and M = total foundation mass per unit length.

Gazetas and Roesset use a semi-analytical approach to obtain the compliance function, F_v , for a homogeneous half-space as a function of the frequency ratio, a_o . The result is presented in [Figure 1.176](#). The response of the foundation can then be evaluated for a set of operational frequencies from [Eq. \(1.141\)](#).

The results presented in Figure 1.176 are for an elastic material with a Poisson's ratio, ν , of 0.4 and a critical damping ratio, β , of 0.05. For this example, we assume that the s -wave velocity of the material is 1000 ft/sec, and the unit weight is 128.8 pcf (mass density is 4 slugs/ft³). For $\nu = 0.4$, the shear modulus G is 4.0×10^6 psf. The half-width of the footing foundation is 10 ft.

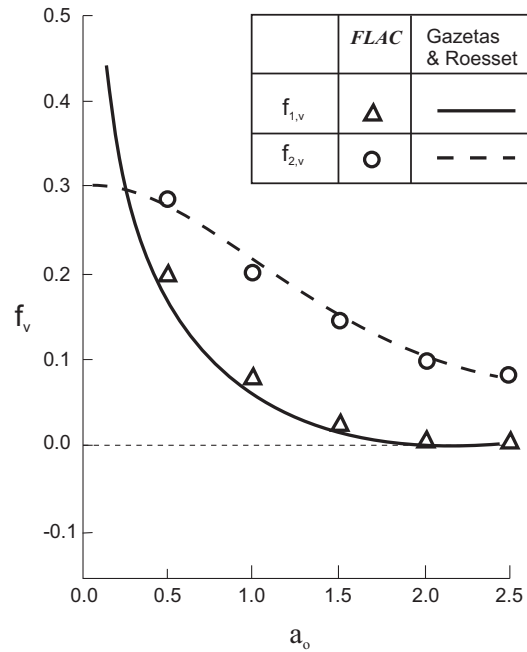


Figure 1.176 Vertical compliance function for a homogeneous half-space

The *FLAC* model consists of an 80×40 zone grid with the foundation represented by structural-beam elements. A vertical symmetry plane is assumed through the center of the foundation. Viscous boundaries are located on the bottom and right side of the mesh. Figure 1.177 shows the model grid, beam elements and boundary conditions.

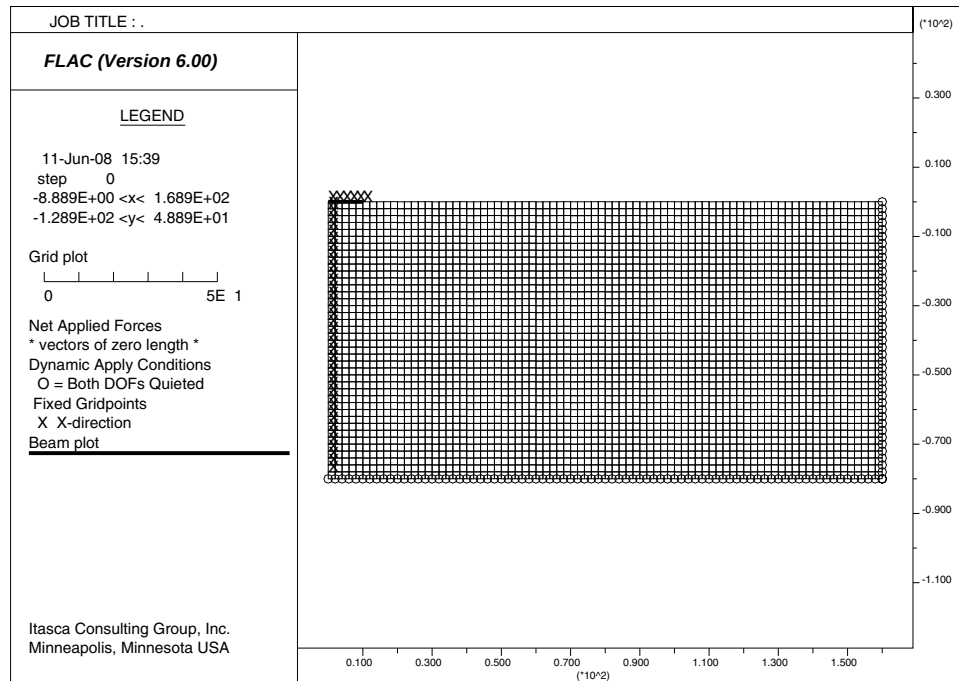


Figure 1.177 FLAC model for a vibrating machine foundation

The foundation is made rigid by slaving the structural nodes together. For the case of an oscillating rigid footing, we fix the x and rotational degrees-of-freedom, and we slave all y degrees-of-freedom together. Hence, the final structural model consists of only one degree of freedom. The density is set to a low value so that the structural mass is insignificant, and the Young's modulus is set low because it is now irrelevant: this allows the timestep to be that of the soil. The machine forcing function is applied as a stress because this is more convenient.

Calculations are performed over a range of frequency ratios: a_o is varied from 0.5 to 2.5. Several constants are computed prior to each calculation. These constants and the displacement results are stored for comparison to the analytical solution. The data file for this example is contained in [Example 1.43](#).

The values for $f_{1,v}$ and $f_{2,v}$ at each a_o are computed in the *FISH* function "COMPL.FIS" (see [Example 1.44](#)). The average phase angle and displacement amplitude are also calculated. The *FLAC* results for $f_{1,v}$ and $f_{2,v}$ are plotted in [Figure 1.176](#) for comparison with the Gazetas and Roesset solution.

The *FLAC* results compare well with the analytical results. Note that the agreement is better at the higher values of a_o . For lower frequencies, a larger grid is required. For example, for $a_o = 0.5$, the wavelength is 125 ft. The distance to the boundaries should be several wavelengths from the foundation, particularly for the top surface, because Rayleigh waves are not damped very efficiently by the quiet boundaries.

The foundation could also be simulated without structural elements. An oscillating velocity could be applied to a line of fixed gridpoints, and the cyclic reaction force measured. The same information (compliance) could be determined from the ratio between displacement and force, and their phase angle.

Example 1.43 Vertical vibration of a machine foundation

```
def setup
; a0      = 0.5
; a0      = 1.0
; a0      = 1.5
; a0      = 2.0
a0        = 2.5
frq_cent  = a0 * 100.0 * 0.5 / pi      ; Forcing frequency
per        = 1.0 / frq_cent            ; Period
omega     = 2.0 * pi * frq_cent
count     = 0
final_t    = per * 10.0
sh_mod    = 4e6                        ; Shear modulus
po_rat    = 0.4                        ; Poisson's ratio
bu_mod    = sh_mod*(2.0*(1.0+po_rat))/(3.0*(1.0-2.0*po_rat)); Bulk modulus
end
setup ; set constants
conf dy
g 80 40
gen 0,-80 0,0 160,0 160,-80
m e
pro den=4 shear=sh_mod bulk=bu_mod
struct prop=1 e=1 i=1 a=1 den=1e-3
struct beam beg grid 1 41 end grid 2 41
struct beam beg grid 2 41 end grid 3 41
struct beam beg grid 3 41 end grid 4 41
struct beam beg grid 4 41 end grid 5 41
struct beam beg grid 5 41 end grid 6 41
def ggg ;--- collapse structure to 1 dof in y ---
loop nn (1,6)
command
struct node nn fix x r
end_command
nn1 = nn - 1
if nn > 1 then
command
struct node nn slave y nn1
end_command
end_if
end_loop
```

```

end
ggg
def s_wave
  s_wave = sin(omega * dytime)
end
def dummy ; ... count number of history points
  count = count + 1
end
set dy_damp rayl 0.05 frq_cent
fix x i=1
app xquiet yquiet i=81
app xquiet yquiet j=1
app syy=0.05 hist=s_wave i=1,6 j=41 ; pressure of amplitude 0.05 (F=1.0)
hist dytime
hist syy i 1 j 40
hist syy i 5 j 40
hist ydis i 1 j 41
hist dummy
set ncw=50 clock=10000000 step=10000000
solve dytime=final_t
save mach.sav
call compl.fis
save mach2.sav

```

Example 1.44 Real and imaginary parts of compliance function – “COMPL.FIS”

```

;--- Extract real and imaginary parts ---
; ... assumes disp history is #4, time is #1 and first motion is positive
set echo off
his write 4 vs 1 table 1 ; copy disp vs time history to Table 1
def get_extremes
  xxx      = ' (normalized)'
  sense    = 'none'
  old_value = 0.0
  old_time  = 0.0
  num_phase = 0
  frac      = 0.0
  pk_to_pk  = 0.0
  num_ptp   = 0
  loop n (2,count)
    section
      new_value = ytable(1,n)
      if sense = 'up' then
        if new_value < old_value then
          sense = 'down'

```

```

        rat      = old_time / per
        frac     = frac + rat - int(rat) - 0.25
        num_phase = num_phase + 1
        upper_val = old_value
    end_if
    exit section
end_if
if sense = 'down' then
    if new_value > old_value then
        sense     = 'up'
        rat      = old_time / per
        rmi      = rat - int(rat)
        if rmi < 0.5 then
            rmi = 1.0 + rmi ; (overflowed one period)
        end_if
        frac     = frac + rmi - 0.75
        num_phase = num_phase + 1
        pk_to_pk = pk_to_pk + upper_val - old_value
        num_ptp  = num_ptp + 1
    end_if
    exit section
end_if
if new_value > old_value then
    sense = 'up'
else
    sense = 'down'
end_if
end_section
old_value = new_value
old_time  = xtable(1,n)
end_loop
degr      = frac * 360.0 / num_phase
u_tot     = pk_to_pk * 0.5 / num_ptp
re_part   = u_tot * cos(degr * degrad)
im_part   = u_tot * sin(degr * degrad)
ii = out(' Average phase angle = '+string(degr)+' degrees')
ii = out(' Displ. amplitude      = '+string(u_tot))
ii = out(' Real part              = '+string(re_part * sh_mod)+xxx)
ii = out(' Imaginary part         = '+string(im_part * sh_mod)+xxx)
end
get_extremes
set echo=on

```

1.7.7 Inert Shock Wave

The analysis of the jump conditions across a shock wave is given in many textbooks (e.g. Zukas and Walters 1997). The main points of such analyses are reproduced here.

Inert shock wave – Consider a frictionless tube containing a polytropic fluid obeying the equation of state given by Eq. (1.143),

$$e = \frac{pV}{(\gamma - 1)} \quad (1.143)$$

where e is the internal energy, p is the pressure, $V = 1 / \rho$ is the specific volume and γ is the ratio of specific heats. The gas in the tube starts with an initial density of $\rho_0 (= 1 / V_0)$ and pressure of p_0 . If one end of the tube contains a piston that moves in the positive coordinate direction with velocity u , then a shock front, moving at velocity D , develops at some distance ahead of the piston. We wish to determine D , the pressure, p_1 , and the density, $\rho_1 (= 1 / V_1)$, behind the shock. The conservation laws (mass, momentum and energy, respectively) applied to the jump across the shock are

$$V_0(D - u) = V_1 D \quad (1.144)$$

$$Du = V_0(p_1 - p_0) \quad (1.145)$$

$$D \left(\frac{p_1 V_1}{\gamma - 1} - \frac{p_0 V_0}{\gamma - 1} + \frac{u^2}{2} \right) = V_0 p_1 u \quad (1.146)$$

The solution to the above equations is given below:

$$p_1 = p_0 \left(1 + \frac{\delta}{\Gamma} \right) \quad (1.147)$$

$$V_1 = V_0(1 - \Gamma) \quad (1.148)$$

$$D = \frac{u}{\Gamma} \quad (1.149)$$

where

$$\Gamma = -\frac{(\gamma + 1)\delta}{4\gamma} + \left[\left(\frac{(\gamma + 1)\delta}{4\gamma} \right)^2 + \frac{\delta}{\gamma} \right]^{1/2} \quad (1.150)$$

$$\delta = \frac{u^2}{p_0 V_0} \quad (1.151)$$

Shock smoothing scheme – The equations derived above imply that there is a step jump in pressure, velocity and density across the shock, but a “standard” *FLAC* simulation shows oscillations in these quantities superimposed on the steps. As Zel’dovich and Raizer (2002) remark:

The increase in entropy indicates that irreversible dissipative processes (which can be traced to the presence of viscosity and heat conduction in the fluid) occur in the shock wave. A theory that does not take into account these processes is not capable of describing either the mechanism or the structure of the very thin layer where the gas undergoes a transition from the initial to the final state.

Thus, we add a mechanism (Eq. (1.152)) that accounts for the necessary entropy increase across the shock, by “leaking” heat from the hot side to the cold side. Note that energy is conserved:

$$\frac{\partial e}{\partial t} = \alpha \frac{\partial p}{\partial x} \frac{\bar{e}}{\bar{p}} \quad (1.152)$$

where $\bar{e}$ is the mean energy and $\bar{p}$ the mean pressure at a point. In discrete form,

$$\Delta e_i = \alpha \frac{p_i - p_{i-1}}{p_i + p_{i-1}} (e_i + e_{i-1}) \frac{\Delta t}{\Delta x} \quad (1.153)$$

where Δt and Δx are timestep and zone size, respectively, and the subscripts refer to gridpoint or zone indices: Δe_i is the increment in energy flowing from zone i to zone $i - 1$ in one timestep. Using the equation of state Eq. (1.143), the energy increment can be written as follows:

$$\Delta e_i = \alpha \frac{p_i - p_{i-1}}{p_i + p_{i-1}} \left[\frac{p_i V_i + p_{i-1} V_{i-1}}{\gamma - 1} \right] \frac{\Delta t}{\Delta x} \quad (1.154)$$

The energy flux is first computed for each gridpoint i . Then all zones are scanned and energy added to, or subtracted from, each zone based on the flux already computed at its two associated gridpoints:

$$\frac{p'_i V_i}{\gamma - 1} = e_i - \Delta e_i + \Delta e_{i+1} \quad (1.155)$$

where p' is the updated pressure.

Thus, zone pressures are updated as follows:

$$p'_i = p_i - \frac{\Delta e_i - \Delta e_{i+1}}{V_i}(\gamma - 1) \quad (1.156)$$

It is found that a value of α between 1000 and 2000 m/s gives generally good results for the systems reported here. The shock-smoothing scheme described above is intended to perform a function similar to the artificial viscosity scheme described by Wilkins (1980), but this approach uses only one parameter and is believed to be related more closely to physical mechanisms.

A user-defined constitutive model is written to implement an incremental form of [Eq. \(1.143\)](#) in *FLAC*. The model, named **EoS**, is written in C++ using the methodology given in [Section 3](#) in **Theory and Background**, and the DLL (dynamic link library) compiled file is named “EoS.dll.” The DLL is loaded in *FLAC* using the command **Model load eos.dll**. Note that *FLAC* must first be configured to accept the DLL model by giving the command **CONFIG cppudm**.

Example of one-dimensional inert shock – We apply various velocities, u , to the base of a one-dimensional column consisting of 10 mm zones, containing gas of density 1.2 kg/m³, $\gamma = 3$ and initial pressure 10⁵ Pa. *FLAC* is operated in Lagrangian mode, with an update frequency of 1 (**SET update=1**), with $\alpha = 1000$ m/s. As an example of the shock front, [Figure 1.178](#) shows a profile, at $t = 3.027$ ms, of pressure versus distance from the original origin, for the case of $u = 750$ m/s. Note that the bottom of the column has moved about 2.27 m (as evidenced by the trace not starting from zero distance) due to the applied velocity, while the shock front has moved about 5 m. There is almost no over-shoot or oscillation in the pressure trace, which confirms that the shock-smoothing scheme described above is working well. [Table 1.9](#) shows the results for five driving velocities (denoted by “num”) compared to results from the analytical expressions [Eqs. \(1.147\), \(1.148\) and \(1.149\)](#), denoted by “exact.” It can be seen that the agreement is generally better than 0.5%.

The data file for this example is listed in [Example 1.45](#). The mechanism that accounts for entropy increase across the shock is implemented as a *FISH* function in this model.

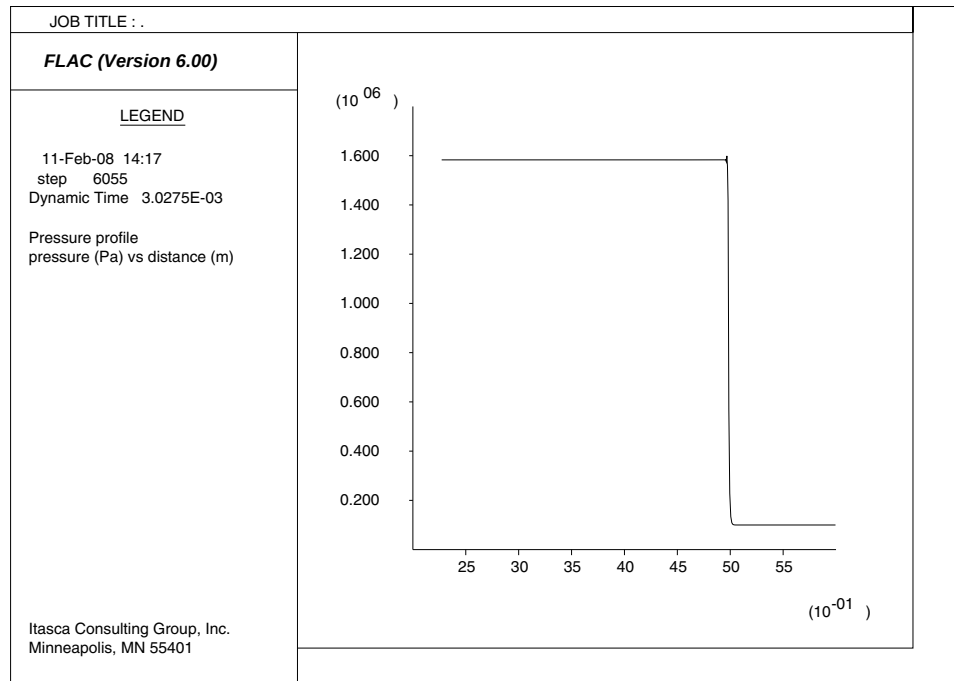


Figure 1.178 Pressure profile in column at time 3.027 ms (note displaced location of base, at 2.27 m)

Table 1.9 Results from inert-shock simulations

u m/s	p_1/p_0 exact.	p_1/p_0 num.	Error %	ρ_1/ρ_0 exact.	ρ_1/ρ_0 num.	Error %	D(exact.) m/s	D(num.) m/s	Error %
250	3.427	3.427	0	1.447	1.447	0	809	809	0
500	8.243	8.233	—0.12	1.708	1.709	0.06	1207	1205	—0.16
750	15.86	15.83	—0.19	1.832	1.835	0.16	1651	1648	—0.18
1000	26.42	26.32	—0.38	1.894	1.901	0.37	2118	2109	—0.42
1250	39.94	39.75	—0.48	1.928	1.937	0.47	2596	2583	—0.50

Example 1.45 One-dimensional inert shock

```

config cppudm dyn ext 10
model load EoS.dll
;
grid 1 600
model EoS
def solveIt
    delta = vfinal^2 * dens0 / pzero
    term1 = (mgam+1.0) * delta / (4.0 * mgam)
    bigGam = -term1 + sqrt(term1^2 + delta / mgam)
    VoD = vfinal / bigGam
    pres1 = pzero * (1.0 + delta / bigGam)
    dens1 = dens0 / (1.0 - bigGam)
    SoSrat = sqrt((1.0+delta/bigGam)*(1.0-bigGam))
end
def setup
    zoneMul = 0.01
    mgam = 3.0
    dens0 = 1.2
    pres0 = -1e5
    vfinal = 750.0
;---
    if vfinal < 700.0
        dtGiven = 1e-6
    else
        dtGiven = 0.5e-6
    endif
    NzFront = 0
    oldTime = 0.0
    NzTold = 0
    pzero = abs(pres0)
    solveIt
    Lfac = 1000 * dtGiven / zoneMul
end
setup
def RunIt
    NcycReq = float(NzTarg-NzTold) * zoneMul / (VoD * dtGiven)
    NzTold = NzTarg
    command
        step NcycReq
    endCommand
end
def getResults
    AvPres = 0.0

```

```

AvDens = 0.0
count = 0
loop j (NzTarg-50,NzTarg-20)
  AvPres = AvPres - syy(1,j)
  AvDens = AvDens + zoneMul^2 * dens0 / area(1,j)
  count = count + 1
endLoop
AvPres = AvPres / float(count)
AvDens = AvDens / float(count)
Dnorm = AvDens / dens0
Pnorm = AvPres / pzero
end
ini y mul zoneMul x mul zoneMul
prop dens=dens0 m_gamma=mgam
ini sxx=pres0 syy=pres0 szz=pres0
set large update=1
fix x
fix y j=601
def ramp
  if step > nsRamp
    ramp = vfinal
  else
    ramp = vfinal * float(step) / float(nsRamp)
  endif
end
;
set nsRamp=500
fix x y j=1
ini yvel=vfinal j=1
hist dytime
hist yvel i=1 j=1
hist yvel i=1 j=50
def leak
  while_stepping
    ex_1(1,1) = 0.0
    ex_1(1,jgp) = 0.0
    loop j (2,jzones)
      pdif = (syy(1,j)-syy(1,j-1))/(syy(1,j)+syy(1,j-1))
      enorm = syy(1,j)/density(1,j)+syy(1,j-1)/density(1,j-1)
      ex_1(1,j) = Lfac * pdif * enorm
    endLoop
    loop j (1,jzones)
      delEn = (ex_1(1,j+1) - ex_1(1,j)) * density(1,j)
      syy(1,j) = syy(1,j) + delEn
      sxx(1,j) = syy(1,j)
      szz(1,j) = syy(1,j)
    endLoop
  endwhile
end

```

```

    endLoop
end
def Dfront
    jymax = 1
    loop j (1,jzones)
        jj = jzones - j + 1
        if abs(syy(1,jj)) > abs(pres0)*1.1
            Dfront = 0.5 * (y(1,jj)+y(1,jj))
            exit
        endif
    endLoop
end
history 6 Dfront
set dydt=dtGiven
set NzTarg=200
RunIt
getResults
set NzTarg=300
RunIt
getResults
set NzTarg=400
RunIt
getResults
set NzTarg=500
RunIt
getResults
his write 6 vs 1 tab 100
def LSfit
    nmax = table_size(100)
    sum1 = 0.0
    sum2 = 0.0
    sum3 = 0.0
    sum4 = 0.0
    nstart = int(3.0*float(nmax)/4.0)
    count = 0
    loop n (nstart,nmax)
        count = count + 1
        sum1 = sum1 + xtable(100,n)
        sum2 = sum2 + ytable(100,n)
        sum3 = sum3 + xtable(100,n)^2
        sum4 = sum4 + xtable(100,n) * ytable(100,n)
    endLoop
    nff = float(count)
    fitVOD = (nff*sum4 - sum1*sum2) / (nff*sum3 - sum1^2)
    oo = out(' best fit VOD = '+string(fitVOD))
end

```

```
LSfit
def pressure_profile
  loop jj(1,jzones)
    xtable(11,jj) = (y(1,jj+1)+y(1,jj))/2.0
    ytable(11,jj) = -(sxx(1,jj)+syy(1,jj)+szz(1,jj))/3.0
  endloop
end
pressure_profile
save eos.sav
;*** plot commands ****
;plot name: pressure profile
label table 11
pressure (Pa) vs distance (m)
plot hold table 11 line alias 'Pressure profile'
```

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